U. S. MILITARY ACADEMY.

A

COURSE OF INSTRUCTION

IN

ORDNANCE AND GUNNERY

TEXT.

BY

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CONTENTS.

PREFACE TO SECOND EDITION.

The great advances which have been recently made in our knowledge of the properties of gunpowder have subjugated the "Spirit of Artillery," as this agent has been termed, to a seemingly docile servitude. These, with corresponding improvements in Metallurgy, have led to such changes in nearly all that relates to fire-arms as to make necessary a comprehensive revision of the course of Ordnance and Gunnery, established by the late Colonel James G. Benton in 1861, and modified from time to time by his successors on the Academic Board.

The subject has outgrown the limits of the small encyclopedia in which Benton comprised all that was then essential for the ordinary officer, as well as for the student, to know of the *materiel* of war.

It has also lost much of the stability which characterized it when spherical projectiles were still generally employed. The labors of men of science and the energy of inventors are continually extending the boundaries of knowledge and undermining positions which appear most fixed.

Therefore, instead of giving to the course a descriptive character, it appears advisable to frame it so as to present as simply as possible such of its principles as are the most important, and appear the best established.

By employing the short time available for this course in teaching such principles, the student, although less familar with existing forms and methods than after the study of the former course, may possibly be better fitted to understand the causes of changes in *materiel* which are now so frequent, and, as his experience increases, to wisely advise the direction that such changes should take.

The selection, enunciation and deduction of such principles in a suitable form is rather embarrassed than assisted by the mass of specialized knowledge to be found in the Government reports and in the periodical press. In fact, had it not been for the admirable text-books used at the "*École d'Application de l'Artillerie et du Génie*," at Fontainebleau, France, for a set of which the author is obliged to superior military and diplomatic authority, it would not have been possible for him to prepare many of the following pages.

Graphical methods have been freely used, both to express abstract relations and to avoid description. In order to relieve the memory and to train the student in reading mechanical drawings, it is intended that the more elaborate shall be recited on from the book.

It has been attempted to give the antecedents of present forms, briefly, but so as to indicate the general lines followed in their evolution and possibly to anticipate the direction in which their improvement tends. In so doing more stress than heretofore has been laid upon the practice of the workshops; since the history of invention shows that this has had as much to do with the march of improvement as a special knowledge of the military necessities of any particular case.

The thanks of the author are due:

To Mr. *Geo. H. Chase*, of the Midvale Steel Works, Philadelphia, for revising Chapter XV.

To Captain *L. L. Bruff*, Ordnance Department, for the appendix to Chapter XIX, relating to the Elastic Strength of Guns.

To Private *C. August Schopper*, of his detachment, for most of the drawings used in illustration.

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HENRY METCALFE.

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REMARK.—The unusual method of paging adopted in this work is intended to facilitate its revision, since new chapters can be inserted without disturbing the sequence of the following pages.

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The heavy figures refer to the number of the chapter, and the lighter figures to that of the page.

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CHAPTER I.

DEFINITIONS.

Ordnance.—A general term usually applied only to the material of Artillery, but embracing also all warlike stores made according to prescribed regulations or ordinances.

Fire Arms.—Offensive weapons used to throw projectiles by means of explosives. They are divided into—

1. *Cannon.*—Heavy fire arms requiring carriages to support and to transport them.

2. *Small Arms.*—Which can be carried by men.

Cannon are divided into—

Guns.—Or relatively long cannon. *Gun* is used as a general term for all fire arms. It has probably the same root as engine, meaning a machine.

Howitzers.—Comparatively short cannon.

Mortars.—Very short cannon.

NOMENCLATURE.

Bore.—That part of a cannon which is bored out. It includes—

1. The *cylinder* or principal portion of the bore.

2. The *seat of the charge*, or the part occupied by the powder. This may be either a continuation of the cylinder terminated by a plane or curved surface; or a *chamber*, the diameter of which differs from that of the cylinder of the bore. For breech loaders, the chamber is divided into the *powder chamber* and the *shot chamber*.

Caliber.—The diameter of the cylinder of the bore. It was formerly expressed by the weight of the inscribed sphere.

For cannon, this was the weight in pounds of the cast iron shot; for small arms, the number of leaden balls required to weigh a pound.

Rifling.—Cutting spiral grooves in the surface of the bore, so as to give to the projectile a motion of rotation at right angles to that of translation.

Lands.—The ridges left between the rifle groves. The caliber of rifles is usually measured between lands.

Chase.—The long conical portion of a cannon in rear of the muzzle.

Reinforce.—The thick portion of the body over and immediately in front of the seat of the charge.

Breech.—The mass of metal in rear of the plane of right section at the base of the charge.

Line of Metal.—The intersection of the upper surface of the piece by an axial plane perpendicular to the axis of the trunnions.

Dispart.—The difference between the semi-diameters at the muzzle and at the breech.

Preponderance.—The difference between the moments of the weight in front of, and in rear of, the trunnions.

Windage.—This is properly the difference between the area of right section of the cylinder of the bore, including the grooves, and the maximum parallel area of right section of the projectile.

It is usually expressed in linear units as the difference between the diameter of the cylinder of the bore and the diameter of the projectile.

VELOCITIES.

Muzzle or Initial Velocity.—Is the maximum velocity of the projectile after leaving the piece.

Remaining Velocity.—Is that at any point of the trajectory.

Terminal Velocity.—Is that at the point of impact.

The greater the velocity at any instant, the more *flat* does the trajectory become; and, therefore, the greater is the probability of striking a vertical object at an unknown distance. (See note 1, page 5.)

The horizontal distance over which, under given conditions, a vertical object would be struck, is called *the dangerous space* for that object.

UNITS OF MEASURE.

Those used in English speaking countries are unfortunately numerous and are apt to cause confusion.

Units of Temperature.

Throughout this work the temperatures are expressed in degrees centigrade.

Units of Length.

Yards.—For ranges of projectiles.

Feet.—For measuring velocities and the chords and altitudes of trajectories.

Inches.—For the internal dimensions of guns and for all dimensions of their projectiles. The decimal sub-division of the inch is generally employed.

Units of Weight.

Tons.—For cannon, 1 ton = 2240 lbs.

Pounds.—For large projectiles, for their charges and for measuring the pressures (per square inch) of the gases of fired gunpowder.

In England, pressures are measured in tons per square inch; in France, in kilogrammes per square centimetre; elsewhere on the continent of Europe in atmospheres.

Grains Troy.—For bullets and powder charges of small arms 7000 grains troy$=$1 lb. avoirdupois.

g is generally taken$=$32.2 lbs., which is nearly its value at London.

Unit of Energy and Work.

Foot tons$=$foot pounds$\div$2240.

USEFUL MECHANICAL FORMULAE.

Uniform motion, $v=\frac{s}{t}$

Varied motion, $v=\frac{ds}{dt}$

Uniformly varied motion, $v=\sqrt{2gh}=gt;\ h=\frac{gt^2}{2}$

Acceleration, $\alpha=\frac{dv}{dt}=\frac{d^2s}{dt^2}$.

Intensity of a motive force, $I=M\alpha$.

Intensity of an impulsive force, $I_i=MV$.

Measure of work, $Q=Wh=\int I\,ds$.

Measure of energy, $E=\frac{WV^2}{2g\ 2240}$

Time of oscillation of a simple pendulum, $t=\pi\sqrt{\frac{l}{g}}$

ABBREVIATIONS.

S. B.—Smooth bore.
R.—Rifle.
M. L.—Muzzle loading.
B. L.—Breech loading.
C. I.—Cast iron.
W. I.—Wrought Iron.
S.—Steel.

The caliber is placed first. **Example:**
8 in. B. L. R. S.

15 in. S. B. C. I., etc.

W. w.—Weight of larger and smaller of two masses considered; as of piece in regard to projectile, or of projectile in regard to charge of powder.

V. v.—Initial and remaining velocities.

p.—Intensity of gaseous pressure per unit of area.

r. d.—Radius and diameter of the cylinder of the bore, or of the projectile according to context.

NOTE 1. From Michies Mechanics, Article 94, we have $\frac{1}{\rho}=\frac{g \cos \varphi}{v^2}$. See also Chapter XX, p. 22.

CHAPTER II.

EXPLOSIVE AGENTS.

Explosion.

Is a name given to a series of phenomena resulting from two general causes.

Causes of Explosion.

I. The rapid *conversion* of a solid or liquid to a gaseous state. This conversion is accompanied by the evolution of heat, due to the nature of the chemical reaction involved.

II. The rapid dilatation of a mixture of gases by the heat evolved in their combination. Such explosives are not yet generally employed in warfare and are not herein considered.

Products.

The gases evolved in conversion are principally CO_2 and CO. These result from the more or less perfect combustion of carbon, which enters into every military explosive under circumstances intended to facilitate its oxidation.

Dissociation.

The tendency at high temperatures of CO_2 and other complex products to occur in simpler forms, as $CO + O$, is supposed by Berthelot to exert a powerful influence upon the corresponding pressures.

Dissociation, as this phenomenon is called, whether it prevents the formation of the complex product or destroys it, increases the specific volume of the gaseous products; but, since Mariotte's law has been proved not to hold for pressures $\frac{1}{20}$ of those found in fire arms, it is supposed that the loss of heat from imperfect combination, or from work done

in breaking up the molecules already formed, exceeds in its effect upon the resultant pressure the increase of specific volume cited. External causes may subsequently decrease the temperature and permit recombination with a relative increase of pressure.

It is found that, when the conversion yields a large proportion of CO, the violence or *sharpness* of the explosion is increased. This is supposed to be due to the rigidity or stability of this gas against dissociation.

H_2O is, by some, supposed to be subject to dissociation at the temperatures found in explosions. (See Bloxam, Arts. 36, 68, 311, Note.)

Firing.

The proximate cause of the reaction resulting in an explosion is always the absorption by some portion of the explosive of heat sufficient to raise its temperature to the point necessary to start the conversion.

The source of heat may be external; or, as in spontaneous decomposition, internal.

The means by which the temperature of the explosive is raised to the critical point may, in general terms, be called *firing*.

Firing generally results from the transformation of kinetic energy into heat as a result of arresting the motion of either molar or molecular masses.

Almost all explosives may be fired by molar shock, if it be concentrated on a mass of the explosive which is sufficiently small. (Bloxam, Arts. 309, 434, 538).

Orders of Explosion.

The energy of molecules in motion depends principally upon their velocity; and the external work done in stopping them, upon their stability of composition. When an explosive is fired by contact with incandescent matter, as by a flame consisting of molecules of CO_2, C, etc., moving with

relatively low velocities, the explosive is said to be *ignited*, and the explosion is called *low* or of the second order.

When fired as by fulminate or gun cotton, the conversion of which yields a large proportion of molecules of CO, moving with a very great velocity, the explosive is said to be *detonated*, and the explosion is called *high* or of the *first order.* Explosives which readily detonate are called *high explosives.*

Example, Gun-cotton when—

			Order.
Ignited...	unconfined; burns quietly..............		—
	confined; explodes like gunpowder.....		2nd
Detonated...	unconfined or confined;	explodes with great violence	1st

The distinction between the two orders of explosion is only conventional; the phenomena in practice appearing often to partake of the nature of both orders.

This may explain certain anomalies observed in mining and in artillery. In mines, when the charges are large, the high pressure resulting from the initial explosion, if at a point considerably beneath the surface of the charge, is supposed to cause the detonation of the remainder. In cannon the mixing of quick with slow powder produces a similar effect.

BERTHELOT'S THEORY OF EXPLOSIVES.

Origin of Reactions.

"Every explosion must be referred to some initial increase of temperature transmitted from particle to particle on the surface of an explosive wave. This wave raises successively all portions of the explosive to the temperature of conversion.

Propagation of Reactions.

Two limiting conditions are supposed to result, viz.:

1. The condition of combustion.
2. The condition of detonation.

These are progressively interchangeable in different degrees, according as the amplitude and velocity of vibration of the particles forming the surface of the explosive wave are increasing or diminishing.

Combustion.

I. The condition of combustion depends upon a reduction of temperature from the free expansion of a portion of the gases resulting from the initial explosion.

Successive portions of the explosive will thereupon be heated to the temperature of decomposition with a velocity depending upon various conditions; this velocity, compared with that of detonation, is slow.

Detonation.

II. On the other hand, the condition of detonation depends upon an initial shock, too sudden (sharp) to permit of molar motion of the particles of the explosive. It is, therefore, transmuted into heat which may raise the contiguous molecules to the temperature of conversion. The resulting gases are projected as a single (not periodic) explosive chemical wave traveling throughout the successive layers of the unexploded mass. This wave transforms its energy into heat at each impact, and, by virtue of its acceleration, raises each of the successive layers more rapidly to the temperature of conversion.

Origin of Orders of Explosion.

The order of the resulting explosion will depend upon the velocity with which the reaction is propagated; *i. e.*, the velocity of the wave surface described.

The velocity of the wave surface will depend—

1. Upon the molecular velocity of the reaction; *i. e.*, the rate of conversion under constant conditions.

2. Upon conditions which prevent the free expansion of the gases formed.

3. Upon the mass and initial temperature of the explosive; these affect the rate of cooling.

The last two conditions may be neglected when, as with the high explosives, the first is fully satisfied.

Influence of Detonator.

It is seen that detonation depends upon a chain of causes which results from the nature of the initial explosion. Herein lies the importance of the nature of the detonator, its mass, and the nature of its own explosion.

Its conversion should be rapid and evolve abundant heat. The mercuric fulminate is the detonator which is preferably employed. It is less violent than NCl, but yields more heat by its explosion and also much CO.

Influence of the Explosive detonated.

Detonation depends upon the physical condition of the explosive. Its sensitiveness generally diminishes as its density and elasticity increase, since the shock is distributed over a greater mass."

SYMPATHETIC DETONATION.

Conditions.

The instability of the high explosives renders their contact unnecessary when the continuous detonation of several charges is desired. The maximum interval permitting "sympathetic detonation," or "detonation by influence," and the order of this detonation, depend upon the elasticity of the intervening medium, the mass of the primitive charge, and the order of its explosion.

Examples.

Calling w the weight of the primitive charge in pounds, and d the maximum interval in feet—

In water, $d=3\ w$.

On a firm soil, $d=5\ w$.

On an iron rail, $d=10\ w$.

When discs of compressed gun-cotton are in contact, the velocity of detonation is said to be over $3\frac{3}{4}$ miles per second when the discs are wet, and less than $3\frac{1}{4}$ miles per second when they are dry. The incompressibility of the water assists in transferring the shock.

STRENGTH OF EXPLOSIVES.

The strength of an explosive, or its mechanical efficiency, may be analyzed by reference to 1, its potential; 2, its force; 3, the molecular velocity of its reaction.

1. Potential.

The potential of an explosive is the maximum work which a unit weight of it can perform. It is measured by the product of the quantity of heat liberated by the reaction, and the mechanical equivalent of heat; or, $Q=J\times H$.

The potential is independent of the process of conversion, provided it be complete and its products be constant.

In practice, these products often vary with the circumstances under which they are formed, so that the potential realized will also vary.

Only a portion of the potential can be realized in practice, depending upon the volumes of the gases produced, their specific heats, and the difference between the temperatures at which they are formed and to which they are expanded.

Examples.*

Theoretical potentials, in foot tons, resulting from the conversion of one pound of each of the following substances:

* It is required that only the general principles illustrated by this and following tables shall be committed to memory.

Name.	Foot Tons.	Proportion.	
Blasting powder............	391	1.0	
Cannon "	509	1.3	
Sporting "	542	1.4	
Gun-cotton................	716	1.8	1.0
Dynamite No. 1............	884	2.3	1.3
Explosive Gelatine.........	1,235	3.2	1.8
Nitro-glycerine.	1,282	3.3	1.8
Chloride of Nitrogen........	216	0.5	
Anthracite coal.............	5,170	13.0	

The greater potential of coal is due to its composition and to there being no loss of energy expended in converting into gas the compounds of oxygen contained in the other substances.

2. Force.

The force of an explosive, or the pressure per unit of area due to the explosion of a unit of weight in a unit of volume, may be calculated on theoretical grounds from the formula,*

$$f = \frac{p_0}{273} \times \frac{v_0 H}{c}.$$

In which v_0 is what is known herein as the *specific volume* of the gas, viz.: the volume in litres of the gases resulting from firing one kilogramme of powder, taken at O° C, and at the pressure p_0 of one atmosphere; and c is the specific heat of the gas.

But the uncertainty attending the application of the laws of Mariotte and Gay-Lussac to such high pressures as exist in cannon, and the doubt as to the nature and state of the products of explosion at the epoch of maximum pressure have caused instrumental measurements of pressure to be preferred.

Examples.

The following table shows in round numbers the relative force of the explosives named.

* For the deduction of this formula see page 11.

The detonation of gunpowder was accomplished by mixing it with dynamite.

Explosive.	Relative Force. 1st Order.	2nd Order.
Gunpowder	4.0	1.0
Gun-cotton	6.0	3.0
Nitro-glycerine	10.0	5.0

The force of a mixture of high explosives is proportional to the sum of the products of the force of each constituent by the corresponding fractional part of the whole mass.

A remarkable property of gunpowder (to be referred to hereafter) is that, however its potential may vary with its composition, the force of all compositions is sensibly constant. The specific volume of the gases formed seems to vary inversely with the quantity of heat evolved in their formation.

3. Rapidity of Reaction.

The temperature increases with the rapidity of the reaction. This depends upon the affinity between the combining molecules, and largely upon the state of aggregation of the exploding mass, in so far as it affects the distances between them.

In certain high explosives, the rapidity of the reaction causes so high a temperature that the gaseous products are, as it were, shot against the walls of the envelope with such velocity that the effect seems due rather to a physical shock, than to the elastic pressure of a confined gas. With such explosives tamping is relatively unneccessary.

VALUE OF EXPLOSIVES.

As a general rule, the value of an explosive depends:—

1. Mechanically; upon its primitive state of aggregation, in so far as this affects the ease of handling it in loading; also upon its density.

2. Chemically; upon the value of the ratio $\frac{v_0 H}{c} = v_0 T_0$

If, when this is great, the conversion is sufficiently rapid, a high and elastic pressure will succeed the initial shock; this pressure will be well sustained, since the cooling effect of the envelope will be relatively small.

The potential of an explosive is thus seen to be the measure of its power of sustaining a given force or pressure.

Examples.

The relative importance of potential, force, and rapidity, depends upon the use made of the explosive.

In order to burst, we use one of high force and density, acting locally like an hydrostatic pressure.

Chloride of nitrogen detonates with such rapidity that it may simply pulverize the surface of the envelope without rupturing its walls.

For mining in rock or coal, blasting powder is better than cannon powder, since the end sought is rather the rupture of the envelope than the dispersion of the fragments. Its force depends on the great specific volume of the gases generated rather than upon their temperature.

For blasting in earth, cannon powder is better than blasting powder as its potential is higher. Its diminished density, compared to high explosives, distributes the effect over a larger area.

MILITARY EXPLOSIVES.

The principal explosives used in warfare are of two general classes:

1. Mixtures.

Gunpowder and its like; these are more or less intimate *mechanical mixtures* of combustibles, such as C, S, Sb, with an oxydizing agent, generally a nitrate or a chlorate.

Explosives of class 1 are relatively stable.

2. Compounds.

Nitro-glycerine and gun-cotton and their derivatives. These are *chemical compounds*, formed by the substitution, in an organic substance of the general form $C_x H_y O_z$, of 3 molecules of NO_2 for 3 atoms of H.

The weak affinity of N renders the NO_2 a readily accessible magazine of oxygen.

Explosives of class 2 are called HIGH EXPLOSIVES, and are relatively unstable. In this class are included the *fulminating compounds*. See Chap. XIV.

GUNPOWDER.

This is formed of a mixture of KNO_3; C, and S, in the proportions of about 75, 15, 10. These proportions are considerably varied in pyrotechnic compositions.

The conversion of gunpowder is approximately expressed by the following reaction:

$$4KNO_3 + C_4 + S = K_2CO_3 + K_2SO_4 + N_4 + 2CO_2 + CO.$$

The reaction is really much more complex, and varies with the circumstances attending the explosion, even though great care be taken to make them constant.

Illustration.

The parts played by the three ingredients may be imagined by reference to the forced combustion of coal in a furnace.

The charcoal, in which form C is introduced, forms the main supply of fuel. The sulphur, owing to the ease with which it is ignited, takes the place of the kindling material. The nitre acts as a bellows forcing in air.

The sulphur also gives coherence to the grain, correcting the friability of a binary mixture of carbon and nitre.

Advantages and Disadvantages.

The facility with which, by varying the proportions and the arrangement of the ingredients of gunpowder its conversion may be controlled, and also its comparative stability against accidental ignition, have hitherto compensated for its defects.

These refer to its bulk, the care required in storage, its sensitiveness to dampness, the large solid residue left from its conversion, and the danger attending its manufacture. While for special purposes, where great force is required, it is being supplanted by the high explosives; its value, as a reservoir of potential energy for purposes of propulsion, increases as our knowledge of its properties extends.

Note to page 7.

1. From the chemistry we have $p\,v = p_0\,v_0 \left(1 + \frac{t}{273} = \frac{T_0}{273}\right)$. If in this we make $v = 1$, then by definition $f = p = \frac{p_0\,v_0\,T_0}{273} = \frac{p_0\,v_0}{273}\,\frac{H}{C}$.

CHAPTER III.

INGREDIENTS OF GUNPOWDER.

COMBUSTIBLES.

1. *Sulphur.*

Preparation.

This is refined by distillation. The product is called "flowers of sulphur," or "rock sulphur" or "brimstone," according to the temperature at which the volatile products are condensed.

Properties.

If below 115°, minute crystals or "flowers" are formed; above that temperature, the vapors condense in a liquid form, which is cast into moulds. Flowers of sulphur are not used for gunpowder, as they contain SO_2 and H_2SO_4 which are hygroscopic.

2. *Charcoal.*

Material.

Charcoal used for gunpowder is made from wood, the composition of which, excluding water and ash, is represented by $C_6H_{10}O_5$, corresponding to the following proportions per cent.:

C,	44
H,	6
O,	50
	100

The object of carbonizing the wood is twofold. 1st. To increase the calorific value of the fuel by increasing the

proportion of carbon. 2d. To increase its calorific intensity by facilitating its reduction to powder.

Composition.

Gunpowder charcoal consists of from 55 to 85 per cent. of carbon with varying proportions of hydrogen, oxygen, and ash. Its imperfect distillation leaves varying amounts of hydro-carbons which increase its inflammability, and, owing to the calorific value of hydrogen, may increase its potential.

Condition.

The uniform action of fired gunpowder and the safety of its manufacture depend largely upon uniformity in the condition of the principal fuel which it contains.

Uniformity is sought by using the same kind of wood, carbonized by the same process; the temperature being raised at the same rate to a point which, for each grade of charcoal, is the same.

PREPARATION.

Preliminaries.

White woods, such as the young willow or alder, which are soft and of rapid growth, are preferably employed, since they yield a charcoal that is inflammable, friable, and free from ash.

The bark is removed, so as to facilitate drying in the open air, and to free the coal from earthy matter and salts.

Distillation.

The wood is usually distilled in iron retorts, surrounded by flame consisting largely of the gases evolved by the process: Figures 1 and 2.

For convenience, the wood is charged in *slips*, which are cylinders of thin sheet iron.

The progress of the operation is judged of by *test sticks*, withdrawn from time to time for examination; by the use of

a pyrometer, or by the appearance of the flame and smoke as follows.

Phenomena of Carbonization.

The rate of distillation being always slow, the characteristics of the product depend principally upon the temperature at which the process ceases. Five stages are recognized, of which three correspond to useful grades of charcoal.

I. Up to 150°, desiccation occurs.

II. At 150°, decomposition begins, and continues as follows:—

1st. H and O are evolved and unite.

2nd. Three acid oxides (carbonic, acetic, and pyroligneous acids.—CO_2; CH_3, CO_2H; $C_2H_4O_2$) and an empyreumatic oil of an analogous composition are evolved.

3rd. Soot comes forth in heavy clouds.

4th. The gases burn with a ruddy flame.

5th. As the proportion of O diminishes, CO replaces CO_2, and at 260°, the flame becomes blue.

The solid products are called *brands* (Fr. *fumerons*), which smoke in burning.

III. From 260° to 270°, *brown charcoal* is formed. It is smokeless but tough.

IV. From 270° to 340° is the period of the formation of hydro-carbons; both gaseous, viz.: olefiant and marsh gases ($C_2 H_4$; $C H_4$), and in various liquid forms, including coal tar. The gases burn with a yellow flame, which, as the proportion of C diminishes, gradually becomes pale.

At 280° the liberation of the hydro-carbons changes the charcoal from brown to *red* (*charbon roux*); it tends to raise the temperature suddenly to about 340°.

The effect of this rise in temperature is to convert the red coal to the next grade, which is black. The redness of the product will, therefore, depend upon the care taken in regulating the temperature. This is done by drawing the fire, and completing the process by the residual heat.

The operation is difficult and the product not uniform.

V. Above 340°, *black* charcoal is formed in proportions increasing with the temperature, as indicated by the increasing whiteness of the flame.

The effect of increasing the temperature upon the proportions of the constituent elements is shown roughly by the following table:

Max. Temperature..	150°	260°	280°	350°
Products...........	Dried Wood.	Brown Coal.	Red Coal.	Black Coal.
Carbon............	44.0	68.0	71.0	77.0
Hydrogen	6.0	5.0	4.5	4.0
Oxygen...........	50.0	27.0	24.5	19.0
Proportion of Weight of Dried Wood...	100.0	50	37	30

Physical Properties.

The physical properties also change. The higher the temperature—

the more—

1. brittle;
2. hard and dense;
3. prone to spontaneous combustion—

the less—

1. hygroscopic;
2. violent as an ingredient of gunpowder—

does charcoal become.

SPONTANEOUS IGNITION OF CHARCOAL.

Cause.

The property of charcoal by which it condenses gases within its pores, particularly the vapor of water, may raise its temperature to the point of ignition. This is facilitated

by the occluded oxygen and by the increased surface resulting from pulverization.

Preservation.

To prevent accident, it is cooled slowly, and kept in the stick for several days. To obtain uniformity in the amount of water occluded, it is prepared only as required for use.

Its power of spontaneous ignition, when pulverized, is destroyed by mixing it with sulphur or nitre.

MANUFACTURE OF BROWN CHARCOAL BY SUPERHEATED STEAM.

Process.

The uniform production of brown charcoal may be accomplished by exposing it for a longer period to a somewhat lower temperature than that above assigned as the maximum. For this purpose superheated steam is used, as shown in figure 3.

Retort.

The retort is a fixed vertical cylinder of boiler iron, jacketed with mineral cotton. (Bloxam, Art. 217.)

Through perforations in the cast-iron top enters a current of steam which has been superheated in a coil to about 230°.

The wood is piled vertically on a perforated false bottom made fast to an axial bar, by which the contents can be removed.

The condensed steam and the water, acids, and tar drain through the pipe shown.

Product.

The process lasts about four hours, being stopped when experience shows that the fibrous structure of the wood is about to disappear.

The fiber, which is retained for its binding effect on the structure of the powder made from this coal, notably increases the difficulty of pulverizing it.

OXYDIZING AGENTS.

1. *Nitre.*

Source.

Only about one-tenth of the supply of nitre is the native Indian product; the remainder comes from the double decomposition of the sodium nitrate with a potassium salt.

Impurities.

The principal impurities are the chlorides, the affinity of which for moisture renders them objectionable. Not over $\frac{1}{3000}$ is allowed in nitre used for government gunpowder.

2. *Sodium Nitrate.*

Advantages.

1. It is cheaper than nitre for equal weights.

2. Owing to the relative atomic weights of sodium (23), and potassium (39), 85 per cent. of the usual proportion of nitre suffices as a supply of oxygen, still further reducing its cost.

3. If the usual proportion of 75 per cent. be retained, the greater volume of gas evolved increases the *force* of the powder and adapts it especially for blasting.

Disadvantages.

1. The deliquescent properties attributed to the salt are detrimental when the powder made from it is to be stored.

2. The salt is more soluble than nitre, and, therefore, powder made from it suffers more than ordinary powder from the segregation of the salt by efflorescence. This is due to the acqueous vapor condensed in the pores of the charcoal which the powder contains. When the powder is made on the spot where it is used, as in the excavation of the Suez Canal, this objection need not apply.

3. *Potassium Chlorate.*

Disadvantages.

1. The low temperature of conversion, due to the affinity of chlorine for the metals, renders the powder dangerous when exposed to shock.

2. Its conversion gives free chlorine, which attacks the bore of the gun and is injurious to the gunners.

3. It is costly.

4. The uncontrollable violence of mixtures containing the chlorates relegates them to the category of the high explosives discussed in Chap. XIV.

They are principally employed for igniting other explosives; themselves being ignited by friction.

4. *Ammonium Nitrate.*

This is becoming extensively used in the so-called *smokeless powders* for heavy cannon.

Advantages.

The products of combustion are gaseous or volatile, so that the smoke is greatly diminished in density, and the entire volume occupied by the powder is available for the expansion of the gases.

Disadvantages.

The deliquescence of this salt requires that powder made from it be hermetically sealed. This prevents the use of the ordinary cartridge bags.

CHAPTER IV.

THE MANUFACTURE OF GUNPOWDER.

ACCIDENTS.

Buildings.

Owing to the danger of explosion the buildings are scattered as much as possible and are separated by traverses or rows of trees. Figs. 1, 2.

The buildings are generally constructed with heavy walls on three sides, the remaining side and the roof being as light as practicable, so as not to increase the violence of explosions by unnecessary confinement. Fig. 3.

Power.

The machines employed are usually automatic, power being conveyed by canals (fig. 1), or wire rope (fig. 2), radiating from a central steam engine. As a general rule safety is enhanced by slowly operating the machines.

Precautions.

The machines are started and stopped from an outside shelter, the completion of the operation being indicated by an automatic signal. Great care is taken to prevent the introduction of foreign matter, the workmen being required to change their clothing before entering, and wearing rubber overshoes within the buildings, at the door of each of which is a wet mat.

All parts of the machines liable to become loose are boxed in. Iron is replaced, wherever possible, by gun-metal, copper, or wood.

Automatic devices are arranged to drench the contents of buildings adjacent to a probable explosion.

The diffusion of dust is avoided by boxing in those machines which produce it.

Powder in barrels is always gently handled. It should never be rolled for transportation.

These details are given to suggest the precautions necessary while handling gunpowder in service.

PRINCIPLES OF THE MACHINES EMPLOYED IN THE MANUFACTURE OF GUNPOWDER.

Types.

In order to derive the benefits of continuous operation, the *tools*, or portions of the machines in contact with the material, are preferably of the rotary type. Reciprocating motion is objectionable, in that it wastes energy in reversing the direction of the motion at each end of the stroke.

Classification.—The tools employed may be classified according to their functions, as follows:

FUNCTIONS.		NAME OF TOOLS.		Nature of Operation.
General.	Special.	General.	Special.	
I. To divide by	1. disintegration	1. *Rolls*	1. single	continuous.
			2. pairs	intermittent.
		2. *Barrels*	tumbling	continuous.
	2. separation	*Sieves*	1. cylindrical	continuous.
			2. flat	intermittent by reciprocating motion.
II. To combine by	1. mixing	*Barrels*	tumbling	continuous.
	2. pressure	*Presses*	1. rolling	continuous.
			2. thrusting (hydraulic)	intermittent by reciprocating motion.
III. To convey by		*Bands*	endless	continuous.

The rotary tools may be classified as to whether the material lies without or within the tool; as—

1st. Rolls.

2d. Tumbling or rolling barrels.

Rolls.

The object of a roller or roll is twofold.

1st. To concentrate a given pressure on a small area of contact.

2nd. To transfer this pressure to successive areas continuously.

Relative motion between the material and the tool is, therefore, necessary.

Reduction of Area.

The reduction of area desired is generally attained by the curvature given to the smooth cylindrical surface of the roll; it may be increased by fluting the surface or by providing it with pyramidal points.

The effect upon a granular material then resembles cracking, rather than the crushing effect of the smooth roller.

Transfer of Pressure.

When the pressure is transferred but slowly, the particles of the material may have time to adjust themselves in their new positions. The effect of the pressure will then be rather to condense the material than to disintegrate it by crushing.

Relative Motion.

1. When the material is at rest, a single roller is used. Example: a rolling-pin. See fig. 8.

In practice, the path of the roller is circular, so that its effects may be repeated.

2. When the material moves, the rollers are in pairs and revolve on fixed axes in relatively opposite directions. Example: A clothes wringer. See fig. 14.

In this case, they act but once upon the material, which is carried through them by friction, and fed to and removed from the rollers by its weight.

To assist in feeding automatically, several pairs of rollers may be placed in tiers, surmounted by a hopper containing

the material to be disintegrated. The upper tiers have the coarsest teeth and are placed farthest apart.

A coffee-mill is a variety of this class. The rolls are vertical, concentric, and conical; the outer roll, which is fixed, being the more obtuse. The funnel-shaped space between them serves as a hopper, and as the material descends, produces the effect obtained by the successive tiers above described.

Fig. 4 shows a charcoal-mill and sieve. The roll is balanced to avoid excessive pressure.

Barrels.

Type.

Tumbling barrels, as represented in fig. 5, are much used in the arts for abrasion. Their utility depends upon the inter-attrition of the contents. In powder making, besides the material, these often consist of balls, *b*, *b*, lifted by ledges, *L*, *L*, and continually falling back upon the material beneath them.

When the operation has proceeded far enough, the door *D* is removed, disclosing a perforated screen through which the finer portions may gradually escape upon the oscillating sieve, *S*.

The product is collected in the drawer *D.'*

Varieties.

The nature of the barrel and of the balls varies with the explosiveness of the material and the character of the operation. Thus, the barrel may be of iron with iron balls where an inexplosive material is to be pulverized*; of a wooden skeleton covered with leather, using bronze or zinc balls, when the operation is dangerous; or covered with wire

*When the material to be disintegrated is very tough, heavy iron cylinders are used instead of balls.

gauze netting, and using wooden balls, where simple comminution of a friable material is desired.

By omitting the balls and varying the size of the netting, such barrels may be used as sieves; and, by slightly inclining their axes to the horizon, both ends may be left open, when they will remove the dust. Fig. 4.

If the barrels be tight and no balls be used, the contents will be merely polished. Such tools are much used in the arts for finishing the surface of rough metallic objects, and, in the manufacture of gunpowder, for *glazing* it.

Mixing Barrels.

Where simple mixture of the ingredients is sought, the barrel may contain paddles revolving independently upon its axle, as in a churn, fig. 6. The action of these paddles is also disintegrating, and, where time is important, may replace the more crude pulverizing apparatus described.

Advantages.

The principal advantage of the rolling barrels consists in their cheapness of construction and operation, by which their number may be multiplied, and the effects of an explosion diminished.

Carrying Bands.

These are endless belts of a suitable width, which serve to carry continuously the material from one part of a machine to another.

If horizontal, a plain band will suffice; but, if inclined, it is furnished with elevator buckets. Fig. 7.

OPERATIONS OF MANUFACTURE.

Processes.

Nature.

All the stages of manufacture may be referred to the following essential processes.

1. Formation of a homogeneous *press cake* of required density.

2. Breaking up the press cake into *grains* of required size and form.

3. Finishing the grains so formed.

Operations.

The necessary operations may be divided into four principal groups, viz.:

I. Preliminary operations.	1. Pulverizing. 2. Mixing. 3. Moistening.
II. Operations relating to press cake.	1. Incorporating. 2. Pressing.
III. Operations relating to graining.	1. Graining 2. Sifting.
IV. Operations relating to finishing.	1. Glazing. 2. Drying. 3. Dusting. 4. Blending. 5. Marking.

I. PRELIMINARY OPERATIONS.

1. *Pulverizing.*

Process.

The nitre is generally in crystals that are sufficiently fine. Otherwise, this and the other materials are pulverized by any suitable process, either separately under single rolls or by a binary process in a barrel, viz.: the charcoal and sulphur together, or the charcoal and nitre together.

Object.

The pulverization should be thorough, so as to reduce the time required for incorporation; the latter, owing to the cost of the plant and the smallness of the "charges" treated, is the most expensive of the operations.

2. *Mixing.*

The three ingredients may be mixed by hand or in the rolling barrel.

3. *Moistening.*

Object.

The object of moistening is generally to assist in the distribution of the nitre; to give consistency to the mass; and to prevent a dangerous rise in temperature during the various operations of manufacture.

Limits.

An excess of moisture may cause segregation of the nitre by crystallization, and its evaporation, as in store, may render the finished powder unduly porous.

On the other hand, extreme desiccation may lead to the re-absorption of hygrometric moisture, which would affect the properties of the powder dried.

The amount of moisture should never exceed 3 or 4 per cent. It is frequently renewed during manufacture, according to the state of the atmosphere and to the special object in view. The amount present is determined by desiccating a weighed sample.

II. OPERATIONS RELATING TO PRESS CAKE.

1. *Incorporating.*

Object.

This is intended to unite the dust of the ingredients as intimately as mechanical means permit, and thereby to facilitate the conversion of the powder into gas. It is the most important of all the operations of manufacture.

Process.

The wheel mill (Figs. 8 and 9), used for this purpose, consists of two cast-iron cylinders *c*, *c*, weighing several tons each and acting as single rolls.

In order that their effect may be exerted throughout the layer of composition, this is made only about one inch thick.

The risk attending this disposition is diminished by frequent careful moistening, and by the eccentricity of the axle; this permits the wheels to rise and fall as obstacles are encountered. The constancy of the resulting pressure increases the uniformity of their effect. This arrangement is shown in Fig. 9.

The arrangement of the wheels upon an axis rotating in a horizontal plane peculiarly adapts them to the requirements of this operation. For, while both edges of either wheel have the same angular velocity, their paths described in the same time are notably different. Hence, it follows that the inner edge will tend to slide backward relatively to the outer edge; giving to the wheel a motion of rotation about an instantaneous vertical axis, combined with that about its permanent horizontal axis.

The effect is to grind the material nearest to the centre more thoroughly than that nearest to the curb of the trough; because in the former case, the sliding of the wheel repeats the effect of its crushing, and, in the latter case replaces it in part.

This effect is distributed by means of ploughs preceding the wheels, and by causing the wheels to travel in different paths.

The process takes about two hours, depending on the quality of the product; it continues day and night, while that of the other machines is confined to daylight.

Product.

The product of the wheel mill, called *mill cake*, unless consolidated by very slow rolling, is friable and of variable thickness and density. These defects are corrected by the next process.

The perfectness of the incorporation may be tested by

flashing a small quantity upon a glass plate. No residue should be left. The stains left by flashing powder on the blue paper used in solar printing are characteristic, and increase the delicacy of the test.

Variations in Process.

In case of necessity the incorporation may be less perfectly performed by the stamp mill (Fig. 13), or by the protracted use of the rolling barrel. (See also page 15.)

2. *Pressing.*

Object.

The object of pressing powder is to increase its density as a fuel, and to give it sufficient hardness to resist the formation of dust in transportation.

Kind of Press.

The intensity and uniformity of the pressure required usually demand the action of an hydraulic press, Fig. 10; although, when quantity rather than quality is desired, single or double rolls may be employed.

Process.

To increase the uniformity of the material pressed, the product of the various wheel mills is coarsely granulated and mixed. Then, having been moistened, it is placed in layers between plates which are kept at about two inches apart until the spaces between them are filled.

The powder is then gradually compressed to about half its former volume; being kept from spreading by hinged side pieces, which, being latched together, form a sort of box. This box is generally vertical, but for convenience is preferably horizontal and on the level of the floor.

Variations in Density.

The resulting density increases within limits, with the duration of the pressure and with the amount of trituration which the powder has received. The proportion of mois-

ture largely affects the density, since it acts as a lubricant between the particles. The density is not uniform throughout the press cake, being always greatest next to the moving surface.

To obtain uniform density, upon which it will be seen that the uniform action of powder greatly depends, one must compress equal masses equally incorporated and containing equal quantities of water at equal rates into equal volumes.

Wiener's Powder.

These requisites are with difficulty attained, owing to the variable hygrometric condition of the atmosphere. It has been attempted to dispense with water for pressing, by heating the powder during this operation slightly above the melting point of sulphur.

This process, invented by Colonel Wiener, of Russia, renders the gunpowder practically waterproof.

Effect of Form of Plates.

The plates between which the powder is pressed are generally flat, in which case the press cake comes from the press in slate-like slabs. The powder, resulting from breaking up these cakes, is called of *irregular granulation*, or simply *grained powder*.

Modern powders for heavy guns are often pressed between plates, the surfaces of which are regularly indented or ribbed after the manner of a waffle iron (Figs. 11, 12). The resulting press cake may be readily broken up into grains of great uniformity of size and shape. Such powders are said to be of *regular granulation*.

Molded Powder.

When the press cake is made exceedingly small, so that each cake shall make one grain, the powder is said to be *molded*. See *molded prismatic*, Figure 12.

Such powders are made by a number of properly shaped punches and dies simultaneously operated. Fig. 15, post.

Concrete Powders.

The structural homogeneity of the product depends much upon the condition of the material compressed. If the soft mill cake, above referred to, be replaced by that which has already been pressed and granulated, a *concrete* powder is produced; the fine grains composing it being cemented together by the pressure into a mass, the porosity of which is greatest in the middle. The burning of this powder is notably different from that of the homogeneous mass generally produced.

OPERATIONS RELATING TO GRAINING.

Object.

The object of graining, like that of splitting fire wood, is to increase the initial surface of combustion.

Operations.

The press cake is broken up by a series of rolls (Fig. 14), and sifted between limiting sieves.

Principle of Gauging.

The use of these sieves illustrates a principle common in manufactures; this principle when it is applied to individual articles, is called *gauging*.

Assuming that no two objects can be made of precisely the same size, a certain *tolerance* is established by the adoption of a maximum gauge, through which each object must pass, and of a minimum gauge, through which no object may pass.

The grains which are too coarse or too fine are reworked.

Special Operations in Graining.

Regular Granulation.

These depend upon the kind of grain required. For

example, the powders of regular granulation require only breaking up as by hand.

Pebble Powder.

The English cubical, or *pebble* powder is made by cutting the flat press cake into prisms between ribbed rolls, and then recutting these prisms across their length.

Flat Powder.

The flat French powder (Castan's) is made by roughly breaking a rather thin press cake, so as to make the thickness of the cake the minimum dimension of the finished grain. (Fig. 12.)

IV.—OPERATIONS RELATING TO FINISHING.

1. *Glazing.*

Object.

The object of glazing is to remove the angles and asperities of the grain; these would form dust in transportation and facilitate the absorption of moisture in store.

It compensates for the diminished density of the interior of the press cake from which most of the grains are formed, by increasing their superficial density by their mutual collision; it also increases the homogeneity of their structure by the heat which is thus evolved.

Process.

Moisture having been added to give some plasticity, the grains are rolled in a wooden barrel without balls.

2. *Drying.*

Object.

The object of drying is to reduce to normal limits, the moisture required in the previous stages of manufacture.

Process.

It is accomplished by passing a current of warm, dry air through successive layers of powder spread on screens or on shallow trays.

The temperature should be increased gradually, to avoid disintegration of the grains.

3. *Dusting.*

Object.

This is intended to remove the dust resulting from the glazing, and detached from the surface of the grains by drying.

4. *Blending.*

Object.

To neutralize unavoidable variations in manufacture, powders of the same size and nature may be blended so as to give certain average results.

Process.

Fine grain powders are mixed according to their densities, and those of larger grain, according to their ballistic properties. Molded powders are blended in charges, grain by grain alternately.

5. *Marking.*

In the U. S., powders receive certain conventional factory marks, of which the first two letters generally relate to the size and use, and the final letters to recorded variations in the manufacture, or to the date at which certain lots are made. Thus, I. K. A. might mean the first lot of I. K. powder used for field guns; E. V. B. the second lot of hexagonal powder for sea-coast guns, etc. Similar symbols are used abroad and are very convenient.

VARIATIONS IN MANUFACTURE.

COCOA POWDER.

History.

The most important improvement in gunpowder, since 1860, is the invention by the Germans of what, from its color, is called *cocoa* or *brown* powder.

It is notable for being the first important modification of the long established composition of gunpowder which has proved practically successful, and, as will be seen, for the paradoxical nature of its results. In this country it has so far been used only in heavy cannon.

Characteristics.

As made in this country by the Du Pont Powder Works, it differs from ordinary powder;—

1. In the composition of the charcoal, which is made by steam heat, as described Chap. III.

2. In the addition during incorporation of gummy carbo-hydrates, such as sugar, dextrine, etc.

3. In the proportion of the ingredients—

Nitre,	81.5 per cent.
Charcoal and Carbo-Hydrates,	15.5
Sulphur,	3.
	100.0

4. It is difficult to ignite, requiring in the gun a few prisms of black powder to be built into the cartridge near the mouth of the vent.

5. When ignited, unconfined, it seems to *fuse* rather than to deflagrate explosively.

This and its want of friability make it safe to transport and handle in store.

6. It is quite hygroscopic, but suffers less from moisture than black powder.

7. Its ballistic properties are extraordinary,

8. It gives comparatively little smoke.

Manufacture.

This resembles that of all the molded concrete powders. The grains compressed are of the size of mortar powder, and are slightly moistened before pressing.

Press for Molded Powders.

Pressing is done by carefully regulating the motion of the plungers of a duplex hydraulic press, which molds about 100 prisms at a time.

In fig. 15, *E* is a fixed mold plate containing a number of apertures of a cylindrical or prismatic form, into which the perforated plungers, *G*, *L*, fit.

Through the axes of the plungers run needles, *H*, simultaneously operated by the toggle-joint, *I*, and the supplementary cylinder, *K*.

A quantity of powder is swept into the apertures, *X*, until they are full. The rams, *B*, *B′*, then approach each other with equal velocities; and as *L* enters *E*, *H* rises into *L*.

After suitable pressure, *L* rises; *H* is withdrawn, and *G* rises; lifting the prism so that it may be swept off into a box.

The resulting prism has very dense ends, separated by a somewhat porous belt.

NORDENFELDT POWDER.

Object.

To increase the intimacy of the incorporation and to avoid the danger of performing it by mechanical means.

Manufacture.

Charcoal.

Straw or cotton-wool is carbonized by exposure to a stream of gaseous HCl.

Sulphur.

Dissolved in CS_2 and added to Charcoal.

Nitre.

In aqueous solution is gradually added to above.

The mass is mixed by paddles while in a liquid state, after which the vehicles are distilled and evaporated. The usual operations following incorporation are then pursued.

CHAPTER V.

INTERIOR BALLISTICS.

Division of Ballistics.

Ballistics, which treats of the motion of projectiles, is divided into interior and exterior ballistics, according as the motion of the projectile within or without the gun is considered.

Interior Ballistics.

The latter science is studied later in the course; but the former is so intimately related to the conversion of gunpowder into gas, that it is expedient to deal with it while the circumstances of this conversion are fresh in our minds.

The Gun as a Machine.

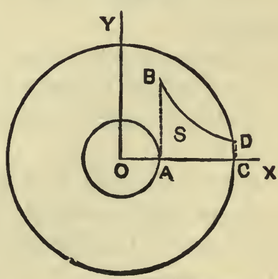
Functionally speaking, the gun is a machine by which the potential energy of the gunpowder is converted into the kinetic energy of the projectile.

It is well to consider in advance certain elementary principles relating to the construction of this machine and to the measurement of the energies received and usefully converted.

FORM OF GUN.

Strength *vs.* Weight of Guns.

Fig. 1.



It will hereafter appear that, considering a gun to be composed of a series of elementary concentric cylinders, the resistance which each of these cylinders offers to a permanent tangential deformation varies inversely with the square of its radius; or, if S represent the stress, AB, on the interior cir-

cumference of an elementary area of cross section of the bore, the radius of which is r; and y be the stress from the same cause at any other radius, x; then $y = \frac{Sr^2}{x^2}$. This is expressed by figure 1.

But the weight of the elementary longitudinal cylinders increases with the square of their radii.

It therefore appears, that after a certain point, an increase in the thickness of the walls of the gun adds rapidly to its weight and but slowly to its strength.

Strength *vs.* Cost.

Also, when the diameters of cannon exceed a certain limit, the difficulties of construction attending an increase in diameter, increase much more rapidly than do those attending an increase in length.

Conclusion as to Form of Gun.

A given amount of energy may, therefore, be most economically transferred from the gunpowder to the projectile, by diminishing the rate of transfer and increasing its duration.

Considerations relating to the weight and cost of a given cannon having thus determined the most suitable diameter, it should be kept constant for the entire length of the gun, provided that the stress to which it is exposed shall also be constant.

It is the object of recent improvements in guns, powder, and projectiles, to make this stress as high as it is safe, and to prolong it as far as possible throughout the length of the bore.

Recent changes in the profile of cannon illustrate the progress which has so far been attained toward realizing the conditions of this ideal gun.

FORM OF PROJECTILE.

Until quite recently, all but experimental cannon were muzzle loaders.

Until about 1860, they were smooth-bores and fired spherical balls.

The success of the rifled field pieces in the war between France and Austria led to the general use of projectiles, oblong in shape, but, like the spherical projectiles, smaller than the bore.

These cannon have been recently replaced by rifled breech loaders, firing projectiles provided with a compressible ring slightly greater in diameter than the bore.

The enlarged chamber, which this form of projectile requires, and the resistance which it offers to motion, considerably modify the circumstances of the conversion.

NOTE.—This chapter is introductory to the seven following chapters.

CHAPTER VI.

VELOCIMETERS.

Object.—In order to study experimentally the transformation of energy from the gunpowder to the projectile and to the gun and carriage, and to measure the kinetic energy residing in the projectile, both as it leaves the gun and when it has done work upon the medium through which it passes, special instruments, known as *velocimeters*, *chronographs*, *chronoscopes*, etc., have been devised.

Importance.—Except where otherwise specified, the following discussion relates to the means employed for measuring the initial velocity of the projectile. This is the great measure upon which all ballistic predictions are based.

Constituent Parts.—All such instruments are *chronometers* and consist essentially of a *register* and a *marker*.

The register has a known velocity relative to that of the marker and receives from it a succession of marks, the time equivalents of the spaces between which measure the periods between certain *events*.

The events are the first visible effects produced upon the velocimeter by the arrival of the projectile at certain *epochal points*. These are often *targets*.

Signal Time.—The interval between an epoch and the corresponding event is called the time of transmission, or the *signal time*. See figure 14.

The time from any origin to an event = time to the epoch, t, + signal time, σ; and the interval between two events, $\theta = (t'' + \sigma'') - (t' + \sigma') = (t'' - t') + (\sigma'' - \sigma')$. If $\sigma'' - \sigma' = O$; $t'' - t'$, or τ, $= \theta$. If $\sigma'' - \sigma' = C$; $\tau = \theta - C$; if then $\tau = O$, $\theta = C$. To diminish accidental variations in C, σ is made as small as possible. Knowing s, the distance between the epochal points, and τ, the mean velocity of the projectile over the intervening path may be determined.

Functions of Velocimeters.—Conceiving times and distances as being each measured from common origins, we may say that the instruments record differences in instrumental distance corresponding to DIFFERENCES IN TIME, which differences in time correspond to differences in distance of the projectile from any point upon its trajectory.

Classification.—The velocimeters may be divided into three general classes according as they are adapted to record:—

I. One difference in time corresponding to one definite difference of distance of the projectile from a common origin.

II. Successive differences in time corresponding to several successive definite differences of distance of the projectile from a common origin.

III. Continuous differences in time corresponding to continuous differences of distance of the projectile from a common origin.

Comparison of Classes.—For each fire the instruments of class I determine the mean velocity of the projectile between one pair of points.

Those of class II determine that between several successive pairs of points.

Those of class III set forth continuously the circumstances of the motion.

By taking the epochal points at constant intervals, either of distance or of time, the indications of the instruments of class II may, by interpolation, serve to determine the variations in velocity corresponding to known values of Δs or $\Delta \tau$ and thus to approximate to the law of motion more fully expressed by the record of instruments of class III.

This method enables the epochal points to be separated further than the construction of the instruments of class III permits.

CLASS I.

Events.—In class I the events are those of the falling of certain masses, either freely or with constrained motion.

The position of the marks indicates indirectly the interval of time separating the events.

Operation.—Calling the masses respectively *a* and *b*, according to their priority of fall, *b* is caused to strike *a* while *a* is falling. The problem resolves itself into determining the difference between two intervals of time, viz.:

t_a=how long *a* was falling before it was struck.

t_b=how long it took *b* to strike *a*.

Then $t_a - t_b = \theta$ =time that *a* was falling before *b* started to strike it=the interval between the starting of *a* and of *b*, which is the time interval required.

a and *b* are generally caused to fall by the demagnetization of electro-magnets in separate circuits, which are broken by the arrival of the projectile at the epochal points. Or they may be made to fall by the cutting of taut threads by which they have been suspended.

Disjunctor.—An essential appendage to machines of this class is the *disjunctor*, by means of which, both circuits being simultaneously broken, the masses *a* and *b* are caused simultaneously to fall.

EXAMPLES OF CLASS I.

1. THE BENTON VELOCIMETER.

See figures 1, 2 and 3.

Description.—This instrument, devised by the late Colonel J. G. Benton, the first Instructor of Ordnance and Gunnery at the U. S. Military Academy, employs either electricity or threads to support *a* and *b*. These are similar pendulums suspended at the centre *d* of the arc *b c o a* so that they are constrained to oscillate in adjacent planes parallel and close to the face of the arc; this arc, being graduated, forms the *register*.

That pendulum which lies nearest to the arc carries at its outer end the *marker;* this is a delicate bent lever pivoting in a plane perpendicular to the arc, and so placed that its inner end, which is lightly covered with printing ink, shall travel close to the register.

As the pendulums pass each other, a projection on the inner face of the outer pendulum strikes the outer end of the marker and causes it to indicate the point of meeting as at *c*, figure 2.

Inspection of the figure shows $\theta = t_a - t_b =$ time of passage over the arc *a o c*, minus time of passage over the arc *b c* $= 2 \times$ time of passage over the arc *o c*.

Disjunctor.—The disjunctor in this instrument serves to determine *C*, the difference in signal times.

It consists of two flat steel blades, *mn*, *m′n′*, secured to the base at *m*, *m′*, and having their free ends, *n*, *n′*, resting upon posts *b*, *b′*, through which and the blades the electric current passes.

Between the blades is a powerful bent spring *r*, provided with a cross piece *p q*, which lies beneath the blades and lifts them when the spring is released from the latch *g*. The button *s*, having been pressed, contact is made; it is broken by pinching the latch.

Determination of Time Value of Record.—To determine the time corresponding to a given reading, let *l* be the length of the equivalent simple pendulum; *v* the velocity of the center of oscillation or point *b; y* the vertical distance passed over by this point; *x* the variable angle which the axis of the pendulum makes with the vertical; and *t′* the time necessary for the point *b* to pass over an entire circumference, the radius of which is *l*, with a uniform velocity *v*. We then have:

$$v = \sqrt{2gy}.$$

Substituting for y its value in terms of x, the above expression becomes:

$$v = \sqrt{2 g l \cos x};$$

from which it is evident that the velocity of the pendulum increases from its highest to its lowest point.

The time t' is equal to the circumference of the circle, the radius of which is l, divided by the velocity v; if this value of t' be again divided by 360, we shall have very nearly the time of passing over any degree at the height y, or—

$$t = \frac{2 \pi l}{360 \sqrt{2 g l \cos x}}.$$

Calling t'' the time of a single vibration of the pendulum of the machine, we have by known laws—

$$t'' = \pi \sqrt{\frac{l}{g}}$$

Substituting this value in the equation above, and representing—

$$\frac{t''}{180 \sqrt{2}} \text{ by } a, \text{ we have}$$

$$t = \frac{a}{\sqrt{\cos x}}.$$

To determine t'', the pendulums are removed from the machine, and the cylindrical journals about which they revolve are replaced by others, the bearing surfaces of which are knife edges. Each pendulum is started vibrating through a very small arc. By means of a stop-watch the time of 1,000 vibrations may be found. By repeating the operation several times and taking the mean, the time of a single vibration may be determined very exactly. This time for pendulums of recent construction is 0.378 of a second.

If now x be made successively equal to 1°, 2°, 3°, &c., and the corresponding values of t be found, we shall have the time of passage of the pendulum over each degree.

By adding the time of passage over the first degree to that over the second, we shall have the time of passage over an arc of 2°. In the same manner, by adding this latter time to that over the third degree, we shall have the time of passage over an arc of 3°, and so on.

The following table has been determined in this manner:

TABLE.

Degrees.	Time in seconds of passage over each degree.	Sum of times in seconds.	Degrees.	Time in seconds of passage over each degree.	Sum of times in seconds.
1	.00148504	.00148504	19	.00152749	.02849909
2	.00148538	.00297042	20	.00153174	.03003083
3	.00148594	.00445636	21	.00153684	.03156767
4	.00148673	.00594309	22	.00154213	.03310980
5	.00148775	.00743084	23	.00154772	.03465752
6	.00148901	.00891985	24	.00155361	.03621113
7	.00149049	.01041034	25	.00155980	.03777093
8	.00149221	.01190255	26	.00156630	.03933723
9	.00149415	.01339670	27	.00157313	.04091036
10	.00149633	.01489303	28	.00158029	.04249065
11	.00149876	.01639179	29	.00158780	.04407845
12	.00150142	.01789321	30	.00159565	.04567410
13	.00150433	.01939754	31	.00160388	.04727798
14	.00150749	.02090503	32	.00161248	.04889046
15	.00151089	.02241592	33	.00162147	.05051193
16	.00151455	.02393047	34	.00163087	.05214280
17	.00151847	.02544894	35	.00164070	.05378350
18	.00152266	.02697160	36	.00165092	.05543442

To Compute a Scale of Velocities.—It should be remembered that the times above determined correspond to but half the difference of the arcs described by the two pendulums; therefore, they should be doubled in order to get the time τ.

2. THE LE BOULENGÉ CHRONOGRAPH.

See Figures 4, 5, 8, 9.

Description.—This velocimeter, invented by Captain Le Boulengé of the Belgian artillery, is the one used generally throughout the world for the determination of initial

velocities. In it the masses *a* and *b* are rods falling freely from electro-magnets *E*, *E'*. These are supported on a stand *s*, so placed that while *a* may fall through the foot of the stand, the fall of *b* is arrested by a trigger *t*, the shock upon which releases a knife-shaped marker *m*. The edge of this marker lies close to the path of *a*, so that a very slight movement of it to the right, under the impulse of a powerful spring which is liberated by the fall of *b*, produces a mark upon that elementary circle of *a* which was opposite to *m* at the moment of impact.

Disjunctor.—Although the operation of the disjunctor is the same as with the Benton velocimeter, its object in the LeBoulengé instrument is quite different.

It serves, by making $\tau = O$, to determine the value of t_b; since then $t_b = t_a$, or the time that *a* was falling before it was struck measures the time required for *b* to strike it.

This instrument does not serve to determine the signal time, but it may be shown that if the difference in signal times remains constant the time recorded between the events = time between the epochs, or $\theta = z$.

Operation.—This mark may be made in three ways, as follows:

1. Release *m* while *a* is at rest; the mark will fall at *O*, which is the *origin* for future measurements.

2. By means of the disjunctor, rupture *E* and *E'* simultaneously; the mark will be at some point *D* at a height *h* above *O*, corresponding to the time required for *b* to fall to *m* and for *m* to mark the rod *a*. This time is $t_b = \sqrt{\frac{2h}{g}}$ or the time required for *b* to strike *a*. The mark *D* is called the *disjunction mark*.

3. *Use as Megagraph.*—Rupture *E* and immediately afterward *E'*; the mark will occur, as at *R*, at a height *h'* above *O*. This is the usual case in practice. Then

$t_a = \sqrt{\dfrac{2h'}{g}}$ = the time during which a was falling before it was struck, and $t_a - t_b = \theta$, as with the Benton velocimeter. The mark R is called the *record mark.*

As a matter of convenience only, the construction of the instrument permits t_b to be made constant $= 0^s.15$, so that a rule may be so graduated, that, for a given interval between targets, the velocity corresponding to a height OR may be obtained by simple inspection, for $v = \dfrac{s}{\tau} = \dfrac{s}{t_a - 0^s.15}$ The instrument so arranged is called a Megagraph. (Greek *μεγας-great.*)

Use as Micrograph.—By raising E' so that the lower end of b may be nearly level with the top of a, D will be made to occur near that section of a which passes m with the greatest velocity. This serves to verify the accuracy of the operation of the disjunctor, of the magnets, and of the marking apparatus; since, if these parts worked with perfect uniformity, successive disjunction marks would be found at the same height above O. The velocity of a at the moment of marking magnifies the visible consequences of deficient uniformity and assists in correcting the causes to which it is due.

With this arrangement, if b is detached before a, the mark will be found as at R and $\theta = t_b - t_a$.

The advantages of this arrangement for the measurement of very small intervals of time give it the name of micrograph. (Greek *μικρος-small.*)

Determination of time limit.—The following reasoning determines the circumstances under which the instrument should be used as a megagraph, or as a micrograph.

Considering for the moment $\theta = \tau$ and remembering that we are measuring the interval $\tau = \dfrac{s}{v}$ we see that, with the in-

strument arranged as a megagraph, τ may be greatly diminished by increasing v and reducing s. The mark R will then be made near D where a has but little velocity, and therefore imperceptible differences in h may correspond to considerable differences in τ and hence in v. Since the instrument was invented, initial velocities have increased from 1,200 f. s. to over 2,000 f. s. while s may be restricted, as at West Point, so as to be reduced from 50 metres (164 feet, for which interval the instrument was made) to but 50 feet and even less.

On the other hand, when using the instrument as a micrograph, if τ increase unduly, the mark will occur in the same neighborhood as before, and the same consequences will ensue.

It becomes therefore necessary to determine a common time limit within which the instrument should be used as a micrograph and beyond which as a megagraph.

For this time the mark will, in both cases, occur at the same height above O. For the megagraph, it will be at the height corresponding to $t_a = \tau + 0^{sec.}.15$.

For the micrograph, since the length of a=about 0.5 metre, the maximum value of $t_b = 0^{sec.}.32$; therefore, $t_a = 0^{sec.}.32 - \tau$.

Equating these two values of t_a, we have $\tau = 0.^{sec.}085$ as the value of the time limit.

Details of the Instrument.

Chronometer.—Referring to the definitions, p. 1, the rod a is seen to be a *register*, the time of fall of which to any distance h below the edge of the knife making the mark O, is known.

This rod in this instrument is called the *chronometer*. It is enclosed by a tightly fitting zinc tube which receives the marks. By turning this tube axially, and finally by reversing it, many records may be made before it need be changed.

Registrar.—The rod b is called the *registrar*. It is much lighter than a.

Adjustment.—To diminish differences in the time of demagnetization, the power of the magnets, E, E', is reduced to a safe minimum, which, by the movable screw-cores contained in the magnets, is determined as follows:—

A definite surplus of power is assured by attaching to each armature a *make-weight* in the form of a tube of $\frac{1}{10}$ the weight of the armature. The weighted armature having been suspended from the magnet, the core of the latter is slowly unscrewed until the armature falls. The make-weight is removed before the armature is again applied.

Disjunction Circle.—When it is desired to read velocities directly from the rule, the value of t_b is made constant by varying the height of fall of b so that the mark shall fall upon a *disjunction circle* previously traced upon the zinc recorder at a height above $O = \frac{g \times \overline{0.15}^2}{2}$.

Levelling.—The instrument is levelled by using the suspended chronometer as a plumb.
between the epochs.

Brégers Improvements.—In order to diminish variations in $\sigma_a - \sigma_b$ resulting from variations in the method of rupture, depending upon whether the circuit is broken by the disjunctor or by impact of the projectile on the target wires. the improvements of Captain Bréger of the French service tend:—

1. To diminish differences in the time and velocity of rupture of the two circuits by the disjunctor, which differences are due to the unequal operating of the parts of the disjunctor.

Such differences are found to make material differences in the times required to demagnetize E, E', in consequence of variations in the intensity of the induced currents following variations in the method of rupture.

2. Differences in the rate of demagnetization have been avoided by making as nearly equal as possible the masses a and b, and therefore the magnetic states of E and E'.

These and other minor mechanical improvements have diminished the mean error to $\frac{1}{30}$ of that formerly found.

CLASS II.

Register.—The register in these instruments generally consists of a revolving polished metallic cylinder, the angular velocity of which is known. The surface of the cylinder is preferably smoked, so as to make visible the marks which it receives.

Marks.—The marks are made in two general ways;—

1st. By the trace of a quill point held lightly against the cylinder.

By giving the quill point relative longitudinal motion during the rotation of the cylinder, the trace may be greatly developed helically.

This trace being developed during the motion of the projectile, the latter's arrival at an epochal point may be signalized by a sudden motion of the quill point along a rectilinear element of the cylinder, causing a jog or offset in the trace. See Fig. 6. The offset is here the *mark*.

This motion may be caused by the action of a spring previously in equilibrium with the attraction of an electro-magnet. This magnet is included in a circuit that is broken by the arrival of the projectile at the epochal point.

If the circuit can be re-established before the next epochal point is reached by the projectile, the quill point will return to the prolongation of the trace, and one quill point will suffice. Otherwise, as in Fig. 6, as many quill points and circuits are needed as there are epochal points.

2nd. The mark may result from the passage of an induced electric spark caused by the rupture of a primary circuit at each epochal point.

Signal Times.—In order to avoid variations in the time of signaling, it is advisable, in both cases above cited, to include all the epochal points in the same circuit and to provide each of them with the means of renewing the broken circuit automatically before the projectile can arrive at the next point. See Targets, Class II, below.

When the times to be measured are exceedingly minute, this may not be feasible. Equality in signal times is then sought by increasing the delicacy of the apparatus and is verified by the simultaneous rupture of as many circuits as there are markers to be operated.

Tuning Fork.—Uniformity in the rotation of the cylinder is either assumed from the accuracy of the apparatus or may be neglected by attaching a tracing quill to one of the tines of a tuning fork, the time of vibration of which is known.

The trace then takes the form of a harmonic curve, the alternate intersections of which with a median trace, formed when the fork is at rest, mark the ends of each double vibration of the fork.

The duration of the double vibration is the unit of measure of time; if the velocity of the surface upon which the trace is formed be constant during any double vibration, fractional parts of the intercepted median line will measure corresponding portions of the unit of time.

The double vibration, instead of the single vibration, is selected as the unit in order to neutralize errors of measurement. The median line gives the most definite intersections with the harmonic curve.

Interrupter.—When the total time of the observation requires it, the vibrations of the fork F, Fig. 7, may be sustained by the use of adjacent electro-magnets, m, m, the attraction of which separates the tines, t, t, until the rupture of the circuit through the spring R, releases them. The spring is

used instead of a rigid contact so as to prolong the influence of the magnets. The reaction of the fork due to its elasticity renews the circuit and makes the process continuous.

This device, as applied to the Schultz Chronoscope, is known in this country as the Russell interrupter. It is due to Captain Russell of the Ordnance Department.

When the total time of vibration is very short, no interrupter is required. In this case the fork may be set vibrating by the sudden withdrawal of a wedge inserted between its tines; it is then abandoned.

CLASS III.

In instruments of this class relative motion is given to the register directly by the motion, either of the piece or of the projectile.

The velocity of the register at any portion of its path is determined by tracing upon it a harmonic curve with the tuning fork, or by giving it a known velocity at right angles to that of the moving part. In either case, a compound curve is traced from which the required relations between space and time may be deduced.

Examples:—

If to a gun about to recoil be fastened a bar upon the smoked surface of which the harmonic curve is traced during the recoil, or if some point of the gun be kept in contact with a cylinder rotating at a known velocity about an axis parallel to the direction of the recoil, we may in both cases determine by interpolation the velocities desired:

For example:—

Travel $\Delta x = c.$	Time.	Δt	$v = \frac{\Delta x}{\Delta t}$
ft.	sec.	sec.	f. s.
0.00	0.0000000		
0.02	0.0018182	0.0018182	11.0
0.04	0.0023772	0.0005590	35.8

Knowing thus the mean velocities between many pairs of epochal points, it is possible by interpolation to determine the accelerations at each of the epochal points and, knowing the mass of the moving object, to determine the intensity of the pressure accelerating it.

TARGETS.

CLASS I.

The epochal points are generally wire screens stretched across the trajectory, as shown in Fig. 9.

Cannon.—In order to prevent injury to the first screen and to allow for the acceleration of the projectile for a short distance after it has left the muzzle, due to the relatively great velocity of the escaping gas, the first screen is put at a distance from the muzzle, which increases with the calibre of the gun.

When they are situated in the bore of the gun, as in Captain Noble's experiments, Figs. 11 and 12, the wire may be severed by the action of a wedge raised by the passage of the projectile. This arrangement requires the walls of the gun to be pierced radially as many times as there are epochal points.

To avoid this piercing, the Létard apparatus, Fig. 10, is devised. It is principally of wood, cemented by resin to the surface of the bore. The head ot the metallic bolt, *a*, and the metallic washer, *b*, are held in contact by the cross pin, *c*. The impact of the projectile on the point of *a* breaks the circuit and sweeps the fragments out to the front.

Small Arms.—For small arms, in order to save the time required in repairing at each fire the distant target, it consists of an iron plate having attached to its back a flat elastic blade, through which the circuit passes as in the disjunctor, Fig. 3. The shock of impact breaks the circuit which is immediately re-established by the elasticity of the blade.

CLASS II.

Instruments of this class that use but one circuit for all the targets have an arrangement, shown in Fig. 13.

The weights, *W*, depress the free ends of the spring wire staples, *b*, *d*, *f*, so that the current may pass from the brass plate, *a*, to the brass plate, *c*, through the staple, *b*; and from *c* to *e* through *d*, and so on.

When one of the threads *t* is broken by the projectile, the free end of one of the tines of *d* flies upward, breaking the circuit for an instant, but renewing it as soon as the upper side of the oblong hole in *c* is reached.

The West Point target, figure 15, resembles that above described, but is applied to instruments of Class I.

A discontinuous copper strip, *c*, *c*, *c*, conveys the current when the flanged copper tubes *t*, *t* (cartridge cases), are drawn into place by the weights *w*, *w*.

When a weight is cut the spring, *s*, lifts its tube and the circuit is broken.

Compared with fig. 9, the advantages are—

1. The circuit is broken in the same manner, both by the disjunctor and by the projectile. See page 10.

2. The tension of the threads and the resistance of the circuit are more constant than when a spliced, continuous target wire is used.

3. The targets are more readily mended, and the "short circuiting" of leading wires by fragments of the target wires is avoided.

ELECTRIC BATTERIES.

These should be as constant as possible. Where storage room permits the employment of a large number of elements, the batteries of the gravity type are preferred, except for producing the sparks referred to on page 11.

CHAPTER VII.

PRESSURE GAUGES.

Object.

From the circumstances of the case the transfer of energy from the gunpowder to the projectile is accompanied by a considerable elastic pressure upon the walls of the gun, the effects of which, to be guarded against, require the intensity of the pressure to be known.

History.

Owing to the want of suitable apparatus, pressures were formerly inferred only from the injury resulting to the gun, and it was not until the time of Rodman that this important requisite was supplied. Since then great improvements have been made, some of which will be described.

Nature of Pressure.

The pressure varies at each instant during the passage of the projectile through the bore, and is generally taken to be constant throughout the volume in rear of the projectile at any point of its path. If we adopt Noble's hypothesis, hereafter to be explained, this is equivalent to saying that the gases in rear of the projectile at any instant are of uniform density.

But the expansion to the front and rear, in consequence of the motion of the projectile and of the piece, diminishes the density of the successive layers, estimated in both directions from that layer containing the center of gravity of the system. This layer, called the *immovable layer*, is practically taken at the bottom of the bore, where experiment shows that the maximum pressures occur. See page 17.

General Methods Adopted.

Two general methods for measuring the intensity of the pressure are employed, viz.:

1. Statical.

The statical, in which the elastic pressure is placed in equilibrio with a known resistance. The objections to this method relate to the magnitude of the forces to be measured and to the rapidity with which they vary. It is principally valuable for determining the intensity of the maximum pressure at the point at which the measurement is made.

2. Kinetic.

In which the intensity of the pressure at any instant is deduced from the acceleration given to a known mass.

The objections to this method relate to the minuteness of the times to be measured and to the consequences of small errors in measuring the spaces, since $\alpha = \frac{\Delta^2 s}{(\Delta t)^2}$

By the use of comparatively simple apparatus it permits the *law* of the variation in pressure to be approximately determined.

I.—THE STATIC METHOD.

Rumford's Plan.

In 1792, Count Rumford, who made the first recorded experiments on powder pressures, sought to measure the total pressure, $P = p \pi r^2$, of a charge of gunpowder fired in the closed bore of the *éprouvette*, Fig. 1, by determining the greatest weight, W, that would be lifted by the gas sufficiently to allow the escape of gas to cause an audible report. Conversely, the weight and the volume being constant, he varied the charge. In either case he assumed $P = W$ or $p = \frac{W}{\pi r^2}$

But if we represent by t, the time during which the gases were lifting the tenon of the stopper through the height h,

the general expression $Ma = M\frac{d^2 h}{dt^2} = P - W$, (whence $h = \frac{P-W}{.W}\frac{gt^2}{2}$) shows that, t being very small, P must have greatly exceeded W in order that h should have had an appreciable value.

Process of Deformation.

This, which is the present method, consists—

1st. In determining with a press the *tarage*, or law connecting known pressures with the observed permanent deformations of similar metallic *specimens.*

2nd. Exposing a similar specimen to the action of powder gases acting over a known area; observing the resulting deformation, and inferring from the tarage the intensity of the total pressure producing the deformation observed.

Methods of Deformation.

The specimen may be deformed in several ways—

1. By making a cut, the length of which increases more rapidly than its width. Fig. 2.

This is General Rodman's plan.

2. By compressing a cylinder between flat surfaces.

This is Captain Noble's crusher gauge, now generally employed.

Both methods are adapted for service either within or without the bore.

Apparatus.

Specimens.

On account of its homogeneity, copper is generally used for the specimen, although lead, and even silver, are employed.

Piston.

The pressures are exerted through a freely moving piston. When firing, a *gas check* of some kind prevents the gas from leaking past the piston.

Gas Checks.—1. *Cup.*

The action of this gas check, which illustrates a principle of frequent application in ordnance, depends upon the excess of the pressure within the cup over that without it;

since any gas that may leak past the edge before it is fully dilated will expand so readily as to have its density greatly reduced in comparison with that of the gas within the cup.

2. *Air Packing.*

Another form of gas check depends upon a number of circumferential grooves, surrounding the stem of a closely fitting piston, Fig. 4. These diminish successively the tension of any intruding gas and delay its progress, until, by the departure of the projectile from the gun, the pressure ceases. This principle also is applied in ordnance construction.

External Housing.

For external use the gauge is contained in a *housing*, screwed into the walls of the gun and communicating by a radial hole with the bore. This method is rarely employed at present.

Internal Housing.

For internal service a housing, Fig. 3, contains all the parts. The drawing represents a Noble internal crusher gauge, full size.

A is the specimen; *B*, the cavity; *C* the body of the housing, closed by the screw *J* and the soft metallic gasket *K*.

I is the piston, the cross section of which is $\frac{1}{10}$ of a square inch in area. It is enlarged at *E* to accommodate itself to the dilatation of *A*.

D is a cup-shaped copper gas-check acting like a metallic cartridge case, and *F* is a spring to keep the specimen in an axial position.

Use of Internal Gauge.

To use the gauge, it is tied to the bottom of the cartridge so that no powder can pass between it and the bottom of the bore.

It is sometimes recessed into the face of the block of breech loading guns, so that full charges may be fired. It has also been similarly recessed into the base of the projectile.

Advantages of Crusher over Rodman Gauge.

The advantages of the Crusher over the Rodman gauge are:

1. Size.

The small diameter of the specimen, which enables the size of the housing to be greatly reduced.

When used internally, the circumstances are therefore more nearly normal, and, when used externally as in Fig. 13, it may be inserted close to the walls of the bore instead of externally to the gun as with Rodman's first gauge, Fig. 2.

In the latter case it was found that the gas developed considerable kinetic energy in its passage through the walls of the gun and struck the piston a blow which vitiated the results of the experiment. This action accounts for many anomalies in the early experiments.

When properly used, the fact that the gases act by a pressure and not by a blow is shown by the sensible persistence of form of a specimen exposed to several similar discharges, and by the experimental verification of calculations based upon this statical hypothesis.

2. Surface.

The flat face of the piston is less liable to injury and admits of duplication more easily than does the Rodman knife.

3. Tarage.

It also admits of giving to the specimen a preliminary compression before it is exposed to the action of the gas.

This, if nearly equal to that expected within the gun, diminishes the velocity which the piston can acquire (see *post*); it also serves to verify the tarage.

EFFECT OF VARYING THE MASS OF THE PISTON.

Discussion.

Let P be the variable gaseous pressure on the piston; R the variable resistance to deformation of the specimen; m the mass of the piston, and v its variable velocity over the path x.

Let d represent the permanent compression.

Suppose the piston to be in contact with the specimen and to be indeformable.

We have:

$$\frac{m\, v^2}{2} = \int_0^x P\, dx - \int_0^x R\, dx. \qquad (1)$$

When x is a maximum, $v=o$; $x=d$, and the equation (1) becomes

$$\int_0^d P\, dx = \int_0^d R\, dx. \qquad (2)$$

Let the curves AFE and BFC in Fig. 5 represent by their ordinates, respectively, the resistances and pressures due to successive values of x, and let x represent only permanent deformation, *i. e.*, that occurring beyond the elastic limit of the specimen.

From the nature of powder pressures as hereafter explained, Chap. XI, the pressure curve will be of the general form $OABC$; and, if $OD=d$, we have area

$$OABCD = \int_0^d P\, dx.$$

From the nature of the resistances to deformation, the curve AFE will present no maximum phase, and the curve will be of the general form AFE, such that the area

$$OAED = \int_0^d R\, dx.$$

Equations 1 and 2 show that the resistance curve, at first beneath the pressure curve, rises above it when x has some value $= OH$.

The statical value of the ordinate ED, corresponding to the maximum compression OD, is determined from the tarage.

The figure shows that, while ED may be greater or less than IB, which represents the maximum gaseous pressure, the difference between these two ordinates will depend upon the angle at which the curves intersect. If this angle be such that the difference, $P-R$, of any two ordinates corresponding to a common abscissa be relatively small, the resistance corresponding to the maximum compression may safely be taken as the maximum gaseous pressure.

$$\text{But } P-R = \frac{d\left(\frac{m v^2}{2}\right)}{dx}, = m\,a \tag{3}$$

So that the difference between ED and IB can be neglected only when the mass of the piston is small and the initial resistance to deformation is great and increases rapidly.

The last two conditions are satisfied by a preliminary deformation of the specimen to nearly the total extent that is expected. Such conditions are represented by figure 6.

The indications of the gauge will be more correct when the pressure curve approaches parallelism with the axis of X. It will be seen that this occurs rather with slow-burning powders, which give a gradual change of pressure, than with those which act more violently.

Tarage.

The preceding discussion shows that the pressures used in determining the tarage must be applied so slowly that the velocity of the piston and of the contiguous parts of the machine may be neglected. Under such circumstances, when the specimen is of pure copper, 8 mm. in diameter and 13 mm. long, the resistance in kilogrammes, T, corresponding to a permanent compression in millimetres, E, is given by the following equation.

$$T=551+531\ E.$$

The tarage would accordingly be represented by a diagram, such as Fig. 7, the initial ordinate being at the elastic limit, and the co-efficient $531=\tan\varphi$, or the reciprocal of the rate of permanent compression.

SARRAU'S DEDUCTIONS.

An elaborate deduction by M. Sarrau, shows by an analysis confirmed by experiment that,—

Slow Pressure.

I. When the pressure increases slowly, as when the crusher is used in the chamber of a gun firing ordinary gunpowder, the maximum pressure is sensibly equal to that indicated by the tarage.

Quick Pressure.

II. When the pressure is instantaneously, or very suddenly applied, as with some of the high explosives; or when, with ordinary gunpowder, the crusher is placed in front of the position occupied by the projectile when at rest so that the pressure shall be very suddenly applied when the base of the projectile has passed the mouth of the hole; then the maximum pressure is sensibly equal to that corresponding to half the tarage.

The correction increases with the mass of the piston employed.

THE MANOMETRIC BALANCE.

Object.

This avoids the perturbations due to the mass of the piston, and permits certain relations between pressures and time to be determined.

Simple Form.

B, figure 8, is a differential piston, the small end of which is in contact with the bore, and the large base of which enters slightly the air-tight cavity *C*, connected with a manometer

tube, *M*, in which mercury is kept at any given height by means of air pumped into *C*.

A slide, *a*, is held by friction beneath *B* against the tension of a spring, *r*.

The pressure may be determined within limits by finding at two successive similar fires, the heights of the mercury permitting and preventing the motion of *a*.

Compound Form.

Also by providing, say, ten similar pistons of varying area, moving outwardly from *C*; each of them provided, besides the arrangement *ra*, with some apparatus such as described Chap. VI, for recording the intervals of time corresponding to the motion of the slide *a*.

II. THE KINETIC METHOD.

This consists in determining the rate of change of the pressure from the change in rate of motion of some body, the mass of which is known. This body may be either I, the projectile; II, the cannon; or III, a piston or auxiliary projectile, placed in some radial channel communicating with the bore.

I. THE PROJECTILE.

1. *Direct Intermittent Method.*

Mayewski's Experiments. Figure 16.

General Mayewski, of Russia, in 1867, attempted to determine the acceleration of the projectile by attaching to its base a rod which, passing through the breech of the gun, ruptured by means of a projection upon it certain electric currents placed at varying distances from the initial position of this projection. The conditions of each fire were made constant, except as to the portion of the path of the projectile, the duration of which was to be measured.

Supposing that $x=f(t)$, he assumed a development

$$x=At+Bt^2+Ct^3+Dt^4+\text{etc.}, \qquad (4)$$

and determined by trial the values for the co-efficients, A, B, etc., that would satisfy the instrumental values x and t. Then,

$$v=\frac{dx}{dt}=A+2Bt+3Ct^2+4Dt^3+\text{etc.} \qquad (5)$$

$$\alpha=\frac{d^2x}{dt^2}=2B+6Ct+12Dt^2+\text{etc.} \qquad (6)$$

The value of t corresponding to the maximum pressure was found by placing $\frac{d^3x}{dt^3}=0$, and solving with respect to t; then, by substitution in equations (1) and (3), the corresponding values of x and α were found.

The intensity per unit of area of the corresponding pressure, is given by the following equation:

$$p=\frac{m\frac{d^2x}{dt^2}}{\pi r^2}.$$

This pressure is only that giving acceleration to the projectile. The results, found by adding to $P=p\,\pi\,r^2$ the pressure found to be required to force the projectile through the bore, gave reasonably close approximations to the results of the statical pressure gauges, although the apparatus was subject to many instrumental errors.

2. *Direct Continuous Method.*

Sébert's Registering Projectile.

This method, which is applicable only to comparatively short lengths of bore in guns of large caliber, requires a hollow cylindrical projectile, such as is shown in Fig. 9. It

is provided with an axial spindle, *S*, of rectangular cross section and rotating freely at each end; one of the sides of this spindle is covered with a film of soot.

A slide *M* moves freely on the spindle and bears a delicate tuning fork *T* arranged as described, Chap. VI.

When the projectile is fired, the inertia of the slide holds it relatively at rest while the projectile passes by; the points of the tines describe such a trace as is shown in figure 10, in which the parallel straight lines represent the traces when the slide is slipped along the spindle, the fork not vibrating.

The effect of the friction between the slide and the spindle can be shown to be negligible.

Although the path of the slide is limited to less than the length of the projectile, yet it is within this length of travel that is generally found the maximum pressure, the rate of change in reaching which is one of the most important objects of research.

By placing the slide at the bottom of the spindle, it may serve to determine the retardation of the projectile in flight; and, by confining it there by a fragile cross-pin to be broken on impact, it may determine the varying resistance found in penetrating a more resisting medium than the air.

3. *Indirect Intermittent Method.*

Successive Shortening of the Bore.

The mean of the muzzle velocities of a large number of shots fired under conditions, which, excepting the length of the bore, were identical, could be laid off as the ordinates of a curve of which the abscissæ should represent the various paths. The curve would have the form given by Fig. 11.

Calling α the acceleration, we have,

$$\alpha = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v\frac{dv}{dx}, \tag{1}$$

The figure shows that the subnormal $\alpha =$ corresponding ordinate $v \times \tan \varphi = v \times \frac{dv}{dx}$. (8)

Therefore, having plotted the curve expressing

$$v = f(x),$$

the acceleration at the different points along the bore may be determined by finding the corresponding values of the subnormals.

The experiments enabled positive conclusions to be formed:

1st. As to the smallness of the advantage gained by increasing the length of the bore more than 20 calibers, when quick powders were used.

2nd. As to the great advantage of progressive powders in guns of suitable length.

II. THE GUN.

Advantages.

Determining the pressure from the acceleration of the gun in its recoil affords certain advantages owing:—

1st. To the low velocity of the gun compared to that of the projectile; this permits a greater number of observations to be made over a given path.

2d. To the simplicity of the apparatus, which avoids the mutilation of the piece, and permits it to be used with guns of varying calibers.

3d. To the aid given in the study of the pressures prevailing at the bottom of the bore.

1. *Rodman's Velocimeter.*

Construction.

The original instrument of this description was devised by General Rodman. It consisted of a cylinder rotating with a known and uniform velocity about an axis parallel

to that of the gun and close to it. A pointer fastened to the gun traced upon the cylinder during the recoil, a line which, when developed, gave the successive accelerations of the recoil. The gun was hung as a pendulum oscillating in the plane of fire. See figure 17.

Acceleration of Recoil.

For example, let mm', Fig. 12, be the developed circumference traced by the pointer when the projectile is placed at the muzzle, and the charge uniformly dispersed along the bore; and let bb' be the corresponding circumference when the charge is in place. Then taking axes of space and time, S and T

$\alpha = \frac{\Delta^2 s}{(\Delta t)^2}$, as in Chap. VI.

Uniform Pressure.

The dotted line represents the parabola which would be traced under the ideal circumstances discussed, Chap. V, $t\ v$ being straight and parallel to $t'\ v'$.

Rate of Change of Pressure.

It is evident, from the inclination of the initial portions of the curve, that the velocity is actually acquired much more rapidly than is desired.

2. *Sébert's Velocimeter.*

Construction.

The velocimeter of Colonel Sébert of the French service replaces the rotary cylinder by a broad steel tape, one side of which is smoked, and against which rest the tines of a tuning fork set vibrating by the act of recoil. See Chap. VI.

Velocity of Recoil.

The length of any double vibration measured on the tape, divided by the time of the double vibration of the

fork, gives us the mean velocity of recoil over that portion of the path selected; from which, calling,

w = weight of projectile.
W = weight of gun.
w' = weight of powder.
v = velocity of projectile.
V = velocity of gun.

We have $V = \frac{\Delta x}{\Delta t}$ (see Chapter VI).

Velocity of Projectile.

Or else, supposing the center of mass of the powder to be moved to the position occupied by the center of mass of the gases, which is equivalent to supposing that half the mass of the powder is added to that of the projectile.

$$v = \frac{W V}{w + \frac{w'}{2}}. \tag{9}$$

Position of Projectile.

Also, denoting by X the length of recoil at the end of a time t, and x the corresponding path of the projectile.

$$x = \frac{W X}{w + \frac{w'}{2}}. \tag{10}$$

Pressure on Base of Projectile.

The method above described permits the construction of a velocity curve, such as Fig. 11, from which the pressure corresponding to different positions of the projectile along the bore may be deduced.

Pressure on Base of Bore.

Also, the difference between two successive velocities, as determined by the trace of the tuning fork, divided by the common interval of time, would give the mean acceleration; this multiplied by the mass of the gun gives the mean

total pressure on the bottom of the bore during the same interval of time.

III. AUXILIARY PROJECTILES EXPELLED THROUGH THE WALLS OF THE GUN.

1. *Bomford's Method.*

About 1841, Colonel Bomford, of the Ordnance Department, prepared a cannon by boring through its walls a series of small holes at right angles to its axis, as in Fig. 13, and placing in each hole a bullet, the velocity of which was instrumentally determined. The pressure at the various points, deduced from the velocities communicated to the balls, determined the form of the old *Columbiads.*

This method was objectionable, as it treated the powder pressure as an impulsive force and could not take into account the varying accelerations of the projectile, as is done in the following recent inventions.

2. *Ricq's Register, Fig.* 14.

A cylinder C, revolving with known and uniform velocity, is enclosed in a box B, through a groove, D, in which slides a marker F, in contact with the piston L.

The weight per unit of the sectional area, a, of $(F+L)$ may be varied at pleasure.

When the gun is fired, a curve, such as shown, is traced on the cylinder, from which, by finite differences, we have

$$v = \frac{\Delta s}{\Delta t}, \tag{11}$$

$$p = \frac{m}{a} \frac{\Delta^2 s}{(\Delta t)^2}. \tag{12}$$

3. *The French Accelerograph.*

This projects vertically upward a piston, the mass of which may be greatly varied by the addition of weights.

A fixed tuning fork traces the harmonic curve upon a blackened surface on the piston.

III. STATIC AND KINETIC METHODS COMBINED.

Objections to Kinetic Method.

It will be seen that the objection to the kinetic method lies in the liability of error in the measurement of the small spaces by which the time record is expressed.

This limits the method principally to the cases where the pressure changes but slowly, as in those powders known as slow-burning powders.

Noble's Experiments.

In 1869, Capt. Noble, R. A., prepared a M. L. gun as described, Chap. VI. Crusher gauges were placed in the holes leading to the chamber, and the other holes were provided with the apparatus also given in Chap. VI.

By observation and interpolation, a table of spaces and times was formed so as to make Δx constant$=6, 10^{mm}$, as follows:

x mm.	t sec.	Δt sec.	$v=\frac{\Delta x}{\Delta t}$ m. s.	Δv m. s.	log dt *	$\alpha=\frac{\Delta v}{dt}$ m. s.	$p=\frac{w}{\pi r^2 g}\alpha$ kil. per □ cm.
0.00	0.0000000						
6.10	0.0018182	0.0018182					
			3.35				
		0.0005590		7.56	$\bar{4}.88327$	9891	271
12.20	0.0023772		10.91				
		0.0002528		9.02	$\bar{4}.61517$	22082	605
18.30	0.0026330		19.33				

* The values of dt are obtained by interpolation. That of the mean acceleration for the first value of Δv, viz., $\Delta t=\frac{0.0023772}{2}=0.0011886$ would evidently be too large, and the corresponding value for $p=186.5$ would be too small. The method of interpolation is similar to that hereafter described in Exterior Ballistics.

Fig. 15, represents the curves obtained in a 10 in. gun firing a 300 lbs. projectile with fine (R. L. G.), and coarse, (Pebble) powders.

The muzzle velocity of the projectile was in both cases practically equal.

Comparison of Results.

The calculated pressures agreed closely with those observed in the gauges placed near the base of the projectile when at rest; and those observed at the base of the bore considerably exceeded those observed near the base of the projectile. See Chap. XI.

CHAPTER VIII.

PHENOMENA OF CONVERSION.

Phenomena.

For purposes of analysis the conversion of gunpowder into gas may be considered under three heads, viz.: *Ignition*, *Inflammation* and *Combustion.*

Definitions.

By *Ignition* is meant the setting on fire of a particular point of a grain or charge.

By *Inflammation* is meant the spreading of the fire from point to point of the surface of a grain, or from one granular surface to another throughout the charge.

By *Combustion* is meant the passage of the inflamed surface throughout the substance of each grain.

IGNITION.

Gunpowder is the most refractory of the explosives; it ordinarily requires a temperature of 300°. Its ignitibility varies inversely:

1st. With the amount of moisture present.

2d. With the smoothness and sphericity of the surface.

3d. With its density.

It also varies with the character of the charcoal.

COMBUSTION.

Definition.

By *velocity of combustion* is meant the rate of motion of the inflamed surface in a direction normal to that surface. Owing to the impossibility of determining this within the

gun, the velocity of combustion of different kinds of powder in the open air is taken by the manufacturer as a rude means of comparing their combustibility.

Determination.

If the size of the grain permits the time of its burning to be accurately determined, this simple method is preferred; since it resembles most closely the actual conditions of practice. Otherwise, we may extend the time to be measured by burning, like a candle, a prism of press cake having its sides greased to protect them from the flame; or else, we may use a tube rammed with the pulverized mill cake of the same density as that of the powder to be tested. So determined, the velocity is found to be about 0.4 inch per second.

Nature of Combustion. 1. *In Air.*

This experiment proves that the composition burns in parallel layers at a uniform rate; so that the combustion of a spherical grain would resemble the peeling of an onion.

This fact is frequently illustrated on the proving ground, where burning grains of powder are projected from the gun with sufficient force to penetrate deeply into wooden boards. Should they fall in snow, their appearance will plainly indicate the superficial nature of their combustion.

2. *In Gun.*

It is important to remember that the velocity of combustion within the gun is very much greater and less uniform than that in the open air. The process resembles roughly the absorption of water by a porous substance when under variable hydrostatic pressure. The effect may be, not only to accelerate the velocity of combustion, but also, by breaking up the grains, to increase the burning surface; as we crush sugar to facilitate its solution.

The velocity of combustion is supposed to vary directly with the intensity of the gaseous pressure.

CIRCUMSTANCES AFFECTING THE VELOCITY OF COMBUSTION IN AIR.

Varying Conditions.

Under similar circumstances the velocity of combustion of homogeneous powder is constant. It varies however, with the *purity*, *proportions*, *incorporation*, *density* and *condition* of the powder as follows:

1. Purity.

The nitre and sulphur should be pure, or nearly so. The part that charcoal plays depends upon its combustibility. This is determined by finding the velocity of its combustion, when incorporated with a due proportion of nitre in such a tube as above described.

2. Proportions.

By varying the proportions, all velocities up to 0.55 inch per second can be obtained.

The proportions usually adopted are those that give the greatest volume of gas in a given time, because the mass burned is the greatest, and because each unit of mass gives the greatest volume of gas.

3. Incorporation.

Prolonging the incorporation increases the velocity at a rate which increases as the proportions approach those adopted.

4. Density.

With each set of proportions a density is soon reached that corresponds to the maximum velocity. Beyond this density the velocity varies inversely as the density, at a rate which increases as the proportions approach those adopted.

The increase in superficial density due to glazing diminishes the velocity of combustion; provided that the dust formed in the process be removed.

5. Condition.

The velocity increases with the porosity of the powder. See page 2. The porosity may result from the evaporation of water, alcohol, or vinegar, added to the substance before pressing it. When porosity is carried to the point of friability, the consequences described, page 2, may be expected.

When oils, gums, or resins are added, or when an excess of water remains in the composition, the velocity of combustion is diminished. An excess of water permits the nitre to segregate and to neutralize the effects of incorporation.

Remark.

These variations should be carefully studied, as upon them depend the most important characteristics of gunpowder.

Emergency Powder.

For example; during the Franco-German War of 1870, it was found necessary to increase, far above their normal capacity, the product of the powder mills remaining in the hands of the French.

This was accomplished by reducing the time of incorporation under the wheels, besides calling into use the stamp mills and rolling barrels formerly employed for this purpose.

The effect of less thorough incorporation upon the velocity of combustion was neutralized by reducing the density of the powder.

This answered well where the powder was not intended to be stored, and where the capacity of the chambers in which it was to be fired permitted a corresponding increase in the volume of the charge.

The differences of the effects upon the gun and its projectile, resulting from varying the phenomena of combustion, are described in Chapters X and XI.

INFLAMMATION.

Hypothesis.

The inflammation of a single grain is generally assumed to be instantaneous, and so is that of a charge of powder; unless the time of its inflammation bears so considerable a ratio to that of its combustion that the total time required for the conversion of the charge into gas is sensibly increased.

Experiment.

The nature of the process may be studied by determining the time required to inflame trains of powder of known lengths under various conditions.

Varying Conditions.

The velocity of inflammation is found to vary:

1. With the disposition of surrounding bodies.
2. With the size and shape of the grains.
3. With their composition and constitution.

1. Confinement.

The heated gases, evolved by ignition, follow in their expansion the line of least resistance. If they are confined, so that this line coincides with that along which the powder is disposed, its rate of inflammation is increased. Thus, the velocity of inflammation of a train is increased by firing it in a tube instead of in the open air. It is still further increased when the cross-section of the tube is not entirely filled; and when the bottom of the tube, near which the train is ignited, is closed, as in a gun.

2. Size and Shape of Grain.

The size and shape of the grain affect both the force propelling the gases and the resistances which they encounter. In the first case, the size and shape of the grain affect the amount of gas evolved in equal successive times and also the ignitibility of the unburned grain; in the second

case, they affect the size and shape of the spaces between the grains. So that, in fine powder, although the gaseous pressure may be greater, the resistance to the passage of the wave of inflammation may also be greater. In coarse powder the converse may be the case. The velocity of inflammation should therefore be determined by experiment.

It is now much less important than when muzzle loading guns were in use.

If the charge be made of mealed powder compressed, there will be no interstices; and the velocity of inflammation and that of combustion will be the same.

If it be of concrete powder, the velocity of combustion of the entire grain will be that of the inflammation of the constituent grains, and will be greater than that of the compressed mealed powder above referred to.

This ratio was found to be as 1.4 to 1.0.

3. Composition and Constitution.

The velocity of inflammation is affected by variations in the purity and proportions of the ingredients, in the thoroughness of their incorporation, and in the density of the powder, in so far as these affect its velocity of combustion and its susceptibility to ignition.

CHAPTER IX.

NOBLE AND ABEL'S EXPERIMENTS.

From 1868 to 1874, Captain Noble, R. A., and Mr. F. Abel, the chemist of the British War Department, made a series of experiments upon gunpowder that have become historical.

NATURE OF THE EXPERIMENTS.

These experiments were conducted on the principle, general in all experimental comparisons, of *keeping all conditions constant except that of the variable under consideration.*

Although their ultimate object was to determine the behavior of fired gunpowder in the variable volume behind the projectile in a gun, this principle required that their preliminary experiments should be conducted in closed vessels, the capacity of which was invariable and accurately known.

VARIABLES.

They accordingly varied:—

1. The composition of the powder.
2. The size of its grain.
3. The mass of gunpowder exploded in a given volume, or the *density of loading*.

FUNCTIONS.

Under these different circumstances they observed:—

1. The maximum pressures per unit of area.
2. The composition and condition of the products of combustion.

3. The specific volume of the gases formed, viz., at a pressure of one atmosphere and at 0°.

4. The quantity of heat evolved by the combustion.

CONCLUSIONS.

From the observed states of the functions corresponding to particular values of each variable they sought to determine the law expressing the relation between pressures, volumes and temperatures in closed vessels, with the view of applying it to the variable conditions existing in guns.

METHODS FOLLOWED IN THE EXPERIMENTS.

VARIABLES.

1. Composition of Powders.

Four of the six kinds of powders tried were approximately of the usual composition. The others differed notably as seen by the following table:

COMPONENTS.	POWDERS.		
	Four English.	Spanish.	Blasting.
Nitre,	74	75	62
Carbon,	12	9	18
Sulphur,	10	12	15
Water, H, O, Ash, etc.,	4	4	5
	100	100	100

2. Size of Grain.

The principal experiments in which the size of grain entered as a variable were those in which comparisons were made between R. L. G. (Rifle, large grain) and the Pebble powders referred to Chap. VII. The linear dimensions of these powders were about as 1 to 3.

3. Density.

It is evident that the results of the experiments were largely dependent upon the relation existing between the

mass of the charges and the volumes in which they were fired. This requires a discussion of the density of powder which is named under three heads.

1. *Specific Gravity.*

By *density* simply, or δ, we mean the specific gravity of the press-cake, or that of the individual grains, referred to water. This in practice ranges from 1.68 to 1.85. The maximum attainable density calculated from that of the ingredients of gunpowder united in their ordinary proportions, is 1.95.

2. *Gravimetric Density.*

By this term, or γ, we mean the density referred to water of grained powder, including its interstitial volumes; or, calling w, the weight in pounds of one cubic foot of the loose powder.

$$\gamma = \frac{w}{62.425} \tag{1}$$

The gravimetric density is sometimes expressed by the weight in ounces of one cubic foot of the loose powder.

The gravimetric density of powder is important when it is to be used in a limited volume as in the cartridges for breech-loading small arms and in explosive projectiles. It is evident that the form of grain and the amount of settling affect the interstitial volumes and hence its value. For loosely piled powder of irregular granulation it is about 0.9.

Specific and Interstitial Volumes.

The amount of the interstitial volumes, which, as seen in Chap. VIII, affects the rate of inflammation, may be determined as follows:

Let V, represent the volume of the powder when loose; $v_{,}$, its specific volume, or its volume when compressed to a uniform density, δ; and v', the sum of the interstitial volumes: then, since $m = V\gamma = v_{,}\,\delta$.

$$\delta : \gamma :: V : v' \text{ or } v_{,}=\gamma \frac{V}{\delta} \tag{2}$$

$$\text{whence } v'=V-v_{,}=V\frac{(\delta-\gamma)}{\delta} \tag{3}$$

Ordinarily, δ is about 1.8; and, when the powder is loosely piled, γ is about 0.9. In such a case $v_{,}=v'=\frac{V}{2}$.

Noble conducted his experiments with powder so closely packed that γ was sometimes equal to unity: in such a case v' was sensibly equal to 0.44 V.

3. *Density of Loading.*

By this term, or Δ, we express the relation between the mass of a charge of powder and the volume in which it is fired.

If the values of δ and γ were constant, it would suffice to say that the cavity holding the powder was, say, one-half, three-quarters full, etc. This was the method adopted by the early experimenters.

But the quantity of matter in a given volume of grained powder may vary from both the causes named.

The value of Δ is therefore taken as the ratio of the weight of the powder fired, to the weight of water at its maximum density which would fill the volume in which the powder is fired. Calling this volume expressed in cubic feet V, and expressing w, as before, in pounds, we have

$$\Delta=\frac{w}{V\times 62.425} \tag{4}$$

It is usual to give the linear dimensions of guns in inches; therefore calling $v=V\times 1728$, the volume in cubic inches, we have

$$\Delta=\frac{27.68\,w}{v} \tag{5}$$

This value of Δ is of constant application and must be remembered.

APPARATUS.

The vessels employed were strong steel cylinders as shown in Fig. 1. Each one contained a firing plug, *F*, with a conical stopper, *f*, insulated from *F* by a washer, *w*, and by sheet of tissue paper wrapped around its body. Another conical screw plug, *P*, carried a crusher gauge, *C*.

The object of the form given to *F* and *P* was to facilitate their removal; since a very slight motion would free them from the walls.

The charge was ignited by an electric igniter, *I*.

After the firing the vessel was immediately conveyed to a calorimeter; or a smaller vessel, Fig. 2, could be fired under water.

FUNCTIONS.

1. Pressures.

These were determined by the crusher gauge, and the observed results compared and corrected by the methods used in experimental research.

2. Nature of Products.

Small samples of gas were drawn off for analysis through the tube, *E*, opened by slightly unscrewing the valve *e*.

The initial liquidity of the non-gaseous products was determined by tipping the cylinder in various directions soon after the explosion, and by observing the appearance of the solid crust when the vessel was finally opened.

3. Volume of Gases.

The specific volume of the gases was determined by a gasometer, Fig. 3. The long wrench, *w*, passing through the stuffing-box, *sb*, was used to unscrew *P*, immediately after the explosion.

4. Heat.

The quantity of heat evolved by the conversion was determined by immersing the vessel in a calorimeter con-

taining a known weight of water of known temperature, and by noting the resulting rise in temperature.

RESULTS OF THE EXPERIMENTS.

STATES OF THE FUNCTIONS.

1. Pressures.

For all kinds and sizes of powder the pressure was found to be practically constant for equal densities of loading, or the *force* of all the powders was the same. When $\Delta = 1$, the force was about 6,400 atmospheres, or 43 tons, or 96,000 lbs., per square inch.

2. Products.

The following table* gives, by weight per cent, the mean proportions of the products resulting from many experiments:

PRODUCTS.	KINDS OF POWDER.		
Gaseous.	Four English.	Spanish.	Blasting.
CO_2,	26	25	23
CO,	3	1	15
N,	11	11	9
Various,	4	1	4
Total Gaseous,	44	38	51
Non-Gaseous.			
K_2CO_3,	34	22	19
K_2SO_4,	12	30	
K_2S,	6	5	17
Various,	4	5	13
Total Non-Gaseous,	56	62	49

From the appearance of the cavity after firing, the non-gaseous products were supposed to be suspended at the instant of the explosion as a highly heated liquid spray which eventually assumed a solid form. In cooling it was

*NOTE.—The relative proportions of the total gaseous products and of CO, should be learned. See Chapter II.

supposed to shrink from about 0.6 the volume of the entire charge, or 0.6 V, page 3, to about 0.3 V.

Confining our attention to the typical English powders it is significant to observe that very nearly the same proportions were concluded to exist between the volumes occupied by the gaseous and non-gaseous products at the instant of the explosion, as were found to exist between the weights of these products and between the interstitial and specific volumes of the charge.

That the non-gaseous products did not, by their volatilization, augment the volume of the gases was inferred from their behavior when exposed, solid, in a Siemen's furnace, to a temperature of about 1700°. At this temperature which, although the highest available, was about 700° lower than that determined by calculation, the solids swelled to nearly twice their volume, but did not volatilize.

3. Volumes. 4. Heat.

The relations between the specific volumes of the gases and the calorific values of the powders appear from the following table which illustrates the curious fact, noted in Chap. II, that their product is approximately a constant quantity. The volumes are referred to that occupied by the powder when $\Delta = 1$. See note 1, page 13.

Kind of Powder.	Specific Volumes. or v_0.	Heat Units or H.	Products.
English powders,	264	737	194568
Spanish powder,	234	767	179478
Blasting powder,	360	517	186120

Had the experimenters known the specific heat of the products of combustion when at a constant volume, or C_v, the absolute temperature of the conversion, or T_0, might have been determined from the general equation,

$$T_0 = \frac{H}{C_v}$$

but, although the same products were always formed, they occurred in such varying proportions, even when all the conditions were as nearly as possible identical, that no certain conclusions could be made. Chap. II, page 7.

Also, by taking the mean specific heats of the mean of the non-gaseous products, when in a solid form, and also of the gases, a temperature was computed which was manifestly too great. The experimenters accordingly adopted the following course in which the deductions of theory are corrected by experiment.

Temperature of Explosion.

Assuming the general equation for the work of permanent gases subjected to changes in temperature, or—

$$p\ v = r\ t, \tag{6}$$

in which r is a constant, and t is reckoned from absolute zero; let us express p in atmospheres.

The preceding table gives for the English powders a mean value of $r = (264 = v)\ (1 = p) \div (273 = t) = 0.967$.

Substituting the values of v and p for $\Delta = 1$, we have—
$t = (6400)\ (0.4) \div 0.967 = 2646°$ absolute, $= 2373°\ C$.

This was verified for varying values of Δ and by the exposure to the temperature of the explosion of very fine platinum wire which melts at about the temperature above determined.

CONCLUSIONS.

Fundamental Hypothesis.

The remarkable compensation between the volumes of gas generated and heat evolved permitted Noble and Abel to apply to these gases the laws of Mariotte and Gay-Lussac; provided, *that from the volume of the chamber in which the explosion occurred was subtracted the volume occupied by the non-gaseous residue.*

Remarks.

This conclusion, although simplifying the labors of the experimenters, and useful for a general discussion like the present, is now believed to depend upon a compensation of errors.

It is now believed that the solid products are volatilized and probably dissociated, and it is known that Mariotte's law does not apply to the pressures observed in guns.

Still the latest researches lead to practically the same conclusions reached by these experimenters.

EXPERIMENTS IN GUNS.

The experimenters found that when the gases expanded into a varying volume, as in the gun, results similar to those above described were found, viz.:

1. Products.

That the nature and proportions of the products remained the same as in a closed vessel.

2. Working Substance.

That the work on the projectile may be considered to be due to the elastic force of the permanent gases.

3. Source of Energy.

That the heat evolved by the non-gaseous residue maintains the gases at a constant temperature during their expansion, which, therefore, is *isothermal.*

This is essentially the hypothesis of Hutton, made a century ago. For want of suitable apparatus Hutton erred greatly in his deductions from this hypothesis.

4. Theoretical Work.

The total theoretical work of the permanent gases, when indefinitely expanded, was computed to be about 486 foot-tons per pound of powder.

This is nearly the result given by the table on page 7. Only from 13 to 20 per cent. of this work can be realized in practice. See note 2, page 13.

5. Loss of Heat by Absorption.

The quantity of heat lost by absorption was approximately determined by plunging into a calorimeter a field piece, after firing from it a number of rounds in rapid succession.

The loss was found to vary directly with the ratio of the cooling surface to the weight of the charge, and also with the time of travel in the bore.

It varied, per unit of weight of the powder fired, approximately as follows:

Gun.	Loss in H.	Loss Per Cent. in Energy.
10 in. M. L. R.	25	3.5
12 pdr. B. L. R.	100	14.0
0.45 in. B. L. R. musket	250	35.0

6. Pressures.

The experimenters confined themselves to the prediction of velocities. The determination of the actual intensity of the variable pressure during the progressive combustion of the powder in a volume varying with the position of the projectile during combustion was determined in only a few special cases. The important law by which this pressure varies, upon which modern guns are constructed, was left unsolved.

The methods of M. Emil Sarrau, of the "*Département des Poudres et Salpêtres*," which depend rather upon dynamical than chemical laws, corrected, like those of Noble and Abel, by experiment, are now generally followed where accurate prevision, both of pressures and velocities, is required.

The older methods are adopted in this text, as they permit the presentation of some of the more important phenomena of fired gunpowder in a relatively simple form.

DEDUCTION OF THE VARIABLE PRESSURES IN A GUN.

Hypothesis.

It has been shown in Chapter VIII. that the conversion of gunpowder is not instantaneous. Yet, on account of the difficulty of determining the circumstances of the motion of the projectile during the period of combustion, or $x = \varphi(t)$ and the rate of combustion under the varying pressure to which the powder is exposed, or $\sigma = f(t)$ it is best to begin by assuming that the conversion is instantaneous, and to correct the results of computation by experiment. See note 3, page 13.

Assuming then, the proportions of solid and gaseous products previously given, and that the change in pressure is due to the change in volume in rear of the projectile (which, under the isothermal hypothesis, acts like a piston moving with variable velocity under some external force), we may deduce the following relation between the pressure and the mean density of the products of the explosion.

Let p represent the intensity of the gaseous pressure in tons per square inch, and

w, the weight of the charge in pounds;

v, the variable volume behind the projectile in cubic inches;

v', the volume occupied by the non-gaseous products in cubic inches;

d', the density of these products referred to water;

d, the density of the gases referred to water and supposed to remain at a constant temperature.

R, the ratio $\frac{p}{d}$ assumed under Mariotte's law to be constant.

Deduction.

From the general expression for density we have

$$d = \frac{0.44\,w \times 27.68}{v - v'} = \frac{12.18\,w}{v - v'}; \text{ and}$$

$$d' = \frac{0.56\,w \times 27.68}{v'} \therefore v' = \frac{15.5\,w}{d'}; \text{ and}$$

$$p = R\,d = R\,\frac{12.18\,w}{v - \dfrac{15.5\,w}{d'}}.$$

Multiplying both numerator and denominator of the value of p by

$$\frac{27.68}{12.18\,v} = \frac{2.2724}{v}, \text{ we have}$$

$$p = R\,\frac{\Delta}{2.2724 - 1.2726\,\dfrac{\Delta}{d'}}. \qquad (7)$$

The ratio, R, is found, by experiment, not to be absolutely constant; but, by selecting from Noble's experiments in closed vessels, suitable values of p and Δ in pairs, and by substituting these values in Eq. (7), we may obtain two equations, containing two unknown quantities, from which we find (See note 4, page 13.)

$$R = 32.18 \qquad d' = 0.824.$$

These values substituted in Eq. (7) give, after reduction,

$$p = \frac{1}{\dfrac{0.070618}{\Delta} - 0.048}.^{*} \qquad (8)$$

Which, for convenience, may be placed under the form

$$p = \frac{1}{0.0025571\,\dfrac{v}{w} - 0.048}.^{\dagger} \qquad (9)$$

Equations (8) and (9) give remarkably close approximations to all but the very highest pressures found in Noble's experiments in closed vessels.

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*log $0.070618 = \bar{2}.8489170$. †log $0.0025571 = \bar{3}.4077557$.

COROLLARIES.

1. By substituting in Eq. (9) proper values for $x = \frac{v}{\pi r^2}$, we may construct a curve, as in fig. 4, which will give the pressures at different points along the bore of the gun under the assumptions noted, page 11.

Should the piece be chambered, the value of x', the *reduced length of the chamber*=volume of chamber$\div \pi r^2$ must replace its measured length.

2. It is evident that the value of the initial ordinate is determined by the value of the density of loading, Δ.

3. Also that, knowing by experiment the intensity of the maximum pressure, and the charge, we may determine approximately the corresponding position of the projectile.

Since $x = \frac{w\,[1 + (p \times 0.048)\,]}{\pi r^2 p \times 0.0025571}$.

4. Also that we may determine the charge required to burst a closed vessel, like an explosive projectile, when its resistance to rupture is known.

NOTES.

1. Page 13.—The experimenters ascertained that the erosion of the bore, caused by the rush of the gases past the projectile, increases directly with the factor H, and inversely with v_0.

Since modern steel guns fail rather from erosion than from bursting, it is possible that the large values of H, now generally sought, may be ultimately diminished in favor of v_0.

That is, that the guns have a surplus of strength that may profitably be used to favor their endurance under erosion.

2. Page 10.—Taking $J = 1390$ ft.-lbs. for $1^\circ\ C$ we have

$$Q = \frac{J\,H}{2240} = \frac{737 \times 1390}{2240} = 457.3, \text{ or 94 per cent. of 486. foot tons.}$$

3. Page 11. x and σ signify respectively the variable space passed over by the projectile and the variable surface of the burning grains composing the charge. Chap. XI, pp. 2, 3, 4.

4. Page 12.—For recitation at the board the numerical values after Eq. (7) may be represented by symbols.

CHAPTER X.

COMBUSTION IN AIR.

Single Grain.

We know from the experiment in Chap. VIII that in air, gunpowder burns only superficially, so that the burning under these circumstances of a spherical grain may be likened to the exceedingly rapid peeling of an onion.

Considering, for the present, all solid grains to be represented by their equivalent spheres, the radius of any sphere will be equally shortened in equal successive times, but the surface and the volume will vary in a higher ratio to the time.

Accordingly let Fig. 1 represent the central section of a homogeneous spherical grain burning with a uniform velocity of combustion which in the variable time, t, will reduce the original radius R, to r, and the original surface S, to s. Let the time required for the combustion of the entire grain be τ.

Then $S : s :: R^2 : r^2 :: \tau^2 : (\tau - t)^2$, or

$$s = \frac{S}{\tau^2}(\tau - t)^2. \tag{1}$$

By differentiating Eq. (1) with respect to s and t we have

$$\frac{ds}{dt} = \frac{2S}{\tau^2}(t - \tau). \tag{2}$$

It may be shown from Equations (1) and (2) that the curve, Fig. 2, expressing the relation $s = f(t)$, is a parabola referred to a system of rectangular axes; one of which, the axis of times, coincides with the tangent at the vertex of the parabola, and has upon it the origin, O, at a distance from the vertex $= \tau$.

The rate of change of the ordinate of the curve is the same as that of the surface of the burning grain.

The summation of the successive ordinates of the curve, corresponding to any value, t, will be equal to the area $O\,S\,s\,t$; and since the ordinates represent the corresponding successive surfaces, this area will be proportional, either to the mass or volume of the grain which has been burned up to the time t, according as the density of the powder is, or is not considered.

Calling such an area $w = \int s\,dt; \frac{dw}{dt} = s$, or the rate at which the mass of gas is increasing at any instant, or the rate of conversion, is proportional to the corresponding surface.*

The total area $O\,S\,\tau = \frac{S \times \tau}{3}$ will be proportional to the original mass or volume of the grain.

Number of Grains Varied.

Such a relation, once established, would be true for all equal grains composing a charge, and would therefore be true for the whole charge, but the rate of conversion would vary with the size of the charge as shown by curves 1 and 2, Fig. 3. In these, 1, represents such a curve as shown in Fig. 2, for a single grain; and 2, the same for n grains composing a charge.

Size of Grains Varied.

If, in a charge of a given weight composed of spherical grains of a given density, the size only, of the grains be in-

*If the grain be not homogeneous, and burn with a variable velocity the rate of conversion will vary with the product of the surface, s, of the density, δ, and of the velocity of combustion ω, or

$$\frac{dw}{dt} = s \times \delta \times \omega.$$

In this case the curve will no longer be a parabola.

creased, the sum of the granular surfaces, ΣS, will be inversely proportional to the radius of the grain.

For $W = n\, v\, \delta$, and $v = \frac{S r}{3}$ $\therefore$ $W = \frac{n S r \delta}{3}$,

$$\text{or } S = \frac{3\ W}{n \delta} \cdot \frac{1}{r}$$

$$\text{or } \Sigma S = n S = \frac{3\ W}{\delta} \frac{1}{r} = \frac{C}{r} \qquad (3)$$

If we represent by Σ the sum of the initial granular surfaces of a charge, and by σ the sum of the successive granular surfaces of the same charge during its combustion; Fig. 4 may represent by curves 1 and 2, respectively, the relation $\sigma = f\ (t)$ for charges of equal weights composed of grains of different sizes.

Objections to increasing Size of Grain.

The effect of increasing the size of the grain is to make the powder relatively *slow;* or, as it is called with reference to its action in the gun, *mild* or *progressive.* This diminishes the value of $\frac{ds}{dt}$ by increasing the value of τ.

The objection to this will hereafter appear; it will suffice here to say that it may require the gun to be of inconvenient length.

Alternatives.

The following methods have been proposed for regulating the rate of conversion without, in all cases, increasing the value of τ.

Constant Rate.

1. A constant rate would evidently be attained by forming the powder as a prism and confining the burning area to that of its cross section. This result is approached in the Zalinski pneumatic gun, in which compressed air from a large reservoir, expands continuously into the volume behind the projectile. Also in the steam engine.

2. An approximation to a constant rate, with a small value of τ, has been sought by forming the powder into volumes of which two dimensions considerably exceed the third. The French, *Castan*, powder and the American, LX, powder are so formed.

Increasing Rate.

The rate of conversion may be increased by causing the burning surface to increase:

3. By igniting the grain from the interior, and protecting the exterior surface from the flame by forming the grains into hexagonal prisms closely packed together, fig. 5. The perforations are continuous flues, facilitating inflammation.

This is Rodman's powder.

4. By diminishing the density of the grain toward its center, Chap. III., or by facilitating its disruption after ignition.

These cases may be represented by the correspondingly numbered lines on figure 6.

COROLLARY.

Supposing the charge to consist of n equal spherical grains, the proportion of the whole charge that will be burned in the variable time, t, may be determined as follows:

The original volume of the charge is, $V = n\,\frac{4}{3}\,\pi\,R^3$; or, assuming, as before, that the velocity of combustion is unity, $V = n\,\frac{4}{3}\,\pi\,\tau^3$, the unburned volume at the end of the time, t, will be $v_{,} = n\,\frac{4}{3}\,\pi\,(\tau - t)^3 = V\left(1 - \frac{t}{\tau}\right)^3$. Therefore, the volume burned will be,

$$v' = V - v_{,} = V\left[1 - \left(1 - \frac{t}{\tau}\right)^3\right] \qquad (4)$$

Similarly

$$w' = W\left[1 - \left(1 - \frac{t}{\tau}\right)^3\right] \qquad (5)$$

The curve whose ordinates express the relation $w' = f(t)$ will be of the form shown in figure 7, and figure 1, chap. XII.

CHAPTER XI.

COMBUSTION IN A GUN.

PRESSURES.

Comparison to Steam.

For purposes of illustration, the action of gunpowder, when burning in a gun, may be compared to that of steam in the cylinder of a steam engine; and the pressures, p, at different lengths of travel, x, of the projectile in the bore, may be represented by the ordinates of a curve which expresses the relation $p=f(x)$, in the manner used in the indicator diagram of the steam engine.

The operation may be conveniently analyzed by dividing the volume of the bore into two portions, viz.:

1st. That through which the elastic gases are being evolved from the burning powder, called the *combustion* volume.

2d. That through which these gases are expanding under the elastic potential acquired during combustion. This may be called the *expansion* volume.

Thus, the circumstances during the passage of the projectile through the combustion volume correspond to the admission of steam to the cylinder of a steam engine, and the completion of the combustion to the action of the valve which cuts off the supply of steam. The subsequent expansion in both cases is limited by the length of the cylinder.

This important difference exists; that the expansion, which in steam is treated as *adiabatic* (without loss of heat except from external work), and which, therefore, leads to a loss of temperature due to the work done,

is, in the gun, supposed, from Noble and Abel's experiments, to be *isothermal*, and, therefore, under Mariotte's law.

DISCUSSION.

Hypotheses.

In the following general discussion we will, for simplicity, begin by assuming that the projectile starts freely from its seat. We will neglect the variable volume of the liquid residue and that of the powder remaining unburned at any time.

We will also assume that the inflammation is instantaneous. See Chap. VIII.

Notation.

Taking the origin of co-ordinates at the origin of motion; x will represent either the path of the projectile or the volume described by the translation of its maximum cross section.

The volume of the chamber, k, or the initial volume, is composed of two volumes, viz.:

c, the volume actually occupied by the charge of powder including its interstitial spaces.

c', any excess of volume besides that required to hold the charge. Therefore, $k = c + c'$, and, for the *reduced length of chamber*, we have $x_{,} = \frac{k}{\pi r^2}$, in which r is the radius of the bore. We shall first take $c' = O$.

Let w represent the weight of a charge of powder which will be consumed in a time τ, and let w' be the variable weight of w converted into gas at the end of any time t.

Let σ represent the corresponding sum of the burning surfaces as in Chap. X, and Σ the sum of the initial surfaces.

Let p represent the variable intensity of the gaseous pressure per unit of area on the base of the projectile; and

assume any particular value o. p to be uniform throughout the volume occupied by the gases, the density of which is d.

Let p be taken in such units that R, Chap. IX, be equal to unity.

Let W represent the weight of the projectile, the radius of the cross section of which is r; the variable velocity of which, in the bore is v; and, at the muzzle of the gun is V.

Let q and Q represent the quantities of work done upon the projectile to give it the velocities v and V.

FORM OF PRESSURE CURVE.

Upon firing the charge the combustion volume is gradually filled with gas, the density of which will vary directly with w' and inversely with $x+c$; so that we may write

$$p=d=\frac{w'}{x+c}. \tag{1}$$

Differentiating this equation, considering p, w' and x as variables, and dividing through by dx, we have

$$\frac{dp}{dx}=\frac{1}{x+c}\left(\frac{dw'}{dx}-\frac{w'}{x+c}\right). \tag{2}$$

But, since $q=\int p\,dx$,

$$\frac{dp}{dx}=\frac{1}{x+c}\left(\frac{dw'-dq}{dx}\right). \tag{3}$$

Or, dividing both numerator and denominator of the expression in the parenthesis by dt and remembering that $\frac{dw'}{dt}=\sigma$,

$$\frac{dp}{dx}=\frac{1}{(x+c)v}\left(\sigma-\frac{dq}{dt}\right). \tag{4}$$

Similarly,

$$\frac{dp}{dt}=\frac{1}{x+c}\left(\sigma-\frac{dq}{dt}\right). \tag{5}$$

No simple law has yet been discovered connecting σ, x and t, and these equations cannot, therefore, be integrated;

but, remembering that σ is a decreasing function, and q an increasing function of t, a conception may be had of the form of the curve, the ordinates of which express the relation $p=f(x)$.

The inclination to the axis of X will be greatest at first when σ is large, and x and $\frac{dq}{dt}=p\,v$ are small. It will be O, or p will be constant, when the gas is evolved just fast enough to compensate for the increasing volume. From this point the conversion is not rapid enough to keep up the maximum pressure, so that the pressure will fall off until $\sigma=O$, as at a, Fig. 1, from which point the curve will be an hyperbola with the axis of X as an asymptote, since p becomes equal to a constant, w, divided by $x+c$. By the law of continuity, a should be a point of inflexion and a point of tangency between the combustion and the expansion curves.*

The same results will follow when p is taken$=f'(t)$, except that the inclination of the tangent to the curve will vary more gradually.

I. PRESSURES DURING COMBUSTION.

Effect of Size of Grain.

Although we do not know the law which, in the gun, connects $\sigma=f(t)$ and $x=\varphi(t)$; experiments with Noble's and Ricq's apparatus demonstrate that, when nearly equal charges of powder, a and b, in which $\Sigma_a > \Sigma_b$, are fired; for small

* The parenthetical expression refers to the relation between the potential energy of the unburned powder and the kinetic energy of the projectile; for Π, the potential energy of the charge must always be equal to $\pi=f(\sigma)$, that residing in the unburned charge; $+q=f'(v)$, the work already done at any instant; $+e=f''(p)$, the work which the elastic potential of the gases is capable of doing; or $\Pi=\pi+q+e$.

values of x and t, $\varphi(t)$ changes but slowly for considerable variations in $f(t)$.

The small change in the form of $\varphi(t)$ for a given change in the form of $f(t)$ is probably due, on one hand, to the relative constancy of the initial resistances to motion, or the molecular work (Michie, Art. 25), and on the other hand, to the great changes in $f(t)$ resulting from the cumulative influence upon the velocity of combustion of high pressures when σ is large. Chapters VIII., X.

If, therefore, during the critical period of combustion, we assume that $\varphi(t)$ is nearly constant for all sizes of grain; v, and therefore q, may be taken as independent of σ. Consequently, Eq. 4 shows that during combustion, the inclination of pressure curves corresponding to different values of Σ will be an increasing function of σ; or for equal charges, the smaller the grain, the steeper the curve. Similar reasoning shows that it will also be higher.

Experimental Illustration.

This may be represented by fig. 2, derived from Noble's experiments, in which a represents, by its ordinates, the successive surfaces of a charge of fine-grained powder burned in the air, its initial portion only being represented. The curve a' shows the effect produced upon its burning by confinement in the gun. Let b and b' similarly represent the varying surfaces of an equal charge of coarse-grained powder. Let α and β be corresponding curves representing, by their ordinates, the velocities acquired by the projectile at any time, t. For any time t, the area under a' or $b' = \int \sigma\, dt = w'$; and similarly the area under α or $\beta = \int v dt = x + c$.

Under the circumstances named, although $\Sigma_a = 3\, \Sigma_b$, the curves α and β were nearly coincident in their initial portions. These, which we shall term the v curves and the σ curves, have the axis of time in common.

At any time t, which is less than τ_b, the time required for the combustion of the powder b, the ratio, $\frac{w'}{x+c}=p$, is less for the coarse grained powder than for the fine. At τ_a and τ_b expansion begins; at τ_b the pressures from the two powders will be nearly equal to each other, since the same weight of powder in each case is burned in nearly equal volumes.

Similar effects would follow the changes in σ, indicated in Chap. X., from whatever cause the rate of change of σ was varied.

The best results would be attained when both the σ and v curves coincided in such a line as k, fig. 2, since we would then have the constant pressure sought for in the ideal gun.

The effect upon pressures of varying the size of grain, or the rate of burning in charges of equal weight, would be represented by the curves a, b, k, in fig. 3, in which the notation of fig. 2 is preserved.

Additional Illustration.

The principle is illustrated in figure 10, in which curves a and b (which to avoid confusing the drawing are omitted), may be imagined to result from Equation (5), Chapter X, and a' and b' to be constructed from a and b, in the manner indicated in figure 2. The curves α and β express the relation $x=f'(t)$ as in figure 2 they expressed $v=f(t)$.

The ordinate $t\,y'$ represents the proportionate part w', of the original weight of the charge, w_a (represented by $O\,w$), that has been burned in the time $O\,t$; and $t\,z'$ the volume, x, through which the projectile has moved in the same time. Similarly for the curves b' and β.

The line $K\,k$ is parallel to $O\,T$, and at a distance from it, on the scale of the axis X, that is proportional to the volume of the chamber c. Then $\frac{t\,y'}{t\,z'+t\,k}=\frac{t\,y'}{k\,z'}=\frac{w_a'}{x+c}=p_a$: and similarly for the curves b' and β.

In figure 2 it is not possible to represent the constant of integration, c.

2. PRESSURES DURING EXPANSION.

The locus of the pressures at the end of the several combustion periods is the hyperbola H, fig. 3, the intersection of which, with the axis of P, is at a height Op, corresponding to 43 tons per square inch, and the parameter of which depends upon the weight of the charge. Thus, the hyperbola H′ would be the locus for a charge greater than w, and its vertical asymptote would be at a distance from $O = -\overline{x_{,}} = \overline{c}$. See figure 5.

Remarks.

1. The relative constancy of the v curves, in spite of considerable variations in p, may be explained by considering the gunpowder as a reservoir of potential energy. In this case $w' = f\,(q) = f\left(\frac{Wv^2}{2g}\right)$, so that $v = f'(\sqrt{w'})$, while we have seen that $p = f''(w')$.

2. It is probable that the work done during combustion is proportional to the weight of the charge.

ADAPTATION OF POWDER TO GUNS.

The preceding discussion shows that if the size of the grain remains constant, the pressure increases with the size of the charge.

In order to compensate for this, General Rodman proposed to increase the size of the grain as the caliber of cannon of the same class increased.

This is the basis of the modern practice requiring special powders for special guns.

PASSIVE RESISTANCES.

Returning to fig. 1, we see that the area limited by the pressure curve, the axis of X, and the muzzle ordinate at m, will represent the work done by the powder under the circumstances named.

The greater portion of this work appears in the kinetic energy of translation of the projectile; and, for simplicity in the following discussions, all the work will be considered to have been so transformed.

The difference between the work of the pressures and the energy of translation, which, in practice, may amount to about ten per cent. of the former, is due to the work of the *passive resistances*, including the waste.

Resistances.

The work of these resistances is equal to the sum of the following quantities of work:

1. That done in giving rotation to the projectiles in rifled guns and in causing recoil.

2. That done in permanently deforming the projectile and the gun. The former is practically confined to rifled projectiles and is greatest in breech loaders.

3. That done in overcoming the friction of the projectile, and in distributing the charge in the form of gas throughout the bore.

Waste.

4. The waste is due to the absorption of heat by the walls of the gun, and to the escape of the gases past the projectile and through the vent.

Graphical Representation.

So that if we take a pressure curve, as in fig. 4, and draw a line $R\,R'$, so that the area under it, corresponding to any length of path x, shall represent the work of the passive resistances during the motion of the projectile over that path,

the segment $r p$, of any ordinate $x p$, will represent that portion of the total pressure which gives acceleration to the projectile and imparts to it kinetic energy proportional to the area included between the two curves and any limiting ordinate.

Remarks.

Band.

In breech-loading guns the initial resistance is considerable until the rotating band has entered the rifling; thereafter the resistance diminishes rapidly.

Example: In a 9.5 in. B. L. R. a charge of over 4 pounds of powder failed to move the projectile; but a slight increase in the charge gave it considerable velocity.

Waste.

The loss of energy from absorption of heat by the gun increases with the slowness of the powder; since with slow powder the velocity of the projectile is less at the moment of maximum temperature or pressure.

It varies inversely with the calibre, since with charges of the same proportions the weight of the charge varies with r^3, while the surface varies nearly with r^2.

The escape through the vent probably increases with the slowness of the powder.

Instantaneous Pressure.

Variability.

From the discussion, Chap. VII., it is evident that the pressure at any instant throughout the volume in rear of the projectile is not uniform, but increases toward the bottom of the bore, as represented by the variable line $p_0 p$. Fig. 4.

Neglecting the passive resistances, the intensity of the variable pressure at the bottom of the bore, generally known as p_0, can, by analysis, be shown to be very nearly equal to $p\left(1+\frac{w}{2W}\right)$.

Considering the passive resistance p_0 is taken $=p\left(1+\frac{3\,w}{2\,W}\right)$.

Supposition.

For an elementary discussion, like the following, such differences in the instantaneous pressure, and the effect of the passive resistances, once understood, may be neglected. So that the pressure at any instant upon the bottom of the bore will be assumed to be that exerted at the same instant on the base of the projectile, and all of it is supposed to be utilized in giving motion to the projectile.

The difference, $p\,\pi\,r^2 - \frac{W}{g}\alpha$, evidently tends to compress the projectile in the direction of motion, and its effect will be most felt at the base of the column of metal moved.

Except in the next discussion, in which actual free volumes are considered, the origin of co-ordinates is always taken at the origin of motion; viz., at that section of the bore occupied by the base of the projectile when the gun is fired.

WORK OF FIRED GUNPOWDER.

It is not necessary in practice to separate the work of combustion from that of expansion; but the total work which may be expected from a given charge of powder may be determined in the following manner.

Total Potential Work.

In fig. 3 the area included between the ordinate $O\,P$, the axis of X, and the hyperbola H at infinity, will represent the total amount of work which this charge could perform. Calling this Ω, we see, from Chapter IX., that expressing, as is usually done, work in foot-tons, and w in pounds,

$$\Omega = 486\,w.$$

Actual Potential Work.

If, instead of expanding the powder gases to infinity, we limit the useful work of the expansion by placing the muzzle

as at m, we shall have an area which will represent the maximum potential work under the conditions existing in the gun.

This, which in practice is not much over $\frac{\Omega}{6}$, we will call Q.

Effective Work.

Now, if we fire the charge w in a gun, we shall give a certain velocity V to a projectile W. Calling the amount of kinetic energy so realized E, we have

$$E = \frac{W V_2}{2\,g \times 2240}.$$

The ratio, $\frac{E}{Q} = F$, is called the *factor of effect.* It is used, as hereafter explained, in anticipating the results of certain changes in the piece and ammunition.

Fig. 3, shows by the triangular areas above the curves, a, b, k, the principal reason why $F < 1$.

F is further diminished by the passive resistances.

MEASURE OF Q.

To deduce a formula for the potential work of the powder gases when expanded in a gun of a definite length, or the equivalent of the area Q, we use the general equation

$$Q = \int I\,ds = \int \frac{p\,\pi\,d^2}{4}\,dx. \qquad (6)$$

Substituting the value of p, from Eq. 3, Chap. IX, and for brevity replacing 0.0025571 by a, and 0.048 by b, we have, since all linear dimensions are given in inches,

$$Q'' \text{ (inch-tons)} = \frac{\pi\,d^2}{4} \int \frac{dx}{a\,\frac{\pi\,d^2}{4\,w}\,x - b}.$$

$$= \frac{w}{a} \int \frac{dx}{x - \frac{4\,b}{\pi\,a}\,\frac{w}{d^2}}.$$

$$Q'' \text{(inch-tons)} = \frac{w}{a} \int \frac{dx}{x - 23.9 \frac{w}{d^2}}.$$

Taking, *in this case*, the origin of co-ordinates at the bottom of the bore, integrating between $x_{\prime}$, and x', corresponding to $O'\,O$ and $O'\,m$, fig. 1; substituting the value of a, and remembering that $Q = \frac{Q''}{12}$, we have,

$$Q = 75.04\,w, \log \frac{x' - 23.9 \frac{w}{d^2}}{x_{\prime} - 23.9 \frac{w}{d^2}}. \qquad (7)$$

Volumes of Expansion.

The subtractive term above, appears from the form of the equation and can be shown to be the *reduced length* of the residue,* or

$$\rho = 23.9 \frac{w}{d^2}; \qquad (8)$$

so that, if $x'' = x' - \rho$ and $x_{\prime\prime} = x_{\prime} - \rho$; $\frac{x''}{x_{\prime\prime}} = n =$ *number of volumes of expansion*, an important characteristic of a gun. It is convenient to remember that $\rho =$ about $\frac{6}{10}$ the length of the cartridge, if its diameter $= d$ above.

Equation 7 may, therefore, be written under a form convenient for general discussion.

$$Q = 75.04\,w.\log n. \qquad (9)$$

The calculus shows that the curve, the area between which, the asymptote, and two ordinates is proportional to the logarithm of the extreme abscissa, is an hyperbola, which is the result reached, page 4.

* For the smokeless powder referred to in Chapter III, ρ will be practically $= O$, n will diminish, and so will p_0 for equal values of w. The effect, as hereafter discussed under *Air spacing*, will be to make the powder more progressive; unless the powder belongs to the class of high explosives and its explosion is of a high order.

Consequently if, as in fig. 5, we assume axes of P and N, we may construct various hyperbolas depending upon the value of w, such that the areas under them will give the corresponding values of Q.

VARIATIONS.

1. In Weight of Charge.

The hyperbolas intersecting at the point P, Fig. 5, and Equation 7, show the effect upon $x_{\prime}$; x'; $x_{\prime\prime}$; x''; ρ; n; Q; and p, resulting from variations in w.

Inspection of the figure shows that an increase of w to $\overline{w} = \frac{3}{2}\ w$, decreases n from about 12 to 7, or about $\frac{1}{2}$; the total length of the bore $x' = \overline{x'}$ remaining constant, as in a muzzle loader.

Owing to the effect on n of variations in w, Q will not increase in direct proportion to w. But, in a given gun, log n diminishes so much less rapidly than w increases, that we may for simplicity assume that n is constant; and, taking a constant ratio between n and x, we may replace the axis of N by that of X, and complete the figure by drawing upon it combustion curves, as in Fig. 6, so that for the same weight of charge the areas under the combustion curves are equal, without regard to the size of grain. See Remark 2, page 7.

2. In Size of Grain.

The curves a' a'', refer to different weights of the same kind of fine grained powder, which are supposed to be burned through at about the same point of the bore. Curve a'', refers to a weight $\overline{w}$ of coarse grained powder.

Consideration of Fig. 6, shows how, by increasing both the weight of the charge and its inherent progressiveness, we may obtain a pressure curve, the work area under which may equal and even exceed that due to the fine grained powder, without incurring the risk attending the high pressures to which it gives rise.

In other words, we approach the conditions required in the ideal gun, by effectively diminishing the value of n.

For, comparing curves a'' and b'', figure 6, it is evident that the latter is the more progressive, or more nearly parallel to the axis of X, and that this results from expansion beginning further down the bore. Neglecting the areas under the combustion curves, the inclination of which in the diagrams is purposely exaggerated, the effect is practically the same as if the powder had been instantaneously burned in a volume, $c+c'$, greater than c, page 2, by the volume through which the projectile had moved before expansion began. We would then have $n' = \frac{x' - \rho}{x_{,} + c' - \rho} < n$. See *Air Spacing.*

3. In Length of Bore.

While, in all cases in practice, an increase of $O\ m$ increases Q and F, the proportionate advantage from the increase of $O\ m$ increases with the progressiveness of the powder.

The limit of useful increase of $O\ m$ is determined by the intersection of the line of pressures with that of resistances. Fig. 4.*

In many works on Gunnery the importance of $O\ m$, or the path traversed by the base of the projectile in the bore of the gun, is overlooked; or it is left to be inferred from the total length of the bore.

In the more recent and advanced works it has a specific symbol u by which it will be hereafter recognized.

AIR SPACING.

Variations in Δ.

We have so far assumed the powder to be fired in its own volume. If we assign to the charge a volume greater than that required to contain it by the volume c', page 2, the

*This applies to small arms. For heavy cannon the increased weight of piece resulting from the prolongation of the bore, can generally be used to better advantage elsewhere.

value of $\triangle$ will diminish. Eq. 8, Chap. IX., shows that the initial pressure will also diminish, and so, under given conditions will Q. In such a case, Eq. 1, will take the form $p = \frac{w'}{x+c+c'}$, and the curve expressing the relation $p = f(x)$ will be such as shown in fig. 7.

Curve 1 expresses, by its ordinates, the varying pressure when $\triangle$ is large, and curve 2 the same function when $\triangle$ is small, the weight of the charge being the same in both cases.†

It will be observed that the effect of air spacing is principally felt when c' is large, compared with x, that is, when σ and p are relatively large.

Also, since by differentiating Eq. 1, regarding w' as a constant and $x+k=\lambda$, we have $\frac{dp}{d\lambda} = -\frac{w'}{\lambda^2}$; the inclination of curve 2, will be less than that of curve 1. The values of Q and E will therefore both be smaller for curve 2.

Increase of Charge.

If u is fixed, this effect is compensated for, as before, by increasing w. This produces the effect shown in the dotted curve, 3, fig. 7.

APPLICATIONS.

Air spacing is principally applied to muzzle-loading cannon on account of the necessary limit to their length imposed by the requirements of loading. It results spontaneously from

† To familiarize himself with the principles involved, it is recommended that the student construct curves 1, 2, 3, as follows:

1. Assume a maximum pressure of say 24 units and $c=1$; $c'=o$; $w=1$; then for $x=1$, $p=12$; for $x=2$, $p=\frac{24}{3}=8$, and so on.

2. Take $c'=1$ ∴, $k=2$; $w=1$, as above; then for $x=o$, $p=12$; for $x=2$, $p=6$, and so on.

3. Take $c'=1$ and $w=1.5$; then for $x=o$, $p=\frac{36}{2}=18$; for $x=1$, $p=12$, as in No. 1; for $x=2$, $p=9$ (greater than No. 1). No. 3 will continue above No. 1 to ∞. The powder is more progressive, and n is decreased.

the ease with which their projectiles, particularly those which are spherical, take up their initial motion. It was probably to diminish this that the *sabot*, a cylindrical block fastened to the rear of spherical projectiles, was formerly employed, although other reasons are generally assigned for its use.

Until about 1880, when the English government began to adopt exclusively the breech-loading principle for heavy cannon, air spacing was largely employed for *short*, *thick muzzle*-loading cannon, firing *large* charges of *quick*-burning powder. It was secured by making the diameter of the chamber greater than that of the bore. This was objectionable in sponging and it weakened the gun.

It was made constant by providing the projectile with stops, which held it at an invariable distance from the bottom of the bore. But this, although beneficial in preventing the great variations in pressure and velocity which, from careless loading, are apt to occur in ordinary muzzle-loading guns, increases the length of the gun in the region of its greatest diameter.

It is still employed to some extent in breech-loaders, but is yielding in importance to the means now employed for regulating the combustion of gunpowder.

EFFECT OF THE ROTATING DEVICE.

If the initial motion of the projectile be restrained by the compression of the rotating device, p will have an initial value at least equal to the resistance offered to deformation, and, since the powder is burned under higher and more constant pressures, n will be greater and more constant than when the projectile is free to move.

ADVANTAGES OF BREECH-LOADING GUNS.

General Advantages.

1. The simplicity and exactness of the method by which the value of n may be regulated.

2. The ease with which u may be increased without interfering with the operations of loading.

3. As will be hereafter shown, the compressible projectiles used in breech-loaders are more accurate than the loosely fitting projectiles employed in muzzle-loading guns.

In spite of the greater simplicity of construction and of operation of muzzle loaders, these advantages have compelled the adoption of breech-loading cannon.

Tactical Advantages.

The tactical advantages of breech-loaders are also greater. Among these are—

1. Greater facility in securing cover for the piece behind defenses, and for the gunners behind the piece.

2. Less danger and difficulty in loading, since but one charge can be inserted at a time, and the operation of sponging is less important than with muzzle loaders.

3. Greater facility in examining and caring for the bore.

4. Greater facility in adjusting the charge or fuze after loading.

5. Immovability of the projectile in marching. This permits batteries to come into action rapidly, when under fire.

6. The rapidity of fire is increased for large pieces.

The price paid for these advantages is the difficulty of getting officers and men capable of working the cannon with sufficient care.

OBJECTIONS TO INCREASING WEIGHT OF POWDER.

The preceding discussions show that we compensate for small values of n by corresponding increases in the value of w.

The objections to this are as follows.

1. We increase the waste of powder, as the steam engineer does that of his coal by failing to work his steam expansively. This may be of importance when storage capacity is limited, as on ships and in the field, and it tends to diminish the value of η, hereafter explained.

2. The work done in distributing the gas throughout the bore increases with the weight of the charge and the length of u. Some charges now weigh half as much as the projectile.

3. We increase the tension of the gases within the gun at the instant of the departure of the projectile. This tends to accelerate the recoil and to perturb the flight of the projectile; since, owing to their small mass, the gases leave the gun with a higher velocity than that of the projectile.

4. Considering the gun to consist of a number of staves, like those of a barrel, the moment of the pressures about the breach is increased on account of their greater level arm.

5. Variations in p and n, due to accidental variations in the velocity of combustion, may endanger the safety of the piece or affect its accuracy.

IGNITION AND INFLAMMATION IN GUNS.

The importance of these phenomena has largely decreased with the adoption of the breech-loading principle.

When muzzle-loading cannon, firing free projectiles with charges of fine grained angular powder were generally used, the ratio $\frac{\text{time of inflammation}}{\text{total time of conversion}}$ was large.

To reduce this as much as possible, so as to increase the value of n, the charge was ignited near its middle. It was found that ignition in rear tended to waste energy by moving the forward portions of the unburned charge; while that in front reduced the velocity by the premature movement of the projectile.

With breech-loaders the charge is always inflamed before the projectile has moved.

The shape and size of the grain and the use of a special priming of quick powder placed near the vent, reduce the value of the ratio of times above referred to so much, that the position of the vent is determined by other considerations.

MEASURES DEPENDING UPON THE MUZZLE ENERGY OF THE PROJECTILE.

1. MILDNESS OR PROGRESSIVENESS.

$P = p \pi r^2$ is called the *variable total pressure.*

In this, and in subsequent similar expressions, r is expressed in the same linear units as those of the area upon which p is estimated.

When p has its maximum value, as indicated by the pressure gauge, and taken $= p_0$; the *maximum total pressure* is $P' = x' p'$, Fig. 8.

The area under the pressure curve divided by u gives the *mean total pressure* $P_, = \dfrac{E}{u} = m p_,$.

The *effective length* or $u' = \dfrac{E}{P'} = O x''$.

Therefore, if we represent the following ratio by μ, we have, since $E = P' u' = P_, u$,

$$\mu = \frac{P_,}{P'} = \frac{u'}{u} = \frac{E}{P' u} = \frac{W V^2}{2 g p_0 \pi r^2 u}. \tag{10}$$

in which p_0 and W are expressed in the same units, and g, u and V in the same units.

This coefficient μ, which measures the ratio of the area under the curve to that of the circumscribed rectangle, may be taken as the measure of the *mildness* or *progressiveness* of the action of the gunpowder under the circumstances of any particular case.

The limit of the ratio for all ordinary powders is evidently unity, and would be reached only in the ideal gun.

2. ECONOMY.

$\eta = \dfrac{E}{w}$ is a valuable datum for comparing the *economy* or *efficiency* of various powders.

It appears from figure 3 that the greater is the efficiency, the greater is the maximum pressure; or that the violence of gunpowder increases with η. Also from figure 2, that the smaller the value of τ, or the *quicker* is the gunpowder in a given gun, the larger will be the value of η.

It would be more consistent to follow the method adopted for μ, the value of which is independent of any particular metrical system; but in order to avoid dealing with large numbers, and because of the general use of the term "foot-tons of energy per pound of powder," we shall write

$$\eta = \frac{W\,V^2}{2\,g\,2240\,w}. \tag{11}$$

3. GENERAL COEFFICIENT.

The preceding discussions show that all expedients intended to increase the progressiveness of powder decrease the muzzle energy resulting from the conversion of a given weight of the explosive; or decrease η.

Thus, when, as in figure 3, we increase the size of the grain, or vary its form, composition or density so as to increase τ; or when, as in figure 7, we diminish p_0 by decreasing Δ, we decrease the Factor of Effect; and therefore, in order to obtain the muzzle energy required, we must increase the weight of the charge, as shown in figure 6.

In order to compare the performance of different powders fired under the same conditions, or of the same powder fired under different conditions, it is proposed to use a *general coefficient*, known as χ: which, since μ and η are both desirable, will be proportional to their product; and which, since they tend to vary inversely with each other, will have an approximately constant value.

This relation may be expressed by writing

$$\chi = \mu\,\eta = \frac{W^2\;V^4}{g^2\,2240\,p_0\,\pi\,d^2\,u\,w}. \tag{12}$$

It will be hereafter more fully discussed.

4. STRENGTH OF GUN CONSTRUCTION.

$h = \frac{E}{W'}$, in which W' is the weight of the gun (in the same units in which W, the weight of the projectile, is expressed) measures the height through which the gun would have to fall in vacuo to acquire energy equal to that residing in the projectile at the muzzle of the piece.

Thus the old 10 in. S. B. C. I. gun had a value of $h = 300$ ft. When strengthened by a rifled steel tube that reduces its caliber to 8 in. we have the "converted" 8 in. Rifle, for which h is about 350 ft.

For the new 8 in. B. L. R. *Steel*, h is nearly 500 ft.

5. FACTOR OF EFFECT.

The meaning and derivation of this have already been explained.

Use.

It is used for anticipating the effect of changes in the interior form of a piece, or in its ammunition, upon the muzzle energy of the projectile.

It differs from the use of χ and its factors in taking no heed of the maximum pressure involved in the result.

Conditions.

It is necessarily assumed to remain constant during the variations, the effect of which is sought; and consequently, the conditions under which it is employed should be as nearly alike as circumstances will allow.

These conditions relate to the type of gun, of powder, and of projectile employed

Owing to the greater constancy of Δ, and to the high initial pressure required to move the projectile from its seat, it is better adapted for use with breech-loading than with muzzle-loading cannon.

The factor of effect increases with the size of the gun, as seen by the following table, giving its approximate value, in certain individual cases. So much depends upon the kind of powder used that only the most general conclusion can be drawn:

	Factor of Effect Per Cent.	
	M. L. R.	B. L. R.
Muskets,	30	60
Mountain Guns,	50	80–85
Field "	65	
Medium "	75	
Heavy "	85	

APPLICATIONS.

1. Variations in w and $\triangle$.

1. Suppose it be desired to estimate the change in the muzzle velocity to be expected in a given gun from certain charges in w or $\triangle$.

The values of E and Q, under known conditions, have been determined; and therefore,

$$F=\frac{E}{Q} \text{ is known.}$$

Determining, from Eq. (7), Q' under the new conditions, we have $E'=F\,Q'$. On firing we should find approximately—

$$V=\sqrt{\frac{2\,E'\,g\times 2240}{W}}. \qquad (13)$$

2. Untried Gun.

2. Suppose that we desire to estimate the muzzle velocity of a given projectile to be fired from a new and untried gun, of which we have only the drawings.

We select the record of some gun of as nearly the same type as possible, assume $F=F'$, and proceed as before.

3. Dimensions of Guns.

3. The inverse problem may also arise; viz., to determine the interior dimensions of a gun of any required power.

Eq. (7) may be placed under the form—

$$Q=75.04\,w\log\frac{x_{,}+u-23.9\,\frac{w}{d^2}}{x_{,}-23.9,\,\frac{w}{d^2}} \qquad (14)$$

The calibre and the density of loading, Δ, are always assumed, the former depending upon the service, and the latter upon the strength of the gun; therefore, since

$$\Delta=\frac{27.68\,w}{v=\pi\frac{d^2}{4}x_{,},}$$

$$x_{,}=\frac{110.72\,w}{\pi\,d^2\,\Delta}. \qquad (15)$$

We have, therefore, two problems:

1. Assuming u to find the necessary values of w and $x_{,}$.
2. Assuming w to find u.

The difficulty of simplifying an expression of the form of Eq. (14) requires these solutions to be made by trial.

In the first case, taking F from some similar gun, we have $Q'=\frac{E}{F}$. Then, assuming successive values of w, we insert them and the corresponding values of $x_{,}$, determined from Eq. (15) into Eq. (14) until a suitable value of Q' is obtained.

In the second case, we proceed as before, substituting successive values of u.

The initial approximations to the value of u will be facilitated by reference to the value of n, usual in guns of the type proposed.

EXAMPLES FOR PRACTICE.

1. The 3.20 B. L. W. I. Chambered Rifle, in which $x_{,}=$ 12 in.; $u=56.1$ in.; 3 lbs. I. K. powder $=w$, gave to a 12 lb. projectile, $V=1548\,f.\,s.$

Determine its factor of effect;

$$F = \frac{E}{Q} = \frac{199.3}{244.9} = 0.8143.$$

2. Estimate V for a 13 lb. projectile to be fired from the new 3.20 B. L. Steel Chambered Rifle with a charge of 3.75 lbs. I. K. powder.

From the drawings we find that the volume of the chamber, which is a truncated ellipsoid terminated by various cylindrical and conical surfaces, when diminished by the volume of that portion of the projectile which lies within it, $=123.157$ cubic in. Similarly, the length of the rifled portion of the bore, when increased by that of the projectile lying within the chamber, $= 73.24$ in. Therefore, $x_{,} = 15.31$ in., $x' = 88.55$ in; and in the case supposed $n =$ about 12 as before.

We find $Q' = 303.9$ ft.-tons, $E = 247.4$ ft.-tons, and $V = 1657\ f.s.$

By experiment, $V = 1652\ f.s.$

The difference falls within that usually found when all the conditions are as nearly constant as possible.

DISCUSSION OF THE COEFFICIENT χ.

A study of many records shows that when the conditions of loading approach those sanctioned by experience, the value of χ expressed in the units assumed varies from about 24.0, when the powder is so quick* with relation to the gun in which it is to be used, that the weight of the powder is only about one quarter the weight of the projectile; to about 35.0, when the powder is so slow that w may be safely increased to about one half the weight of the projectile.

* The remark on page 6 shows that the same powder may be quick in some guns and slow in others. Thus the powder suitable for a field piece would be too slow for a musket, and too quick for a siege piece; and in two siege pieces of the same caliber, this powder would be quicker in a gun, than in a howitzer or mortar.

It is rare to find a value of χ with any form of black powder greater than 28.0; while with cocoa powder it often approaches 35.0. Furthermore, these values approach constancy, as will be seen from the table on following page.

The approximate constancy of χ enables conclusions to be drawn from otherwise perplexing data. Thus, in the 7.0 in. Howitzer in Table I., it might be difficult to decide which was the better powder, L. X. B. or L. K. K.; but the values of χ show that the former is to be preferred.

If we assume axes of η and μ as in fig. 9, we may refer to them as asymptotes certain hyperbolas which will limit all reciprocal values of η and μ for each kind of powder. Thus powder b, for any assigned value of η or μ will give a higher value of μ or η than powder a.

Of the two principal ballistic data, viz.: V and p_0, the former is much more easily and certainly obtained than the latter. Indeed, unless the pressure gauges are carefully prepared by experienced observers, their indications are frequently inconsistent.

Therefore, a known value of χ may be employed to check the records of the pressure gauges, or to replace them; for, having observed V, we have

$$p_0 = \frac{W^2\ V^4\ w}{\chi\ 2240\ g^2\ \pi\ d^2\ u}. \tag{16}$$

Also, having ascertained by experiment the value of μ or of η for any powder of which we know the coefficient χ, we may estimate the weight of the charge of that powder required to give to a projectile of any desired weight the maximum velocity which the limit of pressure imposed by the construction of the piece permits, or

$$w = \chi\ \frac{2240\ g^2\ \pi\ p_0\ d^2\ u}{W^2\ V^4}. \tag{17}$$

The density of loading will be regulated by this value of p_0.

As will be shown in Chapter XII., the value of χ, for the

TABLE I.

GUN.				POWDER.		Proj.						
Kind.	d in.	u Inches	u Cal.	Kind.	w	W	V	p_0	μ	η	χ	REMARKS.
M. L......	3.17	74.5	23	I. B.	5.6	10.5	1967	26261	0.49	50.0	24.4	Sphero Hex'l powder.
B. L......	3.20	56.6	18	I. K. B......	3.0	13.0	1510	28700	0.42	68.5	28.9	Converted gun.
B. L......	3.20	73.2	23	I. K. B......	3.5	13.0	1663	30590	0.37	71.2	26.4	Steel gun.
B. L......	3.20	73.2	23	I. K. D.	3.5	13.0	1630	29000	0.38	68.4	25.7	
B. L	3.20	73.2	23	I. K. D.	3.5	15.0	1530	30057	0.36	69.5	23.2	
B. L......	3.20	73.2	23	L. X..	3.5	13.0	1649	31000	0.36	76.0	25.2	Flat powder.
B. L......	5.0	119.8	24	Sph. Hex'l..	12.5	43.0	1829	34960	0.32	79.8	25.9	
B. L......	5.0	119.8	24	Black Prism	15.0	45.0	1960	39400	0.35	79.9	27.8	
B. L......	5.0	119.8	24	O. I. N.....	14.0	45.0	1852	33075	0.37	74.0	28.3	Black Prism.
B. L... ..	7.0	73.8	10.5	L. X. B.....	6.0	10.5	840	15000	0.32	85.7	27.7	
B. L......	7.0	73.8	10.5	L. X. B.....	9.25	10.5	1093	28400	0.29	94.0	27.3	
B. L......	7.0	73.8	10.5	L. K. K.....	9.0	10.5	1027	23900	0.30	85.3	25.9	
B. L......	7.0	73.8	10.5	L. K. K.....	9.75	10.5	1085	28200	0.29	87.9	25.3	
M. L......	8.0	96.0	12.0	E. V. J......	35	180.0	1339	27125	0.45	63.9	29.3	Hexagonal.
B. L......	8.0	96.5	12.3	E. V. F.....	45	183.0	1488	32650	0.47	62.5	29.2	
B. L......	8.0	195.75	24.5	P. N.*......	105	289	1825	33662	0.54	63.5	34.4	Reported "good."
B. L......	8.0	195.75	24.5	Q. W. A.*...	105	289	1881	36500	0.53	67.5	35.8	Reported "good."
B. L......	8.0	195.75	24.5	P. A.*......	100	289	1787	33844	0.51	64.0	33.3	Rep. "entirely too quick"
B. L......	8.0	195.75	24.5	Q. E.*......	110	301	1733	28643	0.57	58.0	33.2	Rep. "entirely too slow."

* Cocoa Powder.

same powder fired in the same gun, will increase as the ratio $\frac{W}{w}$ increases; but a nearly constant expression called Π will result from multiplying the computed value of χ by $\left(\frac{w}{W}\right)^{\frac{1}{4}}$ or

$$\Pi = \chi\left(\frac{w}{W}\right)^{\frac{1}{4}} = \frac{W^{\frac{3}{4}}\ V^4}{p_0\ d^2\ u\ w^{\frac{3}{4}}}\ G \qquad (18)$$

log $G = \overline{6}.2161$.

By comparing different values of Π, we may compare the performance of different powders in the same gun, even when fired with different charges, provided the weight of the projectile is constant, which is usually the case.*

THEORY OF COCOA POWDER.

The facts that, although almost all possible combinations of the ingredients of ordinary black powder have at different times been tried without decided advantage over those generally adopted; and that, as we have seen, the changes in manufacture which have had as their object an increase in μ have necessarily correspondingly reduced η, indicate that the difference in the action on the brown powder is due to some marked difference in the chemical composition of its charcoal.

This was for some time a secret for which it is said that the British Government paid a large sum. Without requiring such payment, the Messrs. Du Pont, of Wilmington, Delaware, the manufacturers of a powder that has shown itself to be nearly equal to that made abroad, have furnished the basis of the following theory as to its peculiar behavior.

* So many of the records require the value of u to be assumed from the proportions of the gun, and so doubtful is the accuracy of many of the pressures recorded before the theory of the pressure gauge was well understood, that the constancy of χ for a given gun and powder is best seen from an analytical discussion in Chapter XII. The table is given rather for illustration than for proof.

The charcoal made by superheated steam contains a large proportion of free hydrogen and much more in relatively unstable combination.

The *carbo-hydrates*, as are termed the resin, gum and sugar added during manufacture, are also magazines of hydrogen.

The effect upon the velocity of combustion, due to the presence of the gum and to the high density of the powder, and possibly also some of the phenomena of dissociation under high pressures, prevent the sudden liberation of the hydrogen and its combustion when x, Eq. (1), is small.

The hydrogen combines as the pressure wanes, and tends to sustain the pressure and to increase both η and μ, whereas in black powder they must vary inversely.

The water formed serves to precipitate the smoke, the solid particles of which are entangled in a condensed spray of liquid gum following the projectile.

To this may be added, as a theory more generally accepted, that the large proportion of nitre tends to prevent the formation of CO, thus reducing the volume of the gases first formed, and diminishing the violence of their action, or increasing μ. On the other hand, the excess of nitre may tend to increase η on account of the more perfect combustion of the charcoal and the high calorific value of the hydrogen which it contains.

The precautions usual in manufacture are taken to affect the size, shape and density of the grain and the amount of moisture it contains, so as to increase its progressiveness.

That these precautions alone do not account for its peculiarities appears from the fact, that while a prism of black powder burns in the open air in $1\frac{1}{2}$ seconds, and a similar prism of brown powder burns in 10 seconds, equal charges of the brown powder give equal or higher muzzle energies than the black powder without exceeding their maximum pressures.

English Experiments of 1890.

These furnish the following data from which the effects of the composition of the powder may be observed.

Solid residue per cent:

	Black.	Brown.
$K_2\,CO_3$	66	64
$K\,H\,CO_3$	—	14
S	9	—
$K_2\,S$	15	—
$K_2\,S\,O_4$	10	22
	100	100

Permanent gases per cent of volume:

	Black.	Brown.
CO_2	47	51
CO	16	3
$H_2\,S$	3	—
H and $C\,H_4$	4	4
N	30	42
	100	100
v_0 of permanent gases above	278	198
Heat units per kilogramme	721	837
$f = v_0\,H.$ (Chap. IX, page 7) ratio	1.0	0.83
Specific volumes of water vapor	41	122

CHAPTER XII.

SARRAU'S FORMULÆ FOR INTERIOR BALLISTICS.

The deductions of M. Emil Sarrau permit a very accurate solution of many important problems affecting the interior form and the method of loading cannon.

By methods which are too elaborate for present instruction, Sarrau deduces four general formulæ for pressures and velocities.

Notation.

The units in the following notation are based upon those adopted in the publications of the Ordnance Department, U. S. A. Some changes are made in the notation to make it agree with that previously used in this work. Where Sarrau's notation differs, it is given in brackets.

Let

V. (v) Muzzle velocity, in feet per second.

p_0 (P_0) Maximum pressure on bottom of bore in pounds per square inch.

p (P) Same on base of projectile.

d. (c) Caliber in inches.

u. Length of the travel of the base of the projectile in the bore, in inches. See Chapter XI, page 13.

W (p) Weight of projectile in pounds.

w (π) Same of powder.

Δ Density of loading.

δ Specific gravity of the powder.

N The granulation of the powder, or the number of grains per pound.

f Force of powder when $\Delta = 1$, Chapter II, page 7.

τ Time of combustion of a single grain, referred to a standard grain as unity. See page 5.

S Initial volume in cubic inches; the same as V, Chapter IX, foot page 3. This volume generally differs from the capacity of the powder chamber since the base of the projectile may occupy some of this space.

z The reduced length of the initial air space which is equal to v' (Chapter IX, page 3) + c' (Chapter XI, page 2).

We have $\frac{\pi d^2}{4} z = S - v$, but

$S = \frac{27.68\, w}{\Delta}$ and $v_{,} = \frac{27.68\, w}{\delta}$; therefore

$$z = \frac{110.72\, w}{\pi d^2}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right)$$

a and l (Sarrau uses λ instead of l).

These are two numerical coefficients depending on the form of the grain, which are functions of the ratio of the least dimension of the grain to its other dimensions. See page 3.

α and β. Two very important characteristics depending on the nature of the powder; viz, both on its form and the time of its combustion. Their values are obtained from the following equations :

$$\alpha = \left(\frac{f\,a}{\tau}\right)^{\frac{1}{2}}, \tag{2}$$

$$\beta = \frac{l}{\tau}. \tag{3}$$

Owing to their preponderating effect in the principal equations which follow, α is known as the

pressure characteristic, and β as the *velocity characteristic.*

A, B, M, K. Certain empirical constants to be determined by experiment.

Form of Grain.

If we develop Equation 5, Chapter X, according to the ascending powers of $\frac{t}{\tau}$ the development may be placed under the general form *

$$\frac{w'}{W} = f(t) = a\frac{t}{\tau}\left(1 - l\frac{t}{\tau} + m\frac{t^2}{\tau^2} + \&c. \ldots\right) \qquad (4)$$

This may be shown to apply to other forms of grain besides the sphere, the coefficients of $\frac{t}{\tau}$ varying with the form of the grain and by their values characterizing the mode of combustion in so far as it is affected by the form of grain.

I. For spherical grains it readily appears that

$$a = 3; \qquad l = 1; \qquad m = \tfrac{1}{3}.$$

The coefficient m is neglected as insignificant.

Besides the spherical grain, which includes not only true spheres, but grains the form of which approaches that of a sphere, such as cubes, hexagonal powders and those of irregular granulation; powders are classified as to form, as *parallelopipedons* and *pierced cylinders.* Both classes include the forms most closely resembling the type, *e. g.* *L. X.* powder would belong to the former, and pierced prismatic powder to the second class.

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* In the above equation replace $\frac{t}{\tau}$ by x, then

$$f(t) = 1 - (1-x)^3 = 3x - 3x^2 + x^3$$

$$= 3x(1 - x + x^2) = \frac{3t}{\tau}\left[1 - \frac{t}{\tau} + \frac{1}{3}\left(\frac{t}{\tau}\right)^2\right]$$

For grains of other forms a similar but more extended method is followed.

II. For the parallelopipedon, if x and y represent the ratios of the least dimension of the grain to its other two dimensions the development of the corresponding function of $\frac{t}{\tau}$ will give the following characteristic values for the coefficients a and l.

$$a=1+x+y; \quad l=\frac{x+y+xy}{a}.$$

If the base of the grain be square, $x=y$, and

$$a=1+2x; \qquad l=\frac{2x+x^2}{a}.$$

III. For the pierced cylinder, x represents the ratio of the thickness of the walls of the cylinder to its height, or conversely; the lesser dimension being divided by the greater in either case. The cylinder is supposed to burn all over at once. The following are the values of the coefficients for the pierced cylinder described:

$$a=1+x; \quad l=\frac{x}{a}.$$

Since the ratio x has generally given to it a value of $\frac{1}{2}$ we may form the following table.

TABLE I.

Form of Grain.	Values of a	l	$\frac{a}{l}$
I. Cubical; Spherical; Hexagonal; Irregular granulation	3.0	1.0	3.0
II. Parallelopipedon; flat powder	2.0	$\frac{5}{8}$	3.2
III. Pierced prism or cylinder, one hole	$\frac{3}{2}$	$\frac{1}{3}$	4.5

By substituting these values of a and l in Eq. (4) we may represent graphically, as in figure 1, the variations in the rate $\frac{dw}{dt}=\sigma$ for grains of equal weight but of different forms, burning in the same time τ.

If the rate of conversion is uniform, Eq. (4) becomes $f(t) = a\frac{t}{\tau}$ and l, β and γ (*post*) reduce to 0.

Size of Grain.

The mean diameter of irregular grains results from knowing their specific gravity and granulation as follows*

$$e = \left(\frac{6 \times 27.68}{\pi N \delta}\right)^{\frac{1}{3}} = \left(\frac{52.86}{N\delta}\right)^{\frac{1}{3}}. \qquad (5)$$

For powders of regular granulation a similar method may be preferred to the actual measurement of their dimensions.

VELOCITY FORMULÆ.

Monomial formula for quick powders

$$V = M\alpha \left(\frac{l}{\tau}\right)^{-\frac{3}{8}} \frac{w^{\frac{3}{8}} \Delta^{\frac{1}{4}} d^{\frac{1}{8}} u^{\frac{3}{16}}}{W^{\frac{7}{16}}}. \qquad (A)$$

Binomial formula for slow powders

$$V = A\,\alpha\,(w u)^{\frac{3}{8}} \left(\frac{\Delta}{Wd}\right)^{\frac{1}{4}} [1-\gamma], \qquad (B)$$

in which

$$\gamma = B\,\beta\,\frac{(Wu)^{\frac{1}{2}}}{d}. \qquad (6)$$

The choice of the formula to be employed in any case depends upon the value of γ. With a given gun and projectile this depends upon the value of β and therefore, under the conditions of loading, β measures the quickness of the powder.

The form of the function γ shows that its value depends largely upon the gun as well as upon the powder. Conse-

*Call $v = \frac{\pi e^3}{6}$ the volume of the mean grain, the weight of which in pounds is w: then

$$N w = 1 = \frac{N v \delta}{27.68} \therefore e^3 = \frac{6 \times 27.68}{\pi \delta N}.$$

quently the same powder may be quick in some guns and slow in others. Chap. XI., p. 24.

When $\gamma > 0.273$, Equation (A), should be employed, and conversely for Equation (B). The two equations give but little difference in results when the conditions make γ approach 0.273.

Referring to the value of β, Equation (3), it appears that the value of γ cannot be known until τ has been determined. It is evident that the methods described in Chapter VIII are not sufficiently accurate, so that the following practical method is adopted.

Determination of Constants.

A well defined molded powder is taken as a standard and its values of f and τ accepted as unity. For this powder the values of α and β, Equations (2, 3), reduce to $\sqrt{a}$ and l, which can be measured by the means described, page 3.

To determine the value of M in equation (A) we substitute the value of V obtained as the mean of several fires in a gun in which the standard powder is relatively quick, and solve with respect to M.

In equation (B) we proceed similarly for A and B, selecting two very dissimilar guns and taking their conditions of loading so as to cover as wide a difference of limits as is likely to occur in practice.

Choice of Formula.

Inasmuch as the values of A, B, M, are true for all powders, and since (Chapter IX, page 6) the force of all nitrate powders may be taken as constant, and in this case equal to unity, equation (A) may be written

$$V = M \left(\frac{a}{\tau}\right)^{\frac{1}{2}} \left(\frac{l}{\tau}\right)^{-\frac{3}{8}} \frac{w^{\frac{3}{8}} \Delta^{\frac{1}{4}} d^{\frac{1}{8}} u^{\frac{3}{16}}}{W^{\frac{7}{16}}}$$

and placing

$$M \frac{w^{\frac{3}{8}} \Delta^{\frac{1}{4}} d^{\frac{1}{8}} u^{\frac{3}{16}}}{V\ W^{\frac{7}{16}}} = X \qquad (7)$$

we obtain

$$\tau = a^4 l^{-3} X^8. \qquad (8)$$

If this value of τ substituted in that of γ, Equation (6), makes $\gamma > 0.273$, Equation (A) may be used; but if it makes $\gamma < 0.273$, Equation (B) must be employed.

To determine the true value of τ for use in Equation (B) requires a method of approximation which is too long to be given here, but can be found in the works mentioned in the bibliography. This will not generally be necessary, as the characteristics may be determined directly, as hereafter explained.

The value of τ for the standard powder is approximately equal to the time of its burning in air at the rate of about 0.4 inch (1 decimeter) per second. Other values of τ will therefore have approximately their values in air.

Remark.—The first term in Equation (B) represents the ideal case in which the form of the grain is such that the rate of conversion (Note, foot page 2, Chapter X.,) is uniform. The second term is subtractive and represents the effect of the decrease of the rate of conversion, or of the burning surface, when the grains have the forms required in practice. It is evidently an advantage to have the second term as small as possible.

Empirical Constants.

The numerical values of A, B, M, the determination of which has been incidentally described, depend only npon the units of measure adopted for dimensions and masses.

In the Ordnance Department, the units being respectively the inch, for internal dimensions of guns; the foot, for velocities per second, and the pound, the constants have the following values given by their logarithms:

$\log A = 2.56635$; $\log B = 2.30964$; $\log M = 2.84571$.

The other terms in the formula require no change; since the effect of changes in the units by which the different

elements of loading are measured, is compensated for by the numerical value of the empirical constants.

PRESSURE FORMULÆ.

The following equations are employed to determine the pressures on the base of the projectile and on the base of the bore.

$$p = K \alpha^2 \Delta \, (W w)^{\frac{1}{2}} d^{-2}, \tag{C}$$

in which $\log K = 3.96198$.

$$p_0 = K_0 \alpha^2 \Delta \, W^{\frac{1}{4}} w^{\frac{3}{4}} d^{-2}, \tag{D}$$

in which $\log K_0 = 4.25092$.

Equation (C) is obtained by differentiating the equation for velocity and determining the maximum acceleration of the projectile; it can be verified only by the apparatus described Chapter VII. But equation (D) can easily be verified by the pressure gauge. See Chapter XI, pp. 8—9

PRESSURE CURVES.

In designing guns it is indispensable to know something about the pressure at other points along the bore than that at which the maximum pressure occurs.

In Chapter IX we have considered an approximate solution; but Sarrau's formula furnishes us a method which is much more accurate.

Expansion Curve.

If in equation (B) we call

$$A_1 = A \, \alpha \, w^{\frac{3}{8}} \left(\frac{\Delta}{W d} \right)^{\frac{1}{4}}, \tag{9}$$

$$B_1 = B \, \beta \, \frac{W^{\frac{1}{2}}}{d}. \tag{10}$$

For the same gun, conditions of loading and powder, equation (B), becomes by writing, v, the velocity at any

point of the bore, for V, the muzzle velocity, and calling u the variable length of travel of the projectile.

$$v = A_1 u^{\frac{3}{8}} \left(1 - B_1 u^{\frac{1}{2}}\right). \tag{11}$$

If we differentiate equation (11) with respect to v and u, and divide by dt, we have

$$\frac{dv}{dt} = \left(\tfrac{3}{8} A_1 u^{-\frac{5}{8}} - \tfrac{7}{8} A_1 B_1 u^{-\frac{1}{8}}\right) \frac{du}{dt} = f(u) \frac{du}{dt}, \tag{12}$$

in which $\frac{du}{dt} = v$ and $\frac{dv}{dt} =$ acceleration of the projectile, or calling $p_{\prime}$ the variable pressure on the projectile; $\frac{dv}{dt} = \frac{p_{\prime} \pi d^2 g}{4 W}$.

From this follows

$$p_{\prime} = \frac{4\ W}{g \pi d^2} (f(u))\, v. \tag{13}$$

Combustion Curve.

It is not recommended to depend upon the values of $p_{\prime}$ thus deduced for a travel of the projectile of less than $\frac{u}{2}$; because the velocity formula is not considered reliable for such small values of u as those existing during the combustion period. Chapter XI page 1.

The form of the pressure curve in the initial portion may be determined as follows.

It appears from the following table based upon the analysis of Sarrau that the displacement of the projectile corresponding to the maximum pressure, or U, is equal to $0.6\,z$, equation (1). This gives us the locus of this pressure and equation (C) gives us the intensity. It remains then to find the form of the portions of the curve in the neighborhood of the point of maximum pressure. This is obtained from the following table which gives the proportion of the maximum pressure exerted at points near the displacement, U, above. In this table the variable y_0 represents the ratio

$\frac{u}{z}$, and $\frac{d^2y_0}{dx^2}$ values proportional to the acceleration, since x, in this case represents a certain function of t. It has no connection with the quantity x, on page 3.

TABLE II.

y_0	$\frac{d^2y_0}{dx^2}$	y_0	$\frac{d^2y_0}{dx^2}$	y_0	$\frac{d^2y_0}{dx^2}$
0.1	0.480	0.6	0.710	1.25	0.651
.2	.605	.7	.705	1.50	.621
.3	.665	.8	.700	1.75	.590
.4	.693	.9	.692	2.00	.563
.5	.700	1.0	.680	2.50	.513

That is to say that after the projectile has travelled over a distance equal to the reduced length of the initial air space, the pressure is $\frac{68}{71}$ of the maximum; etc.

It is supposed that the pressure on the wall adjacent to the base of the projectile is to that upon the base, as 10 is to 7; so that by multiplying the pressures just determined by 1.43 it is easy to determine the probable intensity of the corresponding pressure on the walls of the bore.

QUICKNESS OF POWDER.

Sarrau has established for powders fired under various conditions of loading certain *moduli of quickness* which express their relative quickness under these conditions. See page 5.

The modulus of a powder forms an important independent characteristic which is of considerable help in establishing auxiliary equations of condition for the solution of problems in Interior Ballistics.

It may be shown from equation (B) that if, among the variables in the second member, τ alone be caused to vary, the function, V, will pass through a maximum state.

In practice this is not absolutely true; for, as already stated in Chapter XI, the smaller the time of combustion, the greater the number of volumes of expansion in a given gun, and hence the greater the kinetic energy due to a given charge.

Equation (B) is derived by a process of approximation, and its physical significance cannot therefore be rigorously interpreted. It serves to show, however, that there is a limit below which the reduction in τ has but a very slight effect upon the velocity, and which it is inadvisable to pass; because, as τ diminishes past a certain point, the velocity increases very slowly; but the maximum pressure very rapidly.

The value of τ corresponding to the maximum value of V is obtained by placing equal to zero the first differential coefficient of the second member of equation (B) regarded as a function of τ and solving with respect to τ.*

Denoting this value of τ which is called the *time of the maximum* (velocity?) by τ_1 we have

$$\tau_1 = 3\, B \frac{l(W u)^{\frac{1}{2}}}{d}. \tag{14}$$

In a given piece a powder behaves as a slow powder when the time of its combustion, τ, is notably greater than

* Equation B may be written,

$$V = C\,\psi\,(t) = C\,(\tau^{-\frac{1}{2}} - R\,\tau^{-\frac{3}{2}}),$$

in which

$$R = B\frac{l\,(W u)^{\frac{1}{2}}}{d} \qquad C = A\,(f\,a)^{\frac{1}{2}}\,(w\,u)^{\frac{3}{8}}\left(\frac{\Delta}{W d}\right)^{\frac{1}{4}}.$$

Hence

$$\frac{d\,V}{d\,\tau} = C\,(-\tfrac{1}{2}\,\tau^{-\frac{3}{2}} + \tfrac{3}{2}\,R\,\tau^{-\frac{5}{2}})$$

by placing $\frac{d\,V}{d\,\tau} = 0$.

$$\tau_1 = 3\,R = 3\,B\,\frac{l\,(W\,u)^{\frac{1}{2}}}{d} \tag{14}$$

that which in the particular arm corresponds to the theoretical maximum of velocity. Further, two powders fired in different pieces should be considered as equivalent as far as regards quickness if their times of combustion are proportional to the times of the maximum for the two pieces employed. Consequently we may call the ratio

$$q = \frac{\tau_1}{\tau} \tag{15}$$

the *modulus of quickness* under the particular circumstances under which the powder is fired; since the more nearly does τ_1 equal τ, the more nearly does q approach unity.*

Under this view we may adopt the following arbitrary scale for the classification of powders:

TABLE III.

Value of Modulus.	Nature of Powder.
1.0	Very quick.
.9	Quick.
.8	Medium.
.7	Slow.
.6	Very slow.

Since the above classification was proposed by Sarrau, it has been found advisable to extend the value of the modulus in both directions. For long Sea Coast guns it now runs as low as 0.4, while it has been found advantageous in the B.L. mortars to increase it to 1.2.

In any case we have

$$q = 3\,B\beta\frac{(W\,u)^{\frac{1}{2}}}{d} = 3\gamma. \tag{16}$$

VELOCITY AS A FUNCTION OF THE MODULUS.

By introducing q in place of τ in equation (B) we may obtain a new and useful monomial equation of the general form

* The modulus of quickness also is designated by Sarrau as x.

$$V = \tfrac{2}{3} A \, (3\,B)^{-\frac{1}{2}} \left(\frac{f\,a}{l}\right)^{\frac{1}{2}} \frac{w^{\frac{3}{8}} \, \Delta^{\frac{1}{4}} \, d^{\frac{1}{4}} \, u^{\frac{1}{8}}}{W^{\frac{1}{2}}} f(q), \tag{17}$$

in which $f\,(q)$ may be taken as $N\,q^{n}$; N being some constant.* See page 28.

Collecting the different empirical constants under one head, which we may call M, and ascertaining that for the particular form of $f\,(q)$ employed we have

$$n = \tfrac{3}{2} \frac{1 - q}{3 - q}; \tag{18}$$

we find that equation (17) reduces to

$$V = M \left(\frac{f\,a}{\tau}\right)^{\frac{1}{2}} \left(\frac{\tau}{l}\right)^{\frac{1}{2} - n} \frac{w^{\frac{3}{8}} \, \Delta^{\frac{1}{4}} \, d^{\frac{1}{4} - n} \, u^{\frac{1}{8} + \frac{n}{2}}}{W^{\frac{1}{2} - \frac{n}{2}}}. \tag{A$'$}$$

In equation (A′) V varies with n; that is, with the modulus, q, upon which by equation (18), n depends. It may be used as an approximation, as on page 6, by giving to n a constant value under conditions of loading which are such that the modulus is comprised within certain limits.

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* This equation shows that under given conditions of loading the initial velocity is proportional to $\left(\frac{f\,a}{l}\right)^{\frac{1}{2}}$.

This factor is called the *ballistic coefficient.* It depends both upon the force of the powder and the form of the grain. If the force be considered constant, the ballistic coefficient depends only on the form of the grain.

By transforming Equations (2, 3) we have

$$C^2 = \frac{f\,a}{l} = \frac{\alpha^2}{\beta}.$$

It will be observed from Equations (C′), (D′), that the pressure varies with the square of the ballistic coefficient. This relation imposes a practical limit to increasing the velocity by the increase of this coefficient.

The ballistic coefficient of the powder must be carefully distinguished from the ballistic coefficient of the projectile to be hereafter discussed.

It is convenient to remember that n increases as q decreases: when $q=\frac{9}{11}$, $n=\frac{1}{8}$; and when $q=\frac{6}{10}$, $n=\frac{1}{4}$.*

It is considered that these are the limits imposed by practically satisfactory conditions of loading. See page 21. By making $q=\frac{9}{11}$ equation (A′) reduces to the form of equation (A), which was thus derived.

Since $q=3\gamma$ and since the value of $q=\frac{9}{11}$ is taken to be about the highest modulus that can be profitably employed, we see why the maximum value of γ on page 5, has been determined $=\frac{9}{11}\div 3=0.273$.*

MAXIMUM PRESSURE AS A FUNCTION OF THE MODULUS.

By substituting for $\frac{1}{\tau}$ in the value of α^2, equation (C), its value $\frac{q}{\tau_{\prime}}$ derived from equation (14), Sarrau finds

$$p = K\,(3\,B)^{-1}\frac{f\,a}{l}\,\frac{\Delta}{d}\left(\frac{w}{u}\right)^{\frac{1}{2}} q, \qquad \text{(C}'\text{)}$$

and similarly

$$p_0 = K_0\,(3\,B)^{-1}\,\frac{f\,a}{l}\left(\frac{w}{W}\right)^{\frac{1}{4}}\frac{\Delta}{d}\left(\frac{w}{u}\right)^{\frac{1}{2}} q. \qquad \text{(D}'\text{)}$$

PRINCIPLE OF SIMILITUDE.

Two guns are similar when all their homologous linear dimensions are proportional to their calibers. Chapter XVI, page 17.

The similitude is extended to the loading when the weights of the powder and of the projectile are proportional to the cube of the calibers, and when the grains of powder have the same form, composition, density, etc., and their

*Although not so named, it is convenient to think of n as the *modulus of slowness*.

dimensions are proportional to the calibers. Consequently the numerical coefficients a, l, must have the same values, and the value of τ must vary proportionately with the caliber.

The principle of similitude enables the following proposition to be proved, viz.:

In similar guns, similarly loaded, the velocities and pressures corresponding to distances passed over, which, measured in calibers, are equal, are respectively equal to each other.

For, let us consider two guns having calibers respectively equal to d and to d' such that $d' = \theta\, d$, and substitute in Eq. (16), (17), the ratio $\theta = \frac{d'}{d}$ raised to powers varying with the quantity considered, as follows:

From the conditions of similitude we have

$$w : w' :: W : W' :: d^3 : d'^3,$$

or

$$\frac{w'}{w} = \frac{W'}{W} = \left(\frac{d'}{d}\right)^3 = \theta^3,$$

and

$$\frac{u'}{u} = \frac{d'}{d} = \theta.$$

In Eq. (17) the factors A, B, Δ, N, and the ballistic coefficient will be eliminated by division, so that substituting for $\frac{w'^{\frac{3}{8}}}{w^{\frac{3}{8}}}$, $(\theta^3)^{\frac{3}{8}} = \theta^{\frac{9}{8}}$ and so on, we have

$$\frac{V'}{V} = \left(\frac{\theta}{\theta}\right)^{\frac{12}{8}} \times \left(\frac{q'}{q}\right)^n = \left(\frac{q'}{q}\right)^n$$

Similarly in Eq. (16) since $\beta = \frac{l}{\tau}, \frac{q'}{q} = \frac{\tau}{\tau'}\,\theta.$

Now if τ varies with the caliber, $\frac{\tau}{\tau'} = \frac{1}{\theta}$, and $\frac{q'}{q} = 1$, or $V = V'$.

Since the muzzle may be taken at any distance the proposition is proved as to velocities and can be shown to be true as to pressures by the similar treatment of Equation (C').

But if the same powder is used in two similar guns of different caliber $\frac{\tau}{\tau'} = 1$ and $\frac{V'}{V} = (\theta)^n = \frac{d'^n}{d^n}$.

Consequently, for the same powder in similar guns, the velocity varies as the n^{th} power of the caliber.

Equation (D) similarly shows that when the same powder is used in similar guns the pressure varies as the caliber.

This is a more exact explanation of the practice of varying the size of the grain to suit the gun than that given Chapter XI, page 7.

INFLUENCE OF THE CONDITIONS OF LOADING UPON VELOCITIES AND PRESSURES.

General Statement.

Let us consider as constant for any gun the quantities d, u, W, and as constant for any powder its force and form of grain, or f, a and l, *i.e.*, its ballistic coefficient.

The quantities which may then be varied so as to affect the velocities and pressures are w, Δ and τ.

There are an infinite number of sets of values of these variables which will give the same velocity with different maximum pressures, or the same pressure with different velocities. The pressures considered are those upon the breech of the gun.

The following practical rules result from differentiating the Napierian logarithms of the above named variables in Equations (17) and (D′). In equation (17) the differential of the Napierian logarithm of the function of q which it

contains can be shown to reduce to the form *

$$d \log_e f(q) = -n \frac{d\tau}{\tau}, \tag{19}$$

and in equation D′,

$$d \log_e q = -\frac{d\tau}{\tau}; \tag{20}$$

therefore

$$\frac{dV}{V} = \tfrac{3}{8}\frac{dw}{w} + \tfrac{1}{4}\frac{d\Delta}{\Delta} - n\frac{d\tau}{\tau} \tag{21}$$

$$\frac{dp_0}{p_0} = \tfrac{3}{4}\frac{dw}{w} + \frac{d\Delta}{\Delta} - \frac{d\tau}{\tau} \tag{22}$$

These equations enable us to determine the variations in velocity and pressure corresponding to very small increments of the variables w, Δ and τ.

The influence of each variable on the value of the velocity and pressure is measured by the coefficient which multiplies the relative variation of each variable in the above equations.

In Equation 21 the coefficients of $\frac{dw}{w}$, $\frac{d\Delta}{\Delta}$, and $\frac{d\tau}{\tau}$ are respectively $\frac{3}{8}$; $\frac{1}{4}$; $n = \frac{3}{2}\,\frac{1-q}{3-q}$.

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* Since $q = \frac{\tau_1}{\tau}$; $dq = -\frac{\tau_1}{\tau}\,\frac{d\tau}{\tau} = -q\,\frac{d\tau}{\tau}$

$$d \log_e q = \frac{dq}{q} = -\frac{d\tau}{\tau}$$

Also $d \log_e f(q) = d \log_e N q^n = \frac{d(Nq^n)}{Nq^n}$

$$= \frac{dq^n}{q^n} = \frac{nq^{n-1}}{q^n}dq = n\frac{dq}{q}$$

$$= n\,d \log_e q = -n\frac{d\tau}{\tau}$$

The increments here discussed are small finite differences made in adjusting practically the conditions of loading. For considerable differences Equations (A, B, etc.), should be employed.

The third coefficient varies with τ and is equal to 0 for $\tau = \tau_1$. Its value increases with τ, but does not exceed $\frac{1}{4}$ except when q is less than $\frac{6}{10}$ which is not likely to happen in the ordinary conditions of practice.

Comparing equations (21) and (22) it appears that while in equation (21) the variables are arranged in the order of their relative importance, in equation (22) the influence of w on the maximum pressure is less than that of Δ or τ.

Let us consider as a *fundamental condition* that the maximum pressure remains at a constant value determined by the strength of the gun, and suppose but two of the quantities w, Δ and τ to vary at a time, the third remaining constant.

First Case. Δ and τ variable, w constant.

The equations reduce to

$$\frac{d\,\Delta}{\Delta} = \frac{d\,\tau}{\tau} \text{ and} \tag{23}$$

$$\frac{dV}{V} = \left(\frac{1}{4} - n\right)\frac{d\,\Delta}{\Delta} \tag{24}$$

Therefore, if Δ is varied by changing the size of the chamber for a given charge, the time of burning must change correspondingly to the density of loading. In such a case, if $q > \frac{6}{10}$, V increases with Δ. Hence the conclusion: In order to obtain the greatest velocities we should use high densities of loading and slow powder.

Second Case. w and τ variable; Δ constant. (M. L. gun.)

Equation (22) becomes $\frac{3}{4}\frac{d\,w}{w} = \frac{d\,\tau}{\tau}$,* (25)

and equation (21) $\frac{d\,V}{V} = \frac{3}{4}\left(\frac{1}{2} - n\right)\frac{d\,w}{w}$, (26)

* That is, that if we increase w by 10 per cent; then, to fulfill the fundamental condition, τ must be increased $\frac{3}{40}$, or 7.5 per cent.

It follows from equation (18) that since when q is equal to 0, $n = \frac{1}{2}$; and that when q is greater than 0, n is less than $\frac{1}{2}$, the factor $(\frac{1}{2} - n)$ is always positive and therefore that the velocity increases with the charge of powder, and that the maximum pressure will not be exceeded provided that the time of its combustion be regulated as required by Equation (25).

Third Case. w and τ variable in a chamber of constant capacity. We have supposed in the preceding cases that the volume of the powder chamber can be increased or decreased at will, and in designing guns to perform certain work the conclusions reached are useful. Suppose however that we desire to improve the conditions of loading of an existing cannon. In this case, since

$$\Delta = \frac{27.68\,w}{S}, \text{ we have } \frac{d\Delta}{\Delta} = \frac{dw}{w}, \tag{27}$$

and therefore

$$\frac{dV}{V} = \frac{5}{8}\frac{dw}{w} - n\frac{d\tau}{\tau}, \tag{28}$$

$$\frac{d\tau}{\tau} = \frac{7}{4}\frac{dw}{w}, \tag{29}$$

$$\frac{dV}{V} = \frac{7}{4}\left(\frac{5}{14} - n\right)\frac{dw}{w}, \tag{30}$$

in which $(\frac{5}{14} - n)$ is positive.

Therefore, if the chamber is large enough, we may increase the velocity without changing the pressure by using a larger charge and a slower powder.

Examples.

1. Suppose with a slow powder $(n = \frac{1}{4})$ we wish to increase V by 10 per cent, $\frac{dV}{V} = \frac{10}{100} = \frac{1}{10} = \frac{3}{16}\frac{dw}{100} \therefore dw =$ 53.3 per cent, and $\frac{d\tau}{\tau} = \frac{7}{4}\,53.3 = 93$ per cent. That is, we

would use nearly double the charge of double the size of grain; assuming that τ is proportional to the size of the grain.

2. Using a quick powder ($n = \frac{1}{8}$) and

$\frac{dV}{V} = \frac{1}{10} = \frac{13}{32}\frac{dw}{100}$ $\therefore dw = 25$ per cent, and $\frac{d\tau}{\tau} = 44$ per cent. Or we would use about one-quarter greater charge of less than one-half greater size.

Fourth Case. w and Δ variable and τ constant.

This corresponds to the use of the same powder in guns having different chambers.

From the conditions we have

$$\frac{d\Delta}{\Delta} = -\frac{3}{4}\frac{dw}{w}, \tag{31}$$

$$\frac{dV}{V} = \frac{3}{16}\frac{dw}{w}. \tag{32}$$

That is to say; that if we fix the size and shape of the grain, and wish to increase the velocity, we must increase both the weight of the charge and the volume of the chamber.

General Remark.

A review of the preceding cases shows that whenever τ varies, V is a function of n and also of either w or Δ according to which one of these is variable.

THE EFFECT UPON PRESSURES AND VELOCITIES OF VARYING THE TIME OF COMBUSTION.

If in equation (21) we allow only τ to vary, we have

$$\frac{dV}{V} = -n\frac{d\tau}{\tau}. \tag{33}$$

The value of n increases as the modulus decreases; consequently the same relative variation of the time of combustion has a greater influence upon the velocity as the powder becomes slower. See Chapter XI, page 18.

Now, suppose the pressure to vary; under the conditions equation (22) reduces to

$$\frac{dp_0}{p_0}=-\frac{d\tau}{\tau}. \tag{34}$$

Combining this, with equation (33) we obtain the very simple relation

$$\frac{dV}{v}=n\frac{dp_0}{p_0}, \tag{35}$$

which expresses a relation between velocities and pressures similar to that between velocities and times of combustion, in Equation (33).

It has been stated page 12, that the values $\frac{9}{11}$ and $\frac{6}{10}$ may be considered as the limits that the modulus should not pass. The choice of these limits is justified as follows. When the modulus is greater than $\frac{9}{11}$ the relative variation of the velocity depends upon n in equation (35) which under these circumstances only becomes $\frac{1}{8}$ of the relative variation of the maximum pressure. Consequently, a sensible increment of the velocity is obtained only with a considerable increase in the pressure and the energy acquired by the projectile is imparted at an increased risk to the gun. This grows less as the modulus diminishes from $\frac{9}{11}$; because the value of n increases; but then, from Equation (34), the relative variation of the velocity corresponding to the same relative variation of the time of combustion increases, as shown by Equation (33), so that the influence of accidental irregularities of the powder upon the velocity continually grows greater.

It is then advisable to fix an inferior limit for the modulus so as to preserve uniformity in velocity.

APPLICATIONS.

1. To determine the characteristics α and β of a powder.

The most practical method is to use according to circumstances either equations (A) or (B) in connection with equa-

tion (D), and to substitute in these equations for V and p_0 the mean of several measured velocities and pressures obtained under invariable conditions of loading.

We have then two independent equations involving but two unknown quantities, α and β; these may then be determined without reference to their separate factors.

By the theory, the characteristics are entirely independent of the gun. In this respect, and also in that they give us *numerically* the influence of all the elements of firing, Sarrau's formulæ are more useful than those, like Noble's, described in Chapter XI.

Having determined accurate values for the characteristics of a powder, we may compute the velocity and pressure to be expected in any gun whose dimensions are known, when the conditions of loading are given; and conversely, the dimensions may be determined.

Within reasonable limits of variation of the quantities entering them, the accuracy of the formulæ has been abundantly verified.

EXAMPLES.

1. To find the characteristics of Du Pont's P. N. (Brown Prismatic) powder from a single firing of the 8 inch B. L. *Steel* Rifle. For its dimensions see Table IV.

Data. $w = 110$; $W = 289$; $\Delta = 0.980$; $v = 1878$; $p_0 = 36000$; $u = 195.75$. Equation (D) gives us

$$\alpha^2 = \frac{p_0\, d^2}{K_0 \Delta\, w^{\frac{3}{4}}\, W^{\frac{1}{4}}} = 0.9706 = \log^{-1}\bar{1}.98704.$$

An application of the test mentioned page 6, will show that the binomial formula is applicable; although this might be assumed for powders of this kind. If then we write equation (B) so as to combine in each term the quantities relating to the gun and the conditions of loading, we may reduce it to the form

$$XV = \alpha - \alpha \beta Y \quad \text{or} \quad \beta = \frac{\alpha - XV}{\alpha Y},$$

in which V is measured and

$$\frac{1}{X} = \frac{A\,(w\,u)^{\frac{3}{8}}}{(W d)^{\frac{1}{4}}}\,\Delta^{\frac{1}{4}} \text{ and } Y = \frac{B\,(W u)^{\frac{1}{2}}}{d},$$

Substituting in the above the known values of α, X, V and Y, we find

$$\log \beta = \bar{1}.33725.$$

Another method is to fire the same powder under very dissimilar sets of conditions in which W, w, u, d shall have different values and to determine the values of V under these conditions.

We may thus obtain two formulæ of the form of the above value of XV; as these involve but two unknown quantities, the characteristics sought may be determined.

This method avoids all uncertainty attending the operation of the pressure gauge; but the former method is generally preferred as the conditions more nearly resemble those of practice, and introduce the customary unit of measurement of pressure,

2. To compute the muzzle velocity to be expected from the 8 inch B. L. Steel Rifle for the preceding powder.

Data. $w = 105$; $\Delta = 0.935$; $W = 289$.

Computation of γ.

$\log B$	=	$\bar{2}.30964$
$\log \beta$	=	$\bar{1}.33725$
$\log W^{\frac{1}{2}}$	=	1.23046
$\log u^{\frac{1}{2}}$	=	1.14585
$\log d^{-1}$	=	$\bar{1}.09691$
$\log \gamma$		$\bar{1}.12011$
γ	=	0.13186
$1-\gamma$	=	0.86814

Computation of V.

$\log A$	=	2.56635
$\log \alpha$	=	$\bar{1}.98704$
$\log w^{\frac{3}{8}}$	=	0.75795
$\log \Delta^{\frac{1}{4}}$	=	$\bar{1}.99276$
$\log u^{\frac{3}{8}}$	=	0.85939
$\log W^{-\frac{1}{4}}$	=	$\bar{1}.38477$
$\log d^{-\frac{1}{4}}$	=	$\bar{1}.77423$
$\log (1-\gamma)$	=	$\bar{1}.93859$
$\log V$	=	3.26108
V	=	1824.2

By actual measurement V was found to be 1825.

3. To compute the maximum pressure on the breech under the same conditions as in No. 2. But with a powder (PO) of which the characteristics are different, viz.:—

$$\alpha = \log^{-1}\bar{1}.97701; \quad \beta = \log^{-1}\bar{1}.28978.$$

$$\begin{aligned}
\log K_0 &= 4.25092 \\
\log \alpha^2 &= \bar{1}.95401 \\
\log \Delta &= \bar{1}.97102 \\
\log w^{\frac{3}{4}} &= 1.51589 \\
\log W^{\frac{1}{4}} &= 0.61523 \\
\log d^{-2} &= \bar{2}.19382 \\
\hline
\log p_0 &= 4.50089 \\
p_0 &= 31688
\end{aligned}$$

From actual firing under the above conditions the mean value of p_0 as determined by two independent pressure gauges was 31700 lbs.

4. In order to avoid injury to valuable cannon, it is customary at the Proving Ground to make a preliminary trial of new powders in what is called the *proof gun*.

Data. $w=35.9$; $W=181$; $\Delta=0.8988$; $d=8$; $p_0=20420$.

Find the value of p_0 to be expected when

$$w=90; \quad W=300; \quad \Delta=0.8018; \quad d=8.$$

The first set of data give in Equation (D),

$$\log \alpha^2 = 0.18085,$$

hence, we find for the second set of data,

$$p_0 = 41174 \text{ lbs.}$$

In actual firing the mean value was found to be 41055 lbs.

Useful Tables.

The following tables give the dimensions of various cannon of the U. S. land service with the characteristics of different powders tried in them and the resulting pressures and velocities both computed and as verified by measurement.

They will be useful in solving problems hereafter.

TABLE IV.

Powder.	Gun.	d inches.	u inches.	W lbs.	w lbs.	Δ	V feet per sec.	p_0 lbs. per sq. in.
LX...	3″.20 B. L. rifle..	3.2	73.2	13	3.50	0.857	1,649	31,000
LXB .	....do..........	3.2	73.2	13	3.75	0.827	1,756	35,150
IKD..	....do..........	3.2	73.2	13	3.50	0.857	1,630	29,100
IKB..	....do..........	3.2	73.2	13	3.50	0.857	1,663	30,500
KHC.	12″ mortar	12.0	91.6	610	50.0	0.821	932	22,000
MW..	....do..........	12.0	91.6	610	48.0	0.788	959	26,250
EVF.	8″ B. L. R Conv..	8.0	98.5	183	45.0	0.792	1,488	32,650
PN...	8″ B. L. R. S.....	8.0	195.75	289	110.0	0.980	1,878	36,000
NM..	12″ B. L. R.C.I...	12.0	273.5	800	265.0	0.827	1,688	26,350
NV_3..	....do..........	12.0	273.5	800	265.0	0.827	1,718	26,890
NR...	....do..........	12.0	273.5	800	265.0	0.827	1,826	32,990
NV_1..	....do..........	12.0	273.5	800	265.0	0.827	1,760	26,625
NV_2..	....do..........	12.0	273.5	800	265.0	0.827	1,756	28,000
IB ...	3″.17 M L.R.W.I.	3.175	74.6	10.5	5.469	0.814	1,933	25,000
OB ...	12″ mortar	12.0	91.6	610	52.0	0.854	987	25,250
OC ...	....do..........	12.0	91.6	610	52.0	0.854	942	19,750

TABLE V.—*Characteristics of different powders.*

Powder.	Form of grain.	e	N	Sp. gr.	x	a	l	τ	f	Log. α	Log. β	Log. α^2
LX...	Square prism......	0.315	270	1.706	0.720	2.442	0.803	0.552	0.967	0.31562	0.16297	0.63124
LXB..	....do............	0.315	270	1.706	0.720	2.442	0.803	0.540	1.055	0.33940	0.17243	0.67880
IKD..	Granulated........	......	2,200	1.725	......	3.000	1.000	0.727	0.970	0.30115	0.13851	0.60230
IKB..	....do............	......	2,200	1.725	......	3.000	1.000	0.714	1.001	0.31273	0.14618	0.62546
IB....	Sphero-hexagonal..	......	123	1.728	......	3.000	1.000	1.183	1.064	0.21561	$\bar{1}.92707$	0.43122
KHC.	....do............	......	123	1.775	......	3.000	1.000	1.267	0.979	0.18257	$\bar{1}.89721$	0.36514
OB...	Hexagonal.........	......	123	1.725	......	3.000	1.000	1.269	1.050	0.19747	$\bar{1}.89671$	0.39493
OC...	....do............	......	123	1.750	......	3.000	1.000	1.565	1.013	0.14412	$\bar{1}.81541$	0.28824
MW..	....do............	......	72	1.725	......	3.000	1.000	1.106	1.096	0.23644	$\bar{1}.95615$	0.47288
EVF..	....do............	......	72	1.750	......	3.000	1.000	1.351	1.043	0.18232	$\bar{1}.86937$	0.36464
Cocoa.	Pierced hexagonal prism.	0.475	$11\frac{1}{2}$	1.863	0.500	1.500	0.333	1.130	0.735	$\bar{1}.95922$	$\bar{1}.39892$	$\bar{1}.91844$
NM	....do............	0.475	$11\frac{1}{2}$	1.833	0.500	1.500	0.333	1.589	0.782	$\bar{1}.93402$	$\bar{1}.32182$	$\bar{1}.86805$
NR...	....do............	0.475	$11\frac{1}{2}$	1.814	0.500	1.500	0.333	1.363	0.839	$\bar{1}.98282$	$\bar{1}.38846$	$\bar{1}.96565$
NV_1..	....do............	0.475	$11\frac{1}{2}$	1.830	0.500	1.500	0.333	1.952	0.968	$\bar{1}.93628$	$\bar{1}.23245$	$\bar{1}.87256$
NV_2..	....do............	0.475	$11\frac{1}{2}$	1.818	0.500	1.500	0.333	1.665	0.870	$\bar{1}.94721$	$\bar{1}.30149$	$\bar{1}.89443$
NV_3..	... do............	0.475	$11\frac{1}{2}$	1.826	0.500	1.500	0.333	1.694	0.851	$\bar{1}.93843$	$\bar{1}.29384$	$\bar{1}.87686$

For form of grain, see Chapter IV. Plates.

TABLE VI.—*Verification of the characteristics deduced for the different powders.*

Gun.	Kind of powder.	d	u	W	w	Δ	Velocities. Calculated from binomial formula.	Velocities. Measured.	Pressures. Calculated pounds per sq. inch.	Pressures. Measured pounds per sq. inch.	Number of rounds considered.
							Feet.	*Feet.*			
12-inch B. L. rifle	Cocoa.	12	273.5	750	265	0.827	1,761	1,761	29,051	32,000	1
12¼-inch M. L. rifle	..do..	12.25	183.4	693	175	.999	1,460	1,460	24,404	23,800	1
12-inch M. L. mortar......	MW	12	91.6	610	26	.427	652	651	8,978	9,500	3
do	..do..	12	91.6	610	45	.739	919	925	23,477	24,750	1
do	..do..	12	91.6	610	48	.788	957	959	26,250	26,250	1
8-inch B. L. rifle..........	EVF.	8	98.5	183	45	.792	1,488	1,488	32,650	32,650	5
do	..do..	8	98.5	183	50	.872	1,585	1,581	38,870	38,325	2
12-inch M. L. mortar......	..do..	12	91.6	610	52	.854	979	980	23,549	25,000	2
do	KHC.	12	91.6	610	50	.821	932	932	22,000	21,750	1
do	..do..	12	91.6	610	52	.854	953	993	23,574	25,000	2
8-inch M. L. rifle	..do..	8	95	180	35	.9	1,368	1,388	31,226	31,100	10
3.2-inch B. L. rifle, steel..	IKD.	3.2	73.2	13	3.5	.857*	1,630	1,630	29,000	29,100	25
do	..do..	3.2	73.2	13½	3.5	.857	1,603	1,593	29,275	29,000	3
do	..do..	3.2	73.2	15	3.5	.857	1,530	1,529	30,657	30,700	1
do	IKB.	3.2	73.2	13	3.5	.857	1,663	1,663	30,590	30,590	3
do	..do..	3.2	73.2	13	3.75	.827†	1,692	1,702	30,958	29,770	12
3.2-inch B. L. converted..	..do..	3.2	56.6	13	3.0	.903	1,510	1,501	28,700	27,700	5
3.2-inch B. L. rifle, steel..	LXB.	3.2	73.2	13	3.75	.827	1,756	1,756	35,150	35,150	8
do	..do..	3.2	73.2	13	3.5	.857	1,726	1,713	34,587	36,500	8
3.17-inch M. L. rifle......	IB.	3.175	74.6	10.5	5.469	.814	1,933	1,933	25,000	24,500	2
do	..do..	3.175	74.6	10.5	5.625	.837	1,967	1,965	26,261	26,750	2
do	..do..	3.175	74.6	10.5	5.813	.865	2,008	2,013	27,812	30,000	2

* De Bange gas check used. † Freyre gas check used.

DISCUSSION OF THE COEFFICIENT χ.

If in Equation (10), Chapter XI, we replace r^2 by $\frac{d^2}{4}$ we have after reduction, $\chi = \frac{W^2 \ V^4}{2240 \ g^2 \ p_0 \ \pi \ d^2 \ u \ w}$, in which u is expressed in feet. But since internal measures are given in inches we may avoid errors in practice by writing this

$$\chi = G \frac{W^2 \ V^4}{p_0 \ d^2 \ u \ w}, \tag{36}$$

in which $G = \frac{12}{2240 \ g^2 \ \pi}$.

If in Equations (17) and (D') we replace the ballistic coefficient by C, and collect the constants in both equations so that

$$H = \frac{2 \ A}{3 \ (3 \ B)^{\frac{1}{2}}}; \quad J = \frac{K_0}{3 \ B};$$

these equations will read

$$V = H \frac{C \ w^{\frac{3}{8}} \ \Delta^{\frac{1}{4}} \ d^{\frac{1}{4}} \ u^{\frac{1}{8}} \ N \ q^{n}}{W^{\frac{1}{2}}}. \tag{37}$$

$$p_0 = J \frac{C^2 \ w^{\frac{3}{4}} \ \Delta \ q}{W^{\frac{1}{4}} \ d \ u^{\frac{1}{2}}}. \tag{38}$$

Substituting these values in Equation (36) we have

$$\chi = Z \ C^2 \ N^4 \ q^{4n-1} \left(\frac{W}{w}\right)^{\frac{1}{4}} \tag{39}$$

in which $Z = \frac{G \ H^4}{J} = \log^{-1} 0.7394$, viz., 5.488.

The factor N, which in Equation (17) was taken as constant, is not absolutely so.* Its value is given in Sarrau as

$$f(q) = N q^n = \frac{q^{\frac{1}{2}} (3 - q)}{2}. \tag{40}$$

By substituting the value of N implied in Equation (40) in

* For values of q between $\frac{9}{11}$ and $\frac{6}{10}$, N varies only from 1.012 to 1.056.

the factor $N^4\, q^{4n-1}$ in Equation (39) and calling the resulting value Q, we have

$$Q = N^4\, q^{4n-1} = q\left(\frac{3-q}{2}\right)^4 = f\left(\frac{l}{\tau}\,\frac{Wu}{d}\right) \tag{41}$$

This reduces Equation (39) to

$$\chi = Z\, C^2\, Q\left(\frac{W}{w}\right)^{\frac{1}{4}}. \tag{42}$$

Discussion of the Factors of χ.

Of the four factors producing χ: Z is a constant; C^2 depends solely upon the powder; that is upon its force and the form of the grain; Q depends upon the suitability to the gun and projectile of the *kind* of powder employed; $\left(\frac{W}{w}\right)^{\frac{1}{4}}$ depends solely upon the circumstances of the particular fire considered. Hence, to compare the intrinsic properties of different powders fired in the same gun, we may compare their respective values of

$$\Pi = \chi\left(\frac{w}{W}\right)^{\frac{1}{4}} = Z\, C^2\, Q. \tag{43}$$

The relation between Q and q is shown in figure 2, from which it appears that while Q is sometimes an increasing and sometimes a decreasing function of q; for values of q between $\frac{6}{10}$ and $\frac{9}{11}$, Q decreases slowly from its maximum value of 1.245, corresponding to $q = \frac{6}{10}$, to a value of 1.159, corresponding to $q = \frac{9}{11}$.*

If the force of all nitrate powders were truly constant, $C^2 = \frac{f\,a}{l} = \frac{\alpha^2}{\beta}$ would depend for its value solely upon the form of the grain; and, since within ordinary limits Q does not vary greatly, we would expect nearly equally good re-

* From this we may conclude that for ordinary approximation the mean of these values, or $Q = 1.2$ may be used. Also that it is not well to depart much from the inferior limit of q established by Sarrau.

sults from black or brown prismatic powder. But, in the following illustration, taken from one of the best black prismatic powders recently tried, we find f to be so small that C^2 does not much exceed the value of 3.0 deduced from Table I, for ordinary powders of irregular granulation.

We may therefore conclude that the advantage of cocoa powder consists in its maintaining its force at nearly unity* without becoming so quick (or slow; figure 2) as to cause its value of Q to become unduly small. These considerations indicate in a general way that its peculiar properties are due to the nature of the fuel it contains.

Illustrations.

The following data, derived from experimental records, illustrate the principles discussed:

COMPARISON OF POWDERS.

Gun	5 in. B. L. R.	8 in. B. L. R.
Powder, kind	Bl. prism.	Br. prism.
Powder, name	O. I. N.	Ger. cocoa.
W	45.0	289.0
Sph. Density	2.9	4.5
V	1852	1875
p_0	33075	35900
u	119.8	195.75
a	1.5	1.5
l	$\frac{1}{3}$	$\frac{1}{3}$
α^2	2.45	0.93
β	0.78	0.21
f	0.70	0.96
τ	0.43	1.59
C^2	3.16	4.45
q	0.70	0.38

* See Table V, in which the last six powders are cocoa.

Q	1.23	1.12
χ observed	28.28	34.32
χ Equation (40)	28.44	34.87
Π	21.26	27.39
Ratio of Π	1.00	1.29

If we exchange powders only, we have—

Powder	Ger. cocoa.	O. I. N.
q	0.19	1.47
Q	0.73	0.50
Π	17.95	8.55
Ratio of Π	2.10	1.00

That is, that while each powder is best suited to the gun in which it is actually used, the cocoa powder would be better for general use, and might profitably be adapted to the siege rifle by reducing the size of the prism so as to diminish τ and increase Q.

MAXIMUM VALUE OF χ.

The value of Π before deduced, enables us to solve some very important problems in internal ballistics.

As an example, let us consider the question of how, with our present knowledge of gunpowder, we may attain the maximum value of χ. Also, let us apply this to a gun the construction of which limits p_0; the spherical density of the projectile being known and the value of u being expressed in terms of the caliber or $u = n\,d$. Let $s = \frac{8\,W}{d^3}$ be the spherical density of the projectile.* The maximum value of C^2 being 4.5, figure 2 shows that the maximum value of Π, and hence of χ, will require $q = 0.6$.

The maximum value of η will depend on that of $\triangle$. The

* See Chapter XVI, page 6.

specific gravity of some powders is now such that a value of $\Delta = 1$, has been reached. We may consider this a maximum, as it is rarely exceeded. After deducing general equations, we will apply them to a typical gun based on the 8-in. B. L. R. Steel, in which $s = 4.5$; $n = 24$, and take the maximum value for p_0 as 36000 lbs. per square inch, that being what the records indicate to be a desirable limit.

1. Proper Weight of Charge.

By substituting the values assigned for W, Δ, d, u, q in Equation (D′), it leads to the following ratio:

$$\frac{w}{W} = \left[\log^{-1} \overline{1}1.1119 \frac{p_0^2\, n}{s}\right]^{\frac{2}{3}} \qquad (44)$$

In which, by substituting the special typical values assumed for p_0, n and s, we have $w = 0.2\ W$.

In the 8-in. rifle this would reduce the charge of powder about one-half.

2. Proper Size of Grain.

If in Equation (16) we place $q = 0.6$ and assign values as above, we have for a general equation, since $l = \frac{1}{3}$

$$\tau = \log^{-1} \overline{2}.0799\ (s\ n)^{\frac{1}{2}}\ d. \qquad (45)$$

This shows that, as before stated, the size of the grain should, in similar guns, vary directly with the caliber. For the 8-in. rifle, this makes $\tau = 1.0$, or, from the preceding table, the size of the grain should be about 63% of its present linear dimensions; the force of the powder being unity, and the form remaining unchanged.

3. Maximum velocity.

The maximum value of $\Pi = 5.488 \times 4.5 \times 1.245 = 30.73$.

But $\Pi = \chi \left(\frac{w}{W}\right)^{\frac{1}{4}} = \frac{G\ V^4\ W^{\frac{1}{4}}}{p_0\ d^2\ u\ w^{\frac{3}{4}}} = 30.73.$

By substituting the value of $w^{\frac{3}{4}}$ deduced from Equation (44)

and reducing, we have*

$$V = \left[\log^{-1} 2.7355 \frac{p_0^2 \, n^{\frac{3}{2}}}{s^{\frac{3}{2}}}\right]^{\frac{1}{4}} \tag{46}$$

Which in the type-gun gives

$$V = 1716, \text{ or } \chi = 46.48.$$

The largest value of χ yet attained with this gun is about 35.0; showing an efficiency of about 80 per cent.

Remark.

For sea coast guns, in which the bulk and weight of the charge is of no special consequence, since the guns are stationary and magazine room is ample; the waste of the powder and the increased volume of the chamber necessitated by the present use of very large charges may be neglected in favor of the high muzzle energies required. But as the caliber of the gun decreases, and its mobility increases, the necessity for reducing the weight of the charge becomes more important. This is especially true in the loading of magazine small arms, the efficiency of which requires the weight of the ammunition to be reduced to a minimum; so that the number of cartridges that the soldier may carry will be as great as possible.

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* This is independent of the caliber as would be expected from the principle of similitude. χ may also be shown to be independent of the caliber, by substituting values of W and w in terms of s, and of u in terms of d.

CHAPTER XIII.

HISTORY OF GUNPOWDER.

Origin.

Knowledge of the properties of nitre as a supporter of combustion are attributed to the accidental kindling of the embers of a camp fire by the salt, often, in India, found effer-escent upon the surface of the ground. As sulphur is not essential, its first employment cannot be conjectured. For its binding properties honey was used at an early date.

Early Use.

The use of gunpowder was at first confined to fireworks and rockets. These are mentioned in Chinese records over 2000 years old, and seem to be indicated in the account of Alexander's invasion of India at about the same epoch.

The transition from its use in a paper tube, or bamboo *cane*, to *cannon* of different sizes is indicated by the etymology of the latter name. The barrel of any fire arm is in French called *canon.*

Early Cannon.

The first use of gunpowder as an agent for propelling projectiles is assigned to the Moors at the siege of Baza in Spain, about 1325; twenty-one years before the battle of Crecy. This is about the time that the chemist monk, Berthold Schwartz, of Freiburg, is said to have discovered its powers by the accidental ignition of a ternary mixture, lying in a mortar and covered with a stone.

Owing to the weakness of the early cannon—which were constructed after the manner of ordinary barrels, sometimes

of iron bars welded together longitudinally and hooped with iron tires, and sometimes even of wood, wrapped with rope—efforts at first were directed to reducing the strength of the new agent.

Early Powder.

Therefore, although the best proportions had long been known, it was often composed of equal parts of the three ingredients, and sometimes mixed with saw-dust, resin, sand, or ashes.

It was often mixed and ground by hand as required, and was used in the form of a fine meal or *powder*, from which its name is derived.

The diminished velocity of inflammation resulting from the use of meal powder favored the end in view; but, since the cartridge was yet unknown, the condition of this powder made it so inconvenient to load the long guns then used that the efficiency of artillery was much impaired.

Early Breech-Loaders.

To overcome this difficulty in loading, cannon at a very early date were made to load through the breech. But the arts at that time afforded no means of preventing the escape of gas through the joint so formed, and such cannon are comparatively rare.

It will be seen hereafter that the practical utilization of this principle depended upon the discovery of the self-sealing *gas check*, the best form of which exists in the metallic cartridge case, now used for small arms.

But for this essential improvement many of the systems now in vogue are but repetitions of these ancient forms, not only in principle, but in many details of construction and operation.

The reciprocal evolution of the gun and its ammunition is a striking illustration of the law of continuity.

Men have probably always been equally ingenious in utilizing the accumulated capital of knowledge at their command; but the successful application of even simple principles requires, in many cases, the parallel development of apparently unrelated arts.

Intermediate Stage.

It was not until near the close of the 16th century that cannon, first of copper or its alloys, and then of cast iron, were made strong enough to resist the pressures due to the use of the grained powder, the use of which had hitherto been confined to muskets. This was called *corned* powder, *vide pepper-corn; barley-corn; corning-mill.*

Until about the middle of the present century no great improvement occurred in gunpowder or in cannon. The reasons for this were the general assumption that gunpowder was instantaneously converted into gas, and the want of any apparatus for measuring pressures.

Use of Éprouvette.

Gunpowder was proved by firing it from the *éprouvette*, a small mortar with its axis carefully fixed at an elevation of 45°. The quality of the gunpowder was determined by the distance to which an accurately fitting ball of a given weight was thrown by a given weight of powder. Although some difference existed in the size of the grain used in different guns, the *proof range* increased as the size of the grain diminished; so that for large guns the size of the grain, as measured by our present standard, was exceedingly small. See Chap. XI.

RODMAN'S IMPROVEMENTS.

1. Pressure Gauge.

The late General Rodman, of the United States Ordnance Department, was the first to investigate the properties of gunpowder in the modern method.

His experiments, conducted with the view of increasing the effectiveness of the system of cannon which bears his name, depended primarily upon his employment of the pressure gauge. This was a pyramidal indenting tool, previously used by him to test the relative hardness of cannon metals, and applied in the manner indicated for the crusher gauge.

Although open to many grave objections of detail, this instrument gave useful relative results and served to draw attention to the very erroneous estimates previously made as to the pressure exerted by gunpowder. When fired in its own volume, this had been variously estimated at from 0.7 to 700 tons per square inch.

2. Powder.

a. Mammoth.

Rodman's first step was to recommend the use of large charges of "mammoth" powder, which was of about three times the diameter of the largest powder previously used.

This gave satisfactory velocities and moderate pressures; and, since its manufacture required less granulation than before, it was cheaper, pound for pound.

b. Perforated.

About 1860, he improved upon this idea by suggesting the use of perforated powder, made for small cannon in cylindrical cakes, and for larger cannon in hexagonal prisms which could be built up into cartridges.

Owing to the great cost and novelty of this powder, and to the intervention of the civil war, the perforated powder was used in this country only for experiments; but the mammoth powder has until lately been exclusively used for heavy guns.

DERIVATIVES FROM RODMAN'S POWDER.

Russian Powder.

The perforated prisms were experimented with in Russia, from 1860–1865, being finally made much smaller than

Rodman's, and pierced with seven small holes. The powder was so made in order to adapt it to the muzzle loading guns then used. See Fig. 13, Chap. IV. This is known as *Russian prismatic* powder.

English Powders.

The English objected to this powder, saying that, owing to the number of perforations it contained and to its diminished density, it was liable to break up in the gun.

About 1875, they returned to General Rodman's original idea, adopting the cubical *Pebble* powder, the cubes, for the largest gun , being about 1½ inches on the edge.

United States Powders.

In the United States, the mammoth powder was improved upon by the adoption about 1873 of the Du Pont *Hexagonal* powder, Fig. 12, Chap. IV.

This and the *Sphero-Hexagonal* powder, Fig. 12, have the advantage of great uniformity in the size and shape of the grains and in the form of the interstices between the grains. They are also progressive, owing to the diminished density of the interior of each grain.

This results from the fact that the effect of compression is not transmitted homogeneously throughout the mass compressed. The density is always greatest next to the moving surface.

For reasons given in the text, *Flat* powders of the *L. X.* type, Fig. 12, are also occasionally used.

Italian Powder.

The *Fossano* powder, made in Italy, consists of an agglomeration of dense grains of medium size, set in a mass of powder meal and pressed to a density less than that of the individual grains. Its operation is distinctly progressive.

The principle is applied to other powders, both molded and of irregular granulation.

MODERN POWDERS.

In order to obtain the most effective combination of gun and powder, each type of gun now requires a special powder, and some cannon, as mortars, require more than one powder for each mortar. This increases greatly the difficulty of supply.

The kind of powder best suited to each type of gun is still (in 1888) undergoing experimental investigation.

The advantage of adapting the size of the grain to the size of the gun, upon which for simplicity so much stress has been laid, is becoming of diminished importance, since the effects due to increased size may be attained in many other ways.

Owing to the great number of conditions which require to be simultaneously satisfied, including the effect of meteorological conditions prevailing during manufacture, the powder makers find it difficult to meet the increasing exactness of the demands made upon them. This applies even to the duplication of satisfactory samples.

Present Custom.

All large guns of the present day use hexagonal prisms like the Russian prismatic, but pierced with a single hole. This is easier to make and its ballistic properties are better.

It is preferably a concrete powder made by consolidating under pressure small grains of powder previously compressed in the ordinary manner. Mealed powder is sometimes used instead of that which has been grained.

CHAPTER XIV.

HIGH EXPLOSIVES.

Classification.

Except the chlorate mixtures, the high explosives used in warfare are all organic nitro-substitution compounds, generally of the third order, in which 3 atoms of H are replaced by 3 molecules of NO_2.

The most important are Gun-cotton, Nitro-glycerine, and their derivatives. The derivatives of picric acid are growing in importance, and so, for special purposes, are the mono-, di-, and tri-nitro-benzines and naphthalines.

Those which in their operation resemble the mercuric fulminate are called *fulminating compounds*, and include, besides their typical salt, the mixtures in which the chlorates are used dry.

The demands of civil engineering and the hope of successfully adapting these explosives to warfare are constantly increasing the number of those for which both safety and efficiency are claimed. On the other hand, many, once famous, are obsolete, so that the following discussion will relate only to those of which long experience has demonstrated the essential properties, and to the most distinguished of recent competitors for the selection of the engineer,

Danger.

Although their composition and violence render the handling of many as compared with gunpowder, dangerous; yet, a knowledge of their properties is demanded by the conditions of the time; and, as with gunpowder and steam, this knowledge comes principally by experience.

The disasters reported with such apparent frequency are the price of progress toward safety, and point rather to the enormous consumption of these explosives, often by ignorant and reckless persons, than to any necessary peril when proper precautions are observed.

Commercial Importance.

The scale on which these explosives are employed, probably, as with gunpowder, much greater in time of peace than in war, appears from the size of blasts fired almost daily in the Californian mines during the period of their greatest activity. These blasts often contained 50,000 pounds apiece.

The great blast at Hell Gate, New York Harbor, in 1885, contained but six times as much.

The economic value of an explosive depends so much upon the net cost of the work performed that it is interesting to note the following relative scale of prices per pound in 1888.

Explosive.	Price.	Proportion.
Gunpowder,	20 Cts.	1.0
Dynamite,	50	2.5
Nitro-glycerine,	80	4.0
Gun-cotton,	1.00	5.0

COMMON PROPERTIES OF GUN-COTTON, NITRO-GLYCERINE, AND THEIR DERIVATIVES.*

Sensitiveness.

When not freed from the acids used in their manufacture, these explosives are prone to spontaneous decomposition and tend to form products of a lower order of substitution.

While undergoing decomposition, their sensitiveness is increased, but their efficiency when exploded is diminished.

When properly prepared, they are not sensitive to moderate

* Cadets are advised to review the articles in the Chemistry which treat of nitro-glycerine and gun-cotton.

shock; but friction, the impact of a projectile, or the shock of discharge may cause their explosion.

Firing.

As a rule, they all explode at about 200°. When ignited by a flame and unconfined, they burn more or less quietly. If confined, their explosion is of a low order unless they are detonated. Their behavior in this respect depends much upon their mass and the resistance of the envelope. See Chap. II.

They possess the remarkable property of exploding violently when gradually heated to about 200°; whereas, if dropped upon a red hot iron, they may simply deflagrate.

Detonation.

Owing to the variety of the means by which the mercuric fulminate may be ignited and to the nature of its product, it is almost exclusively employed for detonation, preferably alone and pure, and sometimes with a *primer* of dry guncotton.

The detonators are commercially known as *blasting caps*, *exploders*, or *fuzes* of various degrees of "force" according to the quantity of fulminate they contain. The fulminate lies in a thin copper tube, one end of which is closed, and is ignited either by a quick-match or by the heating of a fine platinum wire by the electric current. The detonator is placed in immediate contact with the charge, but should be so disposed that, if the quick-match is used, the charge shall not be prematurely ignited.

The mass of the fulminate should bear a certain ratio to the mass and condition of the explosive; this may neutralize the advantages on the score of safety which the sluggishness of the explosive confers.

Long charges may require to have dispersed through them several detonators in order to maintain the energy of the explosive wave.

Products.

Except Nitro-glycerine all the substitution compounds yield a large amount of CO, and hence, where potential is sought, require the addition of an oxydizing agent.

Pressures.

The ordinary gauge being unsuited to measuring the high pressures of detonation, special devices have been contrived.

General Abbott of the U. S. Engineers, in a series of experiments (which bear to the high explosives the same relation as do Noble and Abel's experiments to gunpowder), suspended in water his gauges at definite distances from the submerged explosive.

For experiments in air, charges of given weights are detonated either within or upon similar blocks of lead and the resulting deformations compared. Or the exact charges required to burst similar hollow projectiles may be determined.

Effects.

General Abbott's experiments give the following scale by which to measure the *force*, Chap. II, of explosives. His results apply only to sub-aqueous mining and indicate the paradoxical fact that Dynamite is *more powerful* than Nitro-glycerine.

He found that the pressures registered by a crusher gauge varied as the ⅔ power of the charge and inversely as the 1.4 power of the distance. Or calling p the pressure, w the weight of the charge, d the distance, and k a constant varying for each explosive and for the nadir angle under water,

$$p = \sqrt[3]{\left(\frac{k\,w}{d^{2.1}}\right)^2}.$$

These comparative results are expressed by the following table:

Nitro-glycerine,	81	0.93
Gun-cotton,	87	1.00
Dynamite,	100	1.15
Explosive Gelatine,	117	1.35

a result quite different from that of Chap. II.

On the other hand, extended practice in mining operations under ground confirms the relative useful values of the high explosives as determined by their potentials and stated in Chap. II.

Three spheres surround the center of the explosion:

1. The sphere of pulverization.
2. The sphere of rupture or dislocation.
3. The sphere of fracture or fissure.

The relative dimensions of these spheres vary with the force and potential of the explosive.

Tamping.

The great rapidity of the reaction renders special tamping unnecessary, since the pressure of the atmosphere suffices to produce many of the effects desired. This is the origin of the common idea that such explosives act *downward.* This property is particularly valuable in military operations where time is precious.

The best results, however, are found when they are tamped. Even a thin layer of earth or water greatly increases their effect. For a similar reason the mass of the charge is best placed between the detonator and the object to be destroyed.

Example.

Long iron tubes filled with dynamite have been detonated in air without converting all of their contents. When the tubes were submerged, the entire charge was detonated, Chap. II, page 5. The accidental explosion of charges which have been imperfectly detonated leads frequently to disaster, and so, it may be said, does tamping with an iron bar.

Physical Condition.

The greater the density of the explosive the smaller the bore hole required to receive it, and hence the greater its economy.

Plastic explosives are valuable since they may be used in irregular cavities, and in those opening downward; they may also be rammed after loading so as to increase the value of Δ.

The advantages in this respect of the liquid state of nitro-glycerine made it very popular at first; but its tendency to leak in transportation and to filter through crevices in the rock is very objectionable, since in a thin film it is easily exploded by impact and especially so by friction. Cans containing it have been exploded by twisting the cork. The granular form is advantageous on account of the ease with which it may be loaded into bottle shaped cavities, as in hollow projectiles and torpedoes. Rigid prisms form convenient packages for transportation, but require cavities of a special form to develop the best results.

Cold.

When in a liquid or plastic form, the high explosives have their sensitiveness much impaired by freezing. This occurs at a little above 0°.

The force and sensitiveness of loose dynamite are not impaired by its freezing.

Heat.

In such cases thawing is dangerous unless very gradually performed, as by the heat of the body, of manure, or of luke-warm water.

The nitro-glycerine in frozen dynamite of the solid form tends to exude on thawing.

The sensitiveness of an explosive increases with its temperature.

Water.

Nitro-glycerine and gun-cotton are insoluble. Water tends to displace the nitro-glycerine from dynamite which has been

compressed; but, strangely, has no such effect upon that which is loosely granular. For this reason sub-aqueous torpedoes are charged with loose dynamite.

Owing to its greater density the displaced nitro-glycerine settles to the bottom of the vessel containing the dynamite, whence it may exude and lead to the consequences noted above.

When dynamite or gun-cotton is wet, it ignites with great difficulty but may be detonated by a powerful primer. Any soluble addition is of course removed by water.

Use.

Except gun-cotton and the picrates, all the high explosives have so far been employed only for mining and demolition, and to a limited extent in pyrotechny.

Efforts are constantly making to adapt them to the bursting charges of hollow projectiles, by affecting either their condition, the construction of the projectile, or the source of energy by which it is thrown.

Such attempts have not yet (1888) wholly passed beyond the stage of experiment and, though occasionally successful, have yet to endure the test of long continued firing. In many cases it appears that failure comes less from explosion under the initial shock than from the friction due to the rotation of the projectile. If the initial shock or acceleration be diminished, flatness of trajectory is sacrificed or the gun is made inconveniently long; if the rotation of the projectile is abandoned, inaccuracy results.

The sensitiveness of the explosive tends to cause a premature explosion on impact against armor and its force tends to pulverize the envelope into ineffective fragments.

The sphere of such explosives appears to be confined to the ordinary sub-aqueous mines or to their employment in aerial torpedoes, exploding under water in the vicinity of a vessel, as in the Zalinski system; or against earth works as

in the new gun-cotton shell now employed in Germany. This projectile has been fired with charges as great as 110 lbs. Captain Zalinski has fired a mixed charge of high explosives weighing 500 pounds to a distance of one mile.

Some of the high explosives, notably the gun-cotton class, have been used for fire arms, principally in fowling pieces, for which the reasons assigned, Chap. XI, page 18, particularly adapt them. The absence of smoke is a considerable advantage. They have even been employed by the Austrians for field pieces.

The uncertainty as to the order of the explosion resulting from accidental variations in the value of Δ, has caused their use in cannon to be abandoned. For the former purpose it is still unfortunately common.

GUN COTTON.

Forms.

This occurs in three forms; viz.:

1. In the flocculent or pulverulent form, made from cotton wool as indicated in the chemistry.

2. Prepared from the first form by pulping and compression to a density a little greater than that of water.

3. In grains, made by disintegrating the second form above.

Condition.

The first form is always used dry and is employed only in pyrotechny. The other two are used either wet or dry, and when wet, are sometimes protected by a water-proof coating to retard evaporation.

Firing.

Dry gun-cotton ignites at a lower temperature than any other of the common explosives. Its combustion may be retarded by compression and the addition of a gum.

When it contains from 20 to 30 per cent of water, it can-

not be ignited until the water has been evaporated by the flame. One ton of loose wet gun-cotton has been burned with safety in a bon-fire

Detonation.

When wet and compressed, it may be detonated by using a sufficiently large primer of dry gun-cotton. Its incorporation in a dry state with paraffine is said to yield the same results as to safety as when it is wet, without diminishing its sensitiveness to detonation. This avoids the difficulty of preventing evaporation.

Reaction.

This varies with the value of Δ and with other conditions, but may be represented by the following formula,

$$2C_6H_7(NO_2)_3O_5=7OH_2+3CO_2+9CO+6N.$$

To increase its potential a nitrate or chlorate is often added, the latter being the more energetic.

Gun-cotton mixed with one third its weight of a nitrate forms *Tonite*, an explosive much used in the Californian mines.

Advantages.

Compared with gunpowder, its manufacture is less dangerous and the apparatus can easily be improvised from the paper-mills.

Since it forms no dust and can be kept wet, it is safe in transportation and in store.

In mining, as in fire arms, it yields no solid products, and in sub-marine mining it can be used under water; having even been detonated in a net.

Disadvantages.

Besides those which relate to its sensitiveness and violence, the principal objection to its employment in artillery applies to the absence of smoke which serves to mark the bursting point of a distant shell.

MANUFACTURE OF GUN-COTTON.

Former Method.

Gun-cotton, like nitro-glycerine, was discovered about 1846. It was first made by dipping cotton wool into mixed sulphuric and nitric acid and washing thoroughly the gun-cotton wool so formed. But it was found to be impossible to remove the free acids from the tortuous capillary tubes of which cotton wool is composed, and the resulting product was dangerous in store.

Abel's Method.

The time of manufacture has been much reduced and the quality of the product improved by the following method.

Instead of using raw cotton, often containing impurities which are liable to cause spontaneous decomposition, *cotton waste* is employed. This has been previously spun into yarn for cloth and is therefore mechanically clean.

Preliminary Operations.

Its conversion into gun-cotton follows the method previously taught, the essential points being:—

1. To prevent the continued action of dilute acids and the consequent formation of di-nitro-cellulose (Collodion cotton), by removing the cotton after its first immersion to a fresh mixture of acids in which it is soaked for several hours. After each immersion the excess of acid is removed by wringing.

2. To prevent an undue rise in temperature, by making the first immersion in small quantities at a time, and surrounding the vessels containing the cotton with running water.

3. To prevent the access of water to these contents. A drop of sweat may cause the acid cotton to ignite.

Final Operations.

After the final wringing, it is washed by plunging small quantities of the cotton into large quantities of water.

The cotton is then reduced to a pulp by the rotary knives of the *rag engine* used in paper making. These operate under water.

Being now in short tubes, the washing can be thoroughly performed by means of the paper maker's *poacher*. This is a vertical water wheel working on one side of an oblong trough through which a longitudinal partition extends nearly from end to end.

After a protracted washing in the poacher, the free acids still remaining are neutralized by some alkali; this having been washed out, the pulp is, after draining, ready for the hydraulic press.

After pressing the cylinders they are carefully and slowly dried; or, they may be kept wet as previously stated.

A similar product has been made from bran or straw, and is known as *fulmi-bran*, etc.

NITRO-GLYCERINE.

Manufacture.

The preparation of this explosive has been sufficiently described in the course of chemistry The principal points to be observed are:—

1. To prevent a rise in temperature by pouring the glycerine slowly into the mixed acids, and to preserve a low temperature by a jacket of running water and by agitating the mixture by a current of air.

2. To wash the product thoroughly with cold water and finally with an alkaline solution. The addition of cold water precipitates that portion of the nitro-glycerine which remains suspended in the heavy acid liquid.

Too much importance cannot be attached to the entire removal of free acid. The detection of free acid constitutes one of the most important tests of this product.

When first made, it is white and opaque; it soon assumes an oily appearance which, if well made, it retains. Its density is about 1.6.

Reaction.

The explosion of nitro-glycerine gives the following reaction,

$$2\,C_3H_5(NO_2)_3O_3 = 6\,CO_2 + 6N + O + 5\,OH_2.$$

Following a general law, since its composition furnishes an excess of oxygen, the reaction is sensibly constant and is found to agree with that deduced on theoretical grounds. In this respect it differs from most of the explosives.

Special Properties.

As ordinarily used, this is the most powerful of the explosives, excelling both in potential and force.

It was originally thought to be perfectly safe when frozen; but it has since been found that, when in this condition, it can be exploded by a powerful shock if concentrated upon a mass sufficiently small.

DERIVATIVES OF NITRO-GLYCERINE.

Owing to the dangerous properties of liquid nitro-glycerine, it is no longer employed except with an absorbent *base* or *dope* which will prevent its exudation.

The absorbents are of two kinds:—

I. Those which are chemically inactive, such as kieselgühr (also known as "tripoli" and "electro-silicon"), micaceous scales, and, for its alkaline properties, magnesium carbonate.

II. Those which are chemically active.

These derivatives have a density of about 1.6. They are usually plastic, which gives them great practical utility.

I. MECHANICAL ABSORBENTS.

Dynamites. Giant Powder.

Of these absorbents the best is kieselgühr. This consists of microscopic shells, the cavities in which retain the liquid and protect it from ordinary shock. Kieselgühr has remarkable properties as an absorbent; it can take up three times its weight of nitro-glycerine without exudation, even when under considerable pressure.

Different grades of dynamite are made depending upon the proportion of nitro-glycerine which they contain. The highest is called No. 1.

Owing to the knowledge of the properties of this explosive, gained by the torpedo service and by private industry, it may be called the standard high explosive of the United States. For torpedoes its merits consist in:—

1. Its force.
2. Its permanency under the varied conditions and accidents of service.
3. Its safety and convenience in loading.
4. The readiness with which it may be procured in the market.

This was true in 1881. Since then several explosives have been invented which threaten its supremacy,

Preservation.

Although used for special purposes in the granular form, in which it resembles brown sugar, it is generally put up compressed in cylinders wrapped tightly with paraffined paper. These are packed in sawdust in wooden boxes, preferably made light, without metallic parts and coated inside with a water-proof varnish.

When received, the boxes should be partly opened to facilitate the discovery of the nitrous fumes that accompany the process of spontaneous decomposition. Their contents

should be tested for exudation and acidity, and should be carefully kept from water.

II. CHEMICAL ABSORBENTS.

Properties.

Absorbents of this class reduce the quantity of nitro-glycerine required to produce a given effect and so cheapen the product.

Their judicious selection adds greatly to the energy developed by the nitro-glycerine alone, so that the economic value of the explosive may increase more rapidly than does its percentage of nitro-glycerine.

For sub-aqueous explosions it appears that with any particular base there is an economic gain in increasing the percentage of nitro-glycerine up to a certain point, but that beyond that point the advantage ceases. There appears to be a decided advantage in gelatinizing the nitro-glycerine before its absorption.

See *Forcite* and Explosive Gelatine, *post.*

Classification.

The chemical absorbents may be conveniently divided into two general groups, according as they are simply combustible, or in themselves high explosives.

Class I. A combustible dope.

When finely ground cellulose is treated with super-heated steam, it is converted into a jelly capable of absorbing 19 times its weight of nitro-glycerine. With or without the addition of nitre, it forms *Forcite*, a most powerful explosive.

Gunpowder, preferably made after Col. Wiener's method, Chap. IV, may be coated with nitro-glycerine, the detonation of which detonates powder, Chap. II. This forms the *Judson* powder.

Class II. A high explosive as a dope.

The most famous is known as *Explosive Gelatine*. This consists of about 93 per cent of nitro-glycerine with 7 per cent of collodion gun-cotton (di-nitro-cellulose). The addition of 3 or 4 per cent of camphor greatly diminishes its sensitiveness and adapts it particularly for warfare.

It is generally a transparent jelly, but often becomes hard and opaque. The fulmi-bran, page 11, may replace the collodion cotton.

Although found by General Abbott to be stronger than nitro-glycerine, it is much safer, particularly against shock. It has been found to burn freely without explosion, even when confined, and to resist perfectly the action of water.

It requires an initial primer for its detonation and the weight of the primer required increases as its sensitiveness diminishes.

When the collodion cotton is not thoroughly purified, this explosive tends to decompose spontaneously. Otherwise it is quite stable.

New smokeless powder.

By reversing the proportions of nitro-glycerine, and collodion used in explosive gelatine, and retaining the camphor, the compound becomes plastic when heated. It may then be pressed into sheets or drawn into wires or rods which, on cooling, become horn-like, like the *celluloid* of commerce.

The reduction of w and the increase of μ are reported to give in the 6 in. Rifle a value of $\chi = 100+$. Its tactical advantages adapt it particularly to rapid firing arms of small caliber. The special difficulties to be overcome refer to the volatility of the camphor and to the erosion of the bore resulting from the heat of the explosion. See Chap. IX, Notes.

NITRO-BENZINE OR -BENZOLE.

The preparation of this resembles that of nitro-glycerine,

the mono- (liquid), and di-, and tri-nitro benzines, (crystalline) being formed. (Bloxam, Art. 325.)

These substitution products are in themselves inexplosive, and show by their composition, $C_6 H_x (NO_2)_{6-x}$, the necessity for the addition of an oxydizing agent.

Rack-a-Rock is made at the time of its employment by saturating $K ClO_3$ with crude mono-nitro-benzine, or even with the "dead oil" from the gas works which has been diluted with CS_2 containing a small proportion of sulphur. By exposure to the air the CS_2 evaporates, leaving the finely divided sulphur on the salt but protected by the lubricating property of the oil or nitro-benzine against explosion by friction.

When the dope is finely ground and the charge exploded by a powerful primer it is found to be nearly as powerful as dynamite No. 1.

This is the only chlorate mixture which has been found safe in practice.

Helhofite, as used in Germany for armor piercing projectiles, is another of the Sprengel Safety Mixtures prepared as wanted by dissolving di-nitro-benzine in nitric acid.

Bellite (Latin: *Bellum*—War), is a recent Swedish explosive made of about $\frac{1}{6}$ tri-nitro-benzine and $\frac{5}{6}$ ammonium nitrate, incorporated together.

This is distinguished by its great safety under all conditions and by its greater potential as compared with dynamite,

Only dampness affects it. It is almost incombustible, smouldering only by the continued application of flame. It is so insensitive to shock that the detonation of itself upon a box filled with the explosive, or the explosion of gunpowder in its midst fails to explode it. A wad of it has been fired from a fowling piece against a target without injury to either. It gives no injurious gases, nor flame, which properties,

together with its high potential, particularly adapt it for the coal miner. It is also cheap and indifferent to variations in temperature.

Tamping is necessary to develop its full effect, even when detonated; but when tamped and detonated, it is about 33 per cent stronger than dynamite.

The crystaline form of the two ingredients of Bellite would appear to insure its stability in store and to make of it one of the best high explosives where potential is required, as in torpedo shells. For sub-aqueous mining, dynamite is probably better suited.

PICRIC ACID (TRI-NITRO-PHENOL).

This is made by the action of nitric acid on carbolic acid (phenol). It occurs in slightly soluble plates of a bright yellow and is much used in dyeing. Unconfined, it will not explode by heat, but may be detonated.

When mixed with gun cotton dissolved in ether, it is said to form the new French explosive, *Melinite.*

Emmensite is a recent American explosive, prepared from crystallized *Emmens* acid and a nitrate. The acid results from the solution of picric in nitric acid.

The claims made for this explosive resemble those noted under the description of Bellite. It is (1891) under trial in the United States.

THE PICRATES.

The potassium and ammonium salts are the only ones employed.

The former with the addition of nitre and charcoal forms *Designolle's* powder. This was found too dangerous for use, as it tends to *detonate* on ignition.

The ammonium salt is less sensitive to shock and burns in the air like resin. With the addition of a nearly equal

part of nitre and prepared like ordinary gunpowder, it forms the powder of *Brugère*. In small arms, a charge less than one half the charge of ordinary gunpowder suffices to produce the same effects. This is of importance since it enables the size and weight of the cartridge used in magazine rifles to be greatly reduced.

The powder yields no smoke or fouling but is hygroscopic.

It is believed that the new powder used in the French *Lebel* rifle is a variety of Brugère powder.

THE FULMINATES.

The mercuric salt is the only one of practical value. Its efficiency depends rather upon its *force* and the nature of its product than upon the heat evolved by its decomposition.

This follows from the reaction,

$$C_2HgN_2O_2 = 2CO + Hg + 2N.$$

Its force is said to be nearly 10 times that of gunpowder.

When dry it is easily detonated by shock, friction, a temperature of about 200°, or by the strong acids.

Berthelot finds that even so stable a gas as nitric oxide is dissociated by the detonation of mercuric fulminate. While for detonation it is preferably used pure, for igniting the charges of fire arms it is mixed with an oxydizing agent and often a combustible, in order to increase the length of the flame. Powdered glass is also added when the salt is exploded by percussion.

Its safety in manufacture depends upon its absolute inexplosiveness when wet. If placed upon a metallic surface, it tends to decompose; hence, percussion caps are varnished before they are primed.

Under no circumstances should the fulminate be carried or stored near any of the high explosives.

CHLORATE MIXTURES.

Since their discovery, a century ago, frequent efforts have been made to utilize the chlorate mixtures in mining and as a substitute for gunpowder. Their extreme sensitiveness to friction has almost uniformly caused their employment for such purposes to result in disaster.

Mixed with Sb_2S_3, the potassium chlorate forms the friction composition used in cannon primers. It is also employed in pyrotechny to compensate for the cooling effect of substances employed to give color and brilliancy to the flame.

GENERAL REMARKS ON THE EMPLOYMENT OF THE HIGH EXPLOSIVES.

In Guns.

Their employment has always failed, except for small arms and as noted page 8.

For Bursting Charges of Hollow Projectiles.*

Explosive gelatine has been occasionally fired without premature explosion, by the use of diaphragms within the shell. See also gun-cotton, and the experiments with the Zalinski gun before noted.

For Demolition of Walls, etc.

Unless the wall is very strong, the best results in breaching appear to follow the detonation of the explosive at the base of the wall and at a few inches distant. This distributes the effect, and racks and fissures the wall so as to facilitate its destruction by hand.

Exploded in immediate contact, a smaller hole is said to be made and the energy to be expended in giving motion

* The premature explosion of such bursting charges by the shock of discharge is often attributed to the sensitiveness of the fulminate necessarily used in the detonating fuze.

to but few fragments. About 10 lbs. of dynamite will open a practicable breach in a two foot stone wall. Tamping would reduce the size of the charge required.

Houses, Palisades, etc.

About 5 lbs. of dynamite will wreck a small stone dwelling if exploded near its center, since it tends to overthrow all the walls instead of blowing out through the nearest one. The same quantity suffices for a linear yard of ordinary palisading. A pound of dynamite will shatter a 12 inch bridge timber.*

Disabling Cannon.

When time permits, bring the gun as nearly vertical as possible; fill it with water, plug it, and explode simultaneously two one pound charges of gun-cotton, one at the bottom of the bore and one in the chase. When time is scant, insert a shot within the bore, and place on top of the chase, between the shot and the muzzle, two pounds of gun-cotton, laying over it a filled sand bag or a sod. Such charges are said in the English Manual to suffice for the lighter natures of sea coast guns.

It is found to be more advantageous to attack the cannon themselves, than to waste time and explosive material on their carriages.

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* These directions are intended to apply only to hasty operations. When time permits, the best results will follow from placing the charge under or within the structure to be demolished. The quantities are approximate.

CHAPTER XV.

METALLURGY.

I. TESTING MACHINES.

Necessity.

The physical properties of a metal may sometimes be inferred from a knowledge of its chemical composition, but so many other causes may contribute to modify these properties that chemical analysis should be depended on, rather to indicate the existence of certain limiting or possible conditions than to declare the extent to which these conditions have been approached. Thus, what a metal *is* becomes subordinate to what it can *do*; and its proof is more conclusive than even its chemical inspection.

Requisites.

A complete testing machine should include means for determining the varying strains under tensile, crushing, transverse, torsional, and shearing stresses. But for simplicity, and on account of the comparative ease with which all stresses can be referred to that first named, such machines are generally of the tensile type.

Comparisons.

For establishing comparisons the stresses are usually measured per unit of minimum area of cross section, and the strains per unit of length between established points on the specimen. But if the form and dimensions of the specimens are constant, absolute measures may be compared. In the following discussion stresses and strains will, unless otherwise stated, be taken per unit of section and of length.

In the United States stresses are given in pounds per square inch; in England in tons of 2240 lbs. per square inch; in France in kilogrammes per square centimetre; in Russia and Germany in atmospheres.

Form of Specimen.

Since the deductions from proof are conclusive only as to the actual piece tested and are inferential as to all others, the above precautions for the comparison of properties are not wholly sufficient. The general rule in experimental comparisons, of dealing with but one variable at a time, should be followed by subjecting specimens as nearly as possible in size and shape like those to be actually employed to the same kind of stress that they will be called on to sustain.

But the capacity of the machine rarely permits this, and, as its strength limits the maximum cross section, the length of the specimen in units of the corresponding diameter should be approximately proportional to that of the finished piece. The size of the machine and the cost of preparing specimens limit this; so that the length of the specimen is generally about 4, 6, or 8 diameters, with a tendency to increase.

The specimen is held so that its axis coincides with the action line of the force; otherwise, it will rupture in detail, or *tear* across. This condition is fulfilled by making the specimens truly cylindrical with enlarged concentric heads, figure 3, by which they may be held in the machine.

Form of Record.

This states the strains due to certain stresses. They are functions of each other, and the relation may be expressed

$$e = f(w),$$

in which e represents the strain, or the change of form produced by applying the stress w. The stress is taken as the independent variable since it can be more readily controlled

than the strain. Inasmuch as conditions vary too much, and are not yet sufficiently understood to enable the law of this function to be analytically expressed, that which governs any particular case may be best determined empirically:

1st. By forming successive orders of difference in the observed value of the function for equal increments of the variable w.

2d. By plotting a line constructed from these co-ordinate values. Such a line is called a *strain diagram.*

3d. By constructing a strain diagram automatically during the progress of the test.

The order of preference is as follows:

The first method when great accuracy is required and when a micrometer can be used.

The second and third when general comparisons are to be made.

The second to the third when the expense of the registering apparatus is objectionable.

In general, rectilinear strains can be measured more accurately than they can be registered by any mechanical apparatus.

ELASTICITY.

Elastic Limit.

In operating the machine the stress is very slowly and steadily applied, either directly by hydraulic pressure, or indirectly by a screw acting in combination with levers. The stress is relieved at intervals and the specimen permitted to recoil.

The difference between the *strain* e and the *recoil* r is the *set* s, or $e = r + s$. The set may diminish in time and become the *permanent set,* but the first temporary set is that generally recorded. Sets probably occur under all stresses, but may be too small for measurement. This may be illustrated by the curves, figure 1, which are very much exaggerated.

Let OO', OO'' represent certain stresses resulting in strains $O'p'$, $O''p''$, etc. Each of these strains is by definition com-

posed of the recoil $r = r'p'$ and the set $s = O'r'$, etc. Starting from O, as the stresses increase the recoils and sets both increase, but the sets less rapidly than the recoils.

After a certain stress, L, the line of sets becomes nearly parallel* to that of strains, so that for a given increment of stress the increments of strain and set are nearly equal. The stress corresponding to L is the superior limit of the stresses for which the sets increase less rapidly than the recoils, and the inferior limit of those for which the sets increase more rapidly than the recoils. This stress is called the *Elastic Limit.*

Since below the elastic limit the sets are relatively small, and above it the sets are relatively large, when compared with the recoils, it may be defined as *the limit of the stresses within which sets may be neglected and beyond which recoils may be neglected; or the limit separating the consideration of the elasticity of the metal from that of its ductility.*

It may be determined—

I. By finding the stress corresponding to the first significant term of the second order of differences of strains or sets.

II. By inspection of a diagram such as represented in figure 13.

Coefficient of Elasticity.

If the strains below the elastic limit be considered directly proportional to the stresses, this portion of the line will be straight, and the tangent of the angle included between it and the axis of E will be proportional to the reciprocal of the rate at which the specimen submits to (i.e., directly to the rate at which it resists) the stress. This is called the coefficient of elasticity, or

$$\frac{dw}{de} = E = \frac{W}{A} \times \frac{L}{l}, \quad \left(\begin{array}{c}\text{Wheeler,} \\ \text{Eq. 1.}\end{array}\right)$$

* As a rule, the recoils increase gradually throughout.

Elastic Work, etc.

The area bounded by the diagram, the axis of E, and a line drawn through any point of this axis parallel to the axis of W is evidently proportional to the work done by the corresponding stress.

For a given stress $O\ O'''$, the area, $O\ e\ r'''$, is proportional to the work of permanent deformation corresponding to the stress $O\ O'''$. Similarly the difference between the areas $O\ e'p'''$ and $O\ e\ r'''$ is proportional to the work of restitution, or the elastic work, following the same stress. The term *Elastic Work*, as a measure of this elastic property, also known as *toughness*, is properly applied only to the area under the diagram at the elastic limit.

The total area under the diagram up to the point of rupture is proportional to the *potential work of deformation.* While for mechanical units, such as posts, beams, levers, chains, this property is valuable; in the more complex structures required by the principle of the independence of function, such as wheels, trusses, and built-up guns, the elastic work, which comprises in its measure both the elastic limit and the coefficient of elasticity, is much more important.

In such structures the permanent change of form of one of the units may derange the rest; and generally the elastic work may be counted on repeatedly, while the work of permanent deformation can be utilized but once.

FORMS OF TESTING MACHINES.

Tensile.

This is the form most generally used and upon the indications of which modern gun construction is based.

The sketch, figure 2, shows a simple apparatus extemporized for testing the sheet metal from which small-arm cartridges are made. The strains were taken from the punchmarks *a*,*b*, and plotted.

For testing the metal of which cannon are made, a form of

tensile machine recently devised in England consists of a hard steel cone, which by a blow from a falling weight is driven through a ring cut concentrically from one of the short cylinders composing the gun.

Transverse.

The simplest of all is for transverse stress. The specimen is placed on rollers kept at a constant distance apart. One objection to transverse machines is the difficulty of separating the tensile from compressive strain.

A valuable modification of this form of machine is that which tests the capacity of the metal to endure extreme bending, even to the extent of working its ends back and forth as long as the tenacity of the specimen permits. The bending angle thus determined is one of the readiest and best tests for ductile material.

Torsional.

Although a torsional strain is even more complex than the transverse, yet, owing to the ease with which the power of the lever may be increased; to the simplicity, compactness, cheapness and rapidity of operation of machines of this class; and to the ease with which the relative rotary motion of the parts may be made to record the circumstances of the test, this method is very valuable where great accuracy is not required, nor variation in the form of the specimens expected.

For the machine used in this department of instruction, the specimens are of the standard size shown in figure 3. This requires direct comparison of results.

THURSTON'S AUTOGRAPHIC TORSIONAL TESTING MACHINE. FIGURES 4, 5, 6, 7, 8.

Description.

Two similar wrenches with rectangular jaws, facing each other, are carried by the *A* shaped frames shown in figure 4, and revolve independently about axes which are in the same

straight line. The wrenches are not connected except by the interposition of the specimen, which is supported axially by the conical points shown, and kept by folding wedges from revolving in the jaws. The arm B of one wrench carries a weight W at its lower end. The other wrench is revolved by a worm gear, P.

To the frame A is secured a guide curve G, of such form that its ordinates are proportional to the successive torsional moments exerted by B during its revolution.

The pencil-holder ph is carried on the arm B, to which it is pivoted at a and b so as to oscillate in a plane perpendicular to that in which B rotates. A spring, sp, keeps the pencil-holder in contact with the guide curve.

Operation.

As the worm gear revolves it tends to revolve B and to raise W by means of the specimen S.

As B revolves, the roller r rides on the edge of G so that the pencil is displaced laterally in a plane perpendicular to that of its rotation; the object being to establish as follows a system of rectangular co-ordinate axes of stresses and strains, to which the position of the pencil may be referred:

I. To make the lateral displacement of the pencil proportional to the stress, W. Since W is proportional to the moment of B, which, since the weight of B is constant, is proportional to the sine of the vertical angle ϕ, figure 7, the edge of G is so formed that when B is rotated, the pencil will trace upon the cylinder D a curve the equation of which when developed on a plane surface is $y = a \sin \phi$. In this equation y is the variable ordinate of the curve measured along that rectilinear element of D upon which the pencil rests when the inclination of $B = \phi$, and a is a coefficient depending upon the maximum value of y permitted by the construction of the machine.

II. To make the peripheral displacement of the pencil

proportional to the strain, E. Calling x the developed path of the pencil along a circular element of the cylinder, we have

$$x : \varphi^\circ :: 2\pi r : 360^\circ, \text{ or } \phi = \frac{360}{2\pi r} x \text{ and } \therefore y = a \sin\left(\frac{360}{2\pi r} . x\right).$$

The circumference of the drum is 36 inches and its length is 5 inches; therefore, taking x in inches.

$$y = 5 \sin (10 . x).$$

Such a curve having been constructed upon paper may be wrapped around D, and the edge of G be adjusted so as to make the point of the pencil follow the curve as B is revolved, D being at rest.

The strain is evidently proportional to the rotation of D relatively to that of the pencil; while the stress is proportional to the angular displacement of the pencil. This will be understood by imagining lines traced by specimens which are either perfectly extensible or perfectly inextensible. Such lines are limits for all natural specimens which will cause intermediate lines to be traced that will express the relation $e = f(w)$.

Form of Record.

The record is made on a piece of cross-section paper ruled in inches and tenths wrapped tightly around the cylindrical drum D.

The weight W is so taken that the maximum moment $= 500$ lbs.; therefore, since the ruling is 5 inches wide, one division of the paper measured across its width represents a moment of 10 lbs., and, since $2\pi r = 36$ inches, one division along the length of the paper $= 1^\circ$ of strain.

In raising the arm by the specimen, the moment of W is in equilibrio with the torsional stress plus the frictional moment of the journal J; this last is constant and is allowed for in standardizing the machine.

INTERPRETATION OF THE RECORD.

For torsional test this is facilitated by considering the specimen as consisting of parallel fibres, at first rectilinear, and elongating under stress in a helical form. The general form of strain diagrams, whether made by torsional or tensile test, is so similar, that although the following discussion particularly refers to the results of the torsional test, its application may be considered as general.*

General Case.

The combined effect of stress and strain is seen in the typical diagrams, figure 9.

In curve *I* the elastic limit is plainly shown at *a*. The convexity of the first portion is probably due to the preliminary strain of the exterior fibres occurring in soft materials.

The line then becomes sensibly straight, its inclination determining the coefficient of elasticity, $\frac{dw}{de}$, or the rigidity of the specimen. Beyond the elastic limit it becomes wavy, indicating deficient homogeneity as to structure; the fibres are then supposed to slip. Having adjusted themselves they

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* The following relation between torsional and tensile stress has been approximately determined by experiment. Let T = tensile stress in lbs. per square inch; T_1 = torsional stress in lbs.; θ = angle of torsion in degrees. Then, for steel and probably for other ductile metals,

$$T = T_1\left(300 - \frac{\theta}{3}\right).$$

For cast iron,

$$T = T_1\left(300 - \frac{\theta}{3}\right)\frac{8}{15}.$$

The extension of an external fibre and the reduction in area of cross section corresponding to torsional strain are given in tables furnished with the machine. For ultimate extensions the value of θ corresponding to the maximum and not to the ultimate ordinate is taken.

work together, as shown by the subsequent regularity of the line.

At some point b the stress is relaxed, and the pencil falls to some point c; when the stress is re-applied, the pencil rises in the line $c\,b$ and continues nearly parallel to the straight portion of the line $O\,a$ until it reaches its former height $d\,b$. The ordinates then slowly increase until, by the successive rupture of the concentric fibrous layers, the curve terminates at f.

Note the total work $O\,a\,b\,e\,f\,g\,O$; the elastic work $O\,a\,L$; and the recoil $d\,c$ and set $O\,c$ for the stress $d\,b$. The parallelism of $b\,c$ to $O'a$ shows the practical constancy of the coefficient of elasticity under varying stresses *provided the total elongation be diminished by the set.* By some, this coefficient is considered the most permanent physical characteristic of steel, in various forms of which it has been found to vary less than 8 per cent. in specimens whose elastic limits varied 200 per cent.

The point b is a new elastic limit, and the entire line may be termed a locus of elastic limits. Of these the point a is called the *primitive elastic limit*, and the other points various *special elastic limits*. Notice that as these successively rise, the potential work diminishes. The special elasticity thus produced by stress, as distinguished from the primitive elasticity of the specimen, is treated of in gun construction.

Some metals give a curve like $I\,I$, in which it is difficult by either of the methods given, page 3, to determine the elastic limit. In such cases it is generally taken as the stress corresponding to the point of tangency of a line inclined at 45°.

Graphical Representation of Special Physical Properties.

Considering the diagram, figure 9, to represent that of a tensile instead of a torsional test, the principal properties of the specimen are graphically expressed as follows:

The *Tenacity*, or *the capacity to resist rupture by extension*, is measured by the maximum ordinate at *e*. The correspond-

ing stress may be greater than that at which fracture finally occurs. In such a case the form of the portion of the diagram, ef, indicates probably the progressive rupture of the final layers.

The *Elasticity*, or *the property of resisting permanent extension or compression*, as we have seen may be measured either by an absolute quantity, the elastic limit, or by a rate. When this rate is practically uniform, as in steel, page 10, the elastic limit alone may serve to measure the elasticity.

The *Ductility*, or *the property of submitting to permanent extension*, may also be measured by an absolute quantity, Og, or more exactly, Oh, figure 9; or by a rate. This rate is measured by the cotangent of the angle made by the tangent to the diagram at any point beyond the elastic limit and the axis of strains. This measure, although not generally adopted, is important since it illustrates the phenomenon known as the *flow of metals* under stress. As seen in the following examples, this rate may vary not only in degree but also in its sign.

Ductility, though useful in such arts as the drawing of wire and of metallic cups like cartridge cases, is now regarded only a secondary property of cannon metals. Cannon are so proportioned that the elastic limit is the superior limit of the applied forces, but the ductility of the metal is thought to give an additional safeguard against destructive explosion. But safety then depends rather upon the potential work of deformation of which the metal is capable than upon either its tenacity or its ductility alone.

Particular Cases. 1. Woods.

To illustrate these remarks by reference to materials the physical properties of which are more generally known than those of the useful metals, the torsional diagrams of the principal woods used in ordnance construction are represented in figures 10, 11, 12.

The woods are arranged from right to left in the order of

their coefficients of elasticity. This brings them approximately in the order of hardness, or stiffness, black walnut leading.

These qualities fit this wood to resist the abrasion to which gun-stocks are subject, and to give to the easily bent gun-barrel its necessary support.

The elastic limits of cypress and black walnut are seen to be equal, but the cypress is much the tougher of the two. It appears to be about equal to a poor quality of oak, for which wood, in the construction of gun-carriages, it was formerly used in localities where oak could not be procured.

The forms of the diagrams after passing the elastic limit are very characteristic. In some cases, as in ash and white pine, the line continues for some distance parallel to the axis of strains. This would indicate the use of these woods for pieces which being long and slender are apt to be bent. When lightness is an object, as in the former case for wagon poles, sponge and rammer staves, agricultural-tool handles, and in the latter case for building purposes, particularly of railway carriages, the low density of these woods makes them highly esteemed.

The sudden dip of dog-wood, oak, and hickory occurs in most hard woods. It is supposed to arise from the lateral slipping of the fibres, the cementing substance having given way. When this is brittle, as in the resinous yellow pine, a very sharp depression is sometimes seen.

In some cases, as in dog-wood, hickory, and notably in elm, the line rises again, sometimes exceeding the elastic limit. The rise is supposed to be due to the retwisting of the fibres, separated at the elastic limit, into a consistent whole. On the other hand, the step-like decline of some of the diagrams indicates the brittleness of the corresponding woods.

The surprising qualities of dog-wood show that the small size of this tree is the principal bar to its utility.

The importance of testing machines is very imperfectly appreciated among practical manufacturers. This appears from

one of the oak diagrams which was made by a piece taken from a new gun-carriage the stock of which was broken in two by firing.

2. Metals.

The diagrams in figure 13 will be referred to hereafter in discussing the metals represented upon them. To avoid confusion, curves of similar metals are arranged in groups, a new origin for each group being taken along the axis of strains.

II. ORDNANCE METALS.

The principal metals used in Ordnance manufactures are,

Ferreous.	Steel, { high, low.		Cupreous or Kalchoids.	Brasses,
				Bronzes
	Wrought Iron,			and other alloys
	Cast Iron.			of Cu, Sn, Zn.

Nomenclature.

For clearness of definition by scientific men, the forgeable ferreous metals are proposed to be classified according to their *mode of manufacture* and according to *their capacity to harden*, and are designated as follows:

1st. Those made from a pasty mass, by the prefix, WELD.

2d. Those made by fusion, by the prefix, INGOT.

3d. Those which will harden and temper by the usual treatment of steel, by the suffix, STEEL.

4th. Those which will not sensibly harden, by the suffix, IRON.

5th. The only unforgeable ferreous cannon metal is cast iron, known in the crude state as *pig-iron* and after remelting, as *castings*.

This classification affords the following scheme:

TABLE I. NOMENCLATURE.

	Kind.	Class.	Made by.	Ability to harden.	Name.	Examples.
Ordnance Metals.	Ferreous.	forgeable.	agglutination.	superior.	Weld-Steel.	Blister, shear, puddled steel.
				inferior.	Weld-Iron.	Wrought iron.
			fusion.	superior.	Ingot-Steel.*	Cast or crucible steel: Bessemer or open-hearth steel or *metal*, according to grade.
				inferior.	Ingot-Iron.†	
		not forgeable.	fusion.	superior.	White Cast Iron. (C. combined)	Chilled shot, car wheels.
				inferior.	Gray Cast Iron. (C. graphitic)	Ordinary castings.
					Malleable Iron.‡ (White iron de-carbonized.)	Ordinary white castings heated with iron oxide.
	Cupreous.	not forgeable.§	fusion.	inferior.	Kalchoids.	Bronze or gun-metal = Cu+Sn; Brass = Cu+Zn; Phosphor- or Aluminium-Bronze, etc.

* Generally known as, and called herein, High Steel.
† Generally known as, and called herein, Low Steel; also called Bessemer, or Open-hearth *Metal*.
‡ Forgeable in a slight degree at comparatively low temperatures.

Relative Importance.

Owing to their peculiar adaptability to the demands of construction, the ferreous ingot metals are gradually superseding all others, and the time when they will be altogether employed except for subordinate purposes is delayed only by our imperfect knowledge of their properties. These metals, under the common name of *steel*, already cover in their application the wide range between castings for the frames of iron-clads, weighing many tons, figure 14, and horseshoe nails, the successful making of which was formerly considered the most severe test of the quality of wrought iron. The ages of stone, bronze, cast and wrought iron have been succeeded by the age of steel.

On this account the following discussion will relate principally to steel. As a cannon metal it has only recently come into favor, having been considered brittle and untrustworthy. It is largely due to the patient genius of Krupp and Whitworth that this prejudice has been overcome.

As an example of the quality of modern gun steel may be mentioned the fact that it was found impossible to break a gun hoop under a drop giving a blow of nearly 100 ft.-tons, the arrangement being represented in figure 15. Sixty or seventy blows shortened the vertical diameter only half an inch.

III. PROPERTIES.

The useful properties of metals have regard to their

I. Constitution. (*To be.*)	I. Chemical as to	1, Homogeneity, 2, Composition.
	II. Physical as to	1, Homogeneity, 2, Structure, 3, Strain.

II. Capacity for resisting. (*To do.*)	I. Tensile Stress, or	1, Tenacity,
		2, Ductility.
	II. Compressive Stress,	1, Incompressibility,
		2, Hardness.
	III. Either Stress, or	1, Elasticity,
		2, Homogeneity.
III. Facility for being worked. (*To suffer.*)	Hot, or	1, Fusibility,
		2, Weldability.
		3, Malleability (also cold),
	Cold, or	1, Annealability.

I. CONSTITUTION OF STEEL.

Metals should be homogeneous as to composition and structure so as to be homogeneous as to strain. Those which have been fused are the most homogeneous; but even they may be imperfect, both chemically and physically, as follows:

I. CHEMICAL CONSTITUTION.

1. Homogeneity.

Pure iron can rarely be produced except by the methods of the laboratory, and therefore in exceedingly small quantities. In practice it is combined with the most useful elements by heat, the fusibility of the alloy usually increasing with the number of elements contained. Fused metals in general are imperfect alloys, the constituents of which tend to arrange themselves according to their specific gravities. Due to the property of *liquation*, certain of the most fusible alloys of steel are found near the core of the ingot in greater proportion than elsewhere. Thus, in a steel ingot the parts first solidified represent most nearly its average composition, the centre of the bottom being the softest and that near the top the hardest, since the fusibility and the hardness of the alloy increase with the percentage of C.*

* ORDER OF OXIDATION.

Reactions during fusion depend so much upon the order of oxidation of the constituents of pig-iron that the following approximate relation should be learned:

The segregation of the Kalchoids is very objectionable.

2. Composition.

The following elements occur in iron alloys:

1. *Carbon* and iron as the principal constituents make the steel best suited for general purposes. It has often been tried to replace or supplement the action of carbon by other elements such as silicon, tungsten, chromium, nickel, etc., but for general purposes carbon steel is far the most important.

As a rule, the greater percentage of carbon up to about 1.5 (and even 2 5 per cent.), or the higher the grade,

The more—	The less—
1. hard and elastic;	1. dense;
2. tenacious;	2. ductile;
3. brittle;	3. weldable;
4. fusible;	4. forgeable, does steel become.
5. expansible by heat, does steel become.	

The terms *high* and *low* referring to the *grade* of steel, or per cent. of *C* contained, are loosely applied, but the tendency is to draw the line at 0.35 per cent., where hardening by heating followed by rapid cooling becomes perceptible. The following table exhibits the classification according to use. So much depends upon the percentage of hardening constituents other than *C*, and upon the treatment of the steel in manufacture, that the relation expressed is only approximate.

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1. Silicon.
2. Manganese.
3. Phosphorus, in presence of an oxidizing basic slag.
4. Carbon.
5. Iron.
6. Phosphorus, in presence of an acid slag; *i. e.*, one containing an excess of SiO_2.

These and other following relations depend so much upon existing temperatures and conditions that they are expressed in the most general terms.

TABLE II. GRADES OF STEEL.

Grade.		Per Cent. of Carbon.	Application.
Low.	Extra mild,	0.05—0.20	Boiler plates to be flanged, bridge material.
	Mild,	0.20—0.35	Railroad axles, gun barrels, etc.
High.	Hard,	0.35—0.50	Rails, cannon, etc.
	Extra hard,	0.50—0.65	Springs, saws, etc.
	Tool,	0.75—1.00	Chisels, cutters, etc.
	Extra Tool,	1.00—1.20	Files and very hard tools.
	Die,	2.50	Wire drawing, to resist abrasion.

Carbon is generally supposed to exist in steel in two principal forms: 1. *Cement carbon*, characteristic of annealed or softened steel, and 2. *Hardening carbon*, characteristic of hardened steel. The former is insoluble and the latter is soluble in dilute $H_2\ SO_4$. See page 48.

The following elements are admitted, either of necessity or as a *physic; i. e.*, replacing something more harmful or producing a beneficial effect.

2. *Silicon* tends to displace C from combination, and to confer its properties, although in a less degree.

It restores "burned" or "rotten" steel by forming with the particles of iron oxide, to the dissemination of which this condition is often due, a fusible slag:

$$(Fe_2\ O_x + Si = Fe\ Si\ O_x + Fe).$$

As Si increases the solvent power of steel for gases, and, by reducing the iron oxide present in the liquid steel, prevents the formation of CO it also prevents the honeycombing or vesiculation of the metal from the ebullition of the gases while the metal is becoming solid.

If uncombined, $Si\ O_2$ may remain as *grit*, which is injurious to the strength of the metal and destructive to cutting tools.

When in excess, *Si* makes steel brittle. This is generally true of the non-ferreous ingredients of the alloy.

3. *Manganese*, unlike *Si*, tends to make *C* combine with iron; like *Si*, it tends to replace *C* functionally, but much less energetically.

Mn resembles *Si* as a reducing agent, and forms with it and iron oxide a very fluid, cleansing, slag.

The physical properties it confers vary with the proportion present. With from 3 to 6 per cent., the steel becomes very hard and brittle; but with from 7 to 20 per cent. the steel becomes very tough and strong.

Some *Mn* is necessary to prevent *hot-shortness*, or the tendency to disintegrate when forged, even when no *S* is present. It also acts as an antidote to *S*, by forming *Mn S*, which is insoluble in melted steel.

4. *Phosphorus* make steel *cold* short, or brittle at ordinary temperatures. It can be removed only by some *basic* process, as follows: Since the ordinary silicious, or *acid*, lining would prevent the oxidation of *P*, and, by wasting the iron, would increase the proportion in which *P* remained; in the basic process the furnace is lined with dolomite brick. Iron ore and (for economy) limestone are added to the charge, so that the phosphoric slag that is formed may continue basic.

The presence of *Mn* and *Si* in the pig iron from which washed pig is thus formed, protects from oxidation the *C* that it contains, and therefore maintains the fluidity of the charge, until the *Mn* and *Si* are consumed. When this happens and the bath boils with *CO*, the washed metal is cast into pigs containing only about 0.1 of the original *P*.

So much *C* is retained that (after grading it by analysis) the washed pig is easily remelted in further processes relating to the manufacture of steel.

These processes permit the use of pig iron, which was formerly too high in *P*.

When added to the Kalchoids, P removes their greatest enemy, oxygen.

5. *Sulphur* also is very difficult to remove, although the hot-shortness that it produces may be corrected with Mn or eliminated by $Ca\ F_2$.

6. *Chromium* increases the hardness of steel without impairing those qualities, such as ductility and malleability, which are incompatible with the hardness resulting from high carbon.

7. *Aluminium* is said to increase the fluidity of low steel, and even of weld iron, in a remarkable degree, thus permitting the metal to be cast without danger from vesiculation. The fluidity of the metal in the mold permits the escape of the occluded gases. Its action on iron oxide resembles that of Si.

A new alloy known as *Mitis* metal, which is thus formed, may be cast into the most complex forms.

8. *Nickel* remarkably increases the useful properties of steel; the result varying with the percentage as in Mn. The armor plates now (1891) preferred are made of nickel steel.

9. *Copper.* When thoroughly deoxidized, steel may be improved by the addition of Cu, although it is thought by some that it makes steel *hot-short.*

II. PHYSICAL CONSTITUTION.

1. Homogeneity.

As in chemical composition, no fused metal is naturally physically homogeneous, either in structure or in strain. These properties may be so modified by after-treatment that the following comparisons apply to non-forgeable metals as ordinarily cooled after fusion; and to forgeable metals when annealed. This is the standard condition for their comparison.

2. Structure.

The structure is judged of by the appearance of the fractures; for exact comparison these should be similarly pro-

duced. By varying the method of breaking it, the fracture of a bar of wrought iron may be made either crystalline or fibrous within a few inches of its length.

Crystallization. It is assumed that ingot metals are crystalline, and weld metals fibrous, although the ultimate crystalline forms are doubtful.* Thus, the former may be supposed to consist of normal crystals, and the latter of distorted crystals cemented by a film of slag.

Normal crystals are supposed to be formed like those of soluble salts; the more slowly and quietly they are cooled from fusion, the larger and weaker they are, and conversely. (Bloxam, Art. 38.)

The crystalline axes are found perpendicular to the cooling surfaces, so that surfaces of weakness are formed at the junction of inclined systems. See figures 16, 17. For this reason the corresponding surfaces should be united by gradual curves so as to distribute strains which would otherwise be localized. Sharp re-entrant angles should be avoided in all structural masses, even the non-crystalline.

Vesiculation. Another structural form arises from the occlusion of air and other gases during the casting. This causes "*blow-holes*," which increase with the viscidity of the fluid mass. On the other hand, when the metal is free from blowholes, an axial cavity is formed, due to internal strain. This is known as a *pipe*. Figure 18.

3. Strain.

Differences in the rate of cooling throughout a fluid mass produce internal strain, the parts first solidifying being compressed by their adhesion to the layers cooling subsequently, which last are reciprocally extended. Similar effects follow

* By some the primitive structure of ingot steel is supposed to be that of globules of the alloy imbedded in a mo e highly carbonized cement. This is called the Cellular Theory.

unequal heating. The importance of this principle is frequently apparent, particularly in dealing with iron castings.

II. CAPACITY FOR RESISTING STRESS.

Properties.

Owing to the facility of referring other stresses to a tensile stress, this alone is generally considered, the incompressibility of cannon metals being sufficiently guaranteed by their combined hardness and tenacity.

The principal properties of 1. tenacity, 2. elasticity, 3. ductility, and 4. toughness, have already been discussed.

5. *Hardness* is properly the property of resisting penetration; combined with tenacity, with which it is almost invariably associated, it renders cannon metal incompressible by powder pressure, and in itself resists abrasion, erosion and impact from hostile shot.

Steel may be artificially hardened by heating it, followed by its rapid cooling. Steel and bronze may also be hardened by their compression in a cold state; externally by rolling and internally by mandreling, which consists in forcing through a hole conical plugs of slowly increasing diameter.

It is noteworthy that by heating and rapid cooling, brass, bronze and the high *Mn* steel, p. 19, are *softened.*

III. FACILITY FOR BEING WORKED.

1. Fusibility.

This diminishes the number of joints in a given structure, and, other things being equal, increases its cheapness, homogeneity and strength. Recent advances in mechanical engineering have been principally due to the large units of construction afforded by the capacity and power of modern furnaces. Thus steel is replacing wrought iron, which is formed by agglutination.

2. Malleability,

or to the power to endure hammering or rolling, particularly at high temperatures, enables metals to be forged into special shapes, thereby improving the quality of the metal and reducing the labor of finishing. When combined with fusibility, it gives advantages peculiar to steel.

3. Weldability,

or the power of adhesion at high temperatures between masses is characteristic of wrought iron and low steel. It is upon this property that the manufacture of wrought iron depends. This property in construction is inferior to fusibility and hence is not utilized for steel except in small masses.

The process of electric welding is now (1891) successfully employed. It consists in sending a powerful low-pressure current through the abutting surfaces of the pieces to be united. The resistance at the points of contact raises the neighboring metal to the temperature of incipient fusion; pressure being then applied, fresh surfaces are successively brought into contact. Owing to the high temperature of the first contacts, the current is mainly conveyed through the new ones and so on until a homogeneous joint is formed.

Hollow projectiles are thus made from steel tubing welded to a point and a base. It is even proposed to heat finished pieces locally, so as to permit them to be bent or tempered without injury from the hammer or the fire.

4. Annealability,

or the power to become soft, facilitates reduction to size by cutting tools. All the cannon metals can be softened by annealing, but to steel only can the necessary hardness be restored, except mechanically.

Grinding. It fortunately happens that hardened steel, which can cut all the other useful metals, can itself be abraded by grinding almost as easily as when soft.

This permits the change of form which often follows hardening to be corrected by the use of either natural or artificial grindstones. See Chapter XVII, page 14.

CONCLUSION.

The relative standing of the five cannon metals may be roughly indicated as follows:

		Steel, High	Steel, Low	Iron, Wrought	Iron, Cast	Bronze
Structural qualities.	Elasticity, Tenacity,	1	2	3	4	5.
	Homogeneity, Hardness, normal,	1	2	4	3	5.
	Ductility,	4	3	1	5	2.
Working qualities.	Fusibility,	3	4	—	2	1.
	Malleability,	3	2	1	—	—
	Weldability,	—	2	1	—	—
	Annealability or variable hardness,	1	2	—	—	3.

This indicates why bronze and the irons, which, owing to their workability, were until recently the only cannon metals, are now obsolete.

IV. MANUFACTURE of the FERREOUS METALS.

I. CAST IRON.

Varieties.

The gray pig is known as *foundry* or *melting* iron, the white pig as *forge* iron; the latter is useful only for conversion into wrought iron. *Mottled* pig is an intermediate variety.

Remelting.

To obtain strong castings, the foundry pig is ordinarily remelted and run into molds of the required shape.

The specific gravity of pig-iron is about 7.00, and its tenacity about 16,000 pounds to the square inch, but, when remelted, the specific gravity is increased to about 7.25, and the tenacity about doubled.

The remelting is effected in cupola or reverberatory furnaces, according to the kind of fuel available and the size and quality of the casting required. It is always necessary

to melt as quickly as possible, and with the least consumption of fuel. This usually requires artificial blast.

In Cupolas. The cupola furnace is generally employed; its size depends upon the amount of metal to be melted at a time, and upon the kind of fuel.

A cupola extensively used is of the Mackenzie pattern, figure 19. It consists of the *body B*, of elliptical cross section, made of thick sheet-iron lined with fire-brick; this is surmounted by a conical *hood H*, terminating in the *chimney C*. The blast is admitted through an annular tuyère extending around the bottom part of the furnace. The charge is introduced at the *door D*, and the molten metal, accumulated in the *hearth H*, is drawn off at the *spout S*, and carried to the mold through a channel or by means of ladles.

The elliptical section of the body in combination with the annular tuyère increases the capacity of the furnace for a given intensity of blast; the object being to maintain a high temperature in the vertical plane containing the transverse axis of the ellipse, along which, for regularity of feeding, it is desirable to cause the contents of the furnace to descend.

The cupola furnace saves fuel, labor and time, and furnishes a continuous supply of iron, which, since the carbon in the pig-iron is not diminished by melting, is liquid and therefore of the quality suited to foundry purposes.

The charge consists of pig-iron and generally scraps of cast iron, a flux, and the fuel; for the latter, coke and charcoal are best, though anthracite is generally employed.

In Reverberatory Furnaces. Reverberatory furnaces are principally used for the production of large castings, and are specially adapted to all such as require great strength. Their use is sometimes necessitated if the fuel at disposal contains sulphur.

The name is derived from the arch which beats back the flame on the metal to be heated.

The furnace, figure 20, is built of fire-brick bound strongly together by iron bars or plates; the *hearth H* is of refractory

brick covered with a thick layer of fire-sand; the *grate G* is large, that a great volume of flame from the fuel may be drawn over the *bridge B* and through the furnace; for this purpose the *chimney C* is made very tall when no artificial blast is used. The metal is introduced at the *charging doors D, D′*, and, when melted, is drawn off at the *tap-hole* (*h*).

The dimensions of the furnace depend chiefly on the charge of iron and quality of the fuel. They are of correct proportions if a nearly uniform temperature be produced in all parts of the furnace.

Unlike the cupola, this furnace allows the iron to be kept liquid for any length of time; and, as the fuel is not in contact with the metal, and carbon and silicon are removed by the air, a stronger iron results. On the other hand, it does not admit of constant casting, and involves a great loss of iron by oxidation; owing to these circumstances and to the greater consumption of fuel, such furnaces are used only in large foundries, and, whenever practicable, are replaced by cupolas of large size.

Properties of Iron for Castings.

The color and texture of a casting depend greatly on its size, and on the rapidity with which it has been cooled, and upon its composition. As small castings cool quickly they are almost always white, and the surface of large castings partakes more of the quality of white iron than does the interior.

When gray iron is melted, the particles of graphite to which its color is due are dissolved by the liquid iron, and if it be poured into a cold iron mold so as to solidify quickly, the exterior of the casting will present much of the hardness and appearance of white iron, the sudden cooling having prevented the separation of the graphite. This is particularly apt to follow the presence of manganese in the iron.

At the instant of solidification *gray* iron expands more than white, giving a casting with sharp edges and a convex

surface; and, as it subsequently contracts less, the initial strains due to cooling are less.

White iron gives a casting with a concave surface, and *mottled* iron one with a plane surface, the edges slightly rounded.

SPECIAL CAST IRONS.

Malleable Cast Iron.

By extracting a portion of the carbon from cast-iron its composition is assimilated to that of wrought iron and its toughness increased; the result is known as *malleable* cast iron.

The castings to be softened are packed with powdered hæmatite ore, or scales of oxide of iron, and the temperature raised gradually to a red heat; this is continued from three to five days according to the thickness of the layer of malleable metal required.

When withdrawn from the furnace, articles so heated have the appearance of ordinary malleable iron, but are lighter in color; their fractured surfaces are white and finely granular, occasionally having a silky appearance like that exhibited by soft steel.

The principal application of this process is to such articles as buckles, bits, stirrups, keys, etc.

Case-hardening. The stratum of malleable metal on the surface may be converted into steel by the process of *case-hardening*, which consists in a similar heating in contact with animal charcoal, after which, while still hot, the casting is plunged into water or oil. This process is applied also to articles of wrought iron, such as the parts of small-arms in which it is desired to have a tough, somewhat soft interior protected from friction or blows by a hard surface. The hammer and breech-block of the Springfield rifle are so treated.

SPECIAL ALLOYS.

Varieties.

Spiegeleisen or *Spiegel* (*Sp*) and *Ferro-Manganese* (*FM*) may be regarded as varieties of white cast iron alloyed with a varying proportion of *Mn*. That which contains over 20 per cent. of *Mn* is known as *FM*. When *Mn* amounts to 80 or 90 per cent., it may consume by spontaneous oxidation.

The price of *FM* increases with its richness in *Mn*, for this limits the choice of ores and increases the temperature of reduction and fusion, and the loss by volatilization and oxidation.

Silicon-Spiegel and *Ferro-Silicon* are similar alloys, but contain much more Silicon.

The following table exhibits roughly the ingredients of some of the principal special alloys, and illustrates the statements previously made as to the effects of *Mn* and *Si* upon the proportion of iron in combination.

Note the gain in *C* as *Mn* increases, and its loss as *Si* increases.

TABLE.

Name.	*Si*	*Mn*	*C* (combined.)	*Fe*, etc.
1. Ferro-Manganese,		80	7	13
2. " "		60	6	34
3. Spiegel-Eisen,	1	10	5	84
4. Silico-Spiegel,	10	20	2	68
5. Ferro-Silicon,	10	2		88

Use.

These alloys are manufactured principally for the steel makers, being used by them to improve the quality of steel while in a state of fusion.

Generally speaking, Ferro-Manganese is used when the quantity of *C* necessary is small as compared with the *Mn* required; and conversely with Spiegel-Eisen, although in the

latter case C may be added directly in a pulverulent form, or in a pure pig iron.

The Silicon irons are principally used to prevent vesiculation; No. 4 is preferred to No. 5, as the increase in Mn causes the Si to more thoroughly combine with the steel and improves its structure.

II. MODERN MANUFACTURE OF WROUGHT IRON.

Principles.

The great cost of the hand labor engaged in the ordinary process of puddling has led to the use of mechanical means for accomplishing this result. The two principal processes are those of *Danks* and *Pernot.* Their common feature is the continuous rotation, by mechanical means, of the vessel containing the charge, thus avoiding the loss in time and power due to the reciprocating action of the puddler's *rabble;* and diminishing the number of skilled workmen required.

The principle involved in these processes is that given in Bloxam, Art. 219, viz.: That when cast iron is heated in contact with iron oxide, the C and Si in the iron take O mainly from the iron oxide in the fettling of the furnace. The C passes off as CO and CO_2, and the Si as an iron silicate or slag.

Danks Process.

The furnace, Figure 22, consists of a horizontal drum, revolving on rollers and lined with a *fettling* of lumps of hæmatite ore set in a fused paste of the same ore. The flame from a stationary fireplace plays through one end of the drum and passes off through a movable flue at the other end. The removal of the flue permits the drum to be charged and emptied.

For economy the furnace may be charged with melted iron, either directly from a blast furnace or from a cupola. If charged cold the rate of revolution is slow while melting; it is increased while boiling, during which the fettling and the flame rapidly oxidize the C and Si exposed by the rolling of the pasty mass and the adherent film and drip from that

which is melted. The drum is stopped to tap the cinder. It is then revolved more rapidly than before, draining the pasty mass until it begins to *ball.* The large lumps, carried around by adhesion, fall on those at the bottom and help to work out the cinder. This is more thoroughly done afterward by the usual methods.

Pernot Process.

The pan revolves under a stationary cover, on an axis inclined about 5° or 6° to the vertical, see figure 35. The fettling is thus exposed alternately to the flame and to the metal, the film of oxidized iron thus formed passing under the fluid mass and assisting the reduction. Balling still has to be done by hand; but the process uses less coal than the ordinary one, and the furnace can be more easily repaired.

These processes are losing their importance in consequence of the rapidity with which steel of various grades is supplanting wrought iron.

III. MANUFACTURE OF STEEL.

I. IN SMALL MASSES.

1. Weld Steels.

Puddled Steel. Puddled steel is made by stopping the process of puddling when the de-carbonization of the cast iron has sufficiently advanced. It is principally used for conversion into other kinds of steel.

Blister Steel, which is made by cementation, being full of fissures and cavities, is fit only for a few rough purposes, as for facing hammers; most of that made is used for conversion into other kinds of steel.

Tilted Steel. When bars of blister steel are heated or hammered into bars under a *tilt hammer*, Figure 40, the product is termed *tilted* steel; spring steel is thus prepared.

Shear Steel. Shear steel is produced by cutting bars of blister steel into convenient lengths, and piling, heating, and welding them under a hammer, whereby is obtained a bar of

uniform quality known as *single shear* steel; the quality of the metal is still further improved by a repetition of the process, forming a bar of *double shear* steel. The oftener the process is repeated, the more uniform is the resulting steel.

Shear steel is capable of receiving a better edge and a higher polish than blister or spring steel; when well prepared, it is not much inferior to crucible steel. It is very extensively used in work where steel and iron have to be united by welding, as in axe-bits and scissors.

2. Crucible Steel.

Although blister steel by repeated working under the hammer acquires a tolerably homogeneous structure, it is still further improved by fusion. The process, invented a century ago, still remains in principle unaltered. Fragments of blister steel are melted in crucibles, figure 23, covered to exclude the air, and the liquid poured into cast-iron ingot molds of the shape and size required. These ingots usually contain cavities; they are gotten rid of by heating the mass and hammering it into compact and homogeneous bars.

Most crucible steel is now made direct from bars of the best wrought iron; they are broken and placed in the crucible with a small quantity of charcoal or pig iron, the amount varying according to the grade of steel required; some alloy of manganese is subsequently added. The preliminary fracture of the material charged facilitates its classification and increases the uniformity of the product.

Properties.

In forging, crucible steel should never be raised beyond a certain temperature, varying inversely with the grade, or it will become brittle. It is difficult to weld, as it is usually high in carbon.

If a small quantity of manganese be added to the molten metal, the steel will be more forgeable and may be welded either to itself or to wrought iron.

Remarks.

The manufacture of the *weld steels* and of *crucible steel* is losing its importance, and crucibles are now principally used for small masses in which the desired quality of the product can, from the careful supervision exercised, be most easily maintained.

The size of the crucible charge depends on the strength of the melter and rarely exceeds 80 lbs.; but with well drilled men large numbers of such crucibles may be poured successively into a common ingot mold of any size. Krupp so casts his large cannon, sometimes employing 1200 crucible bearers.

II. IN LARGE MASSES.

Processes.

The principal processes are the *Bessemer* and various forms of the *Open Hearth.* Each of them has its province. The former, owing to its rapidity, excels in cheapness, although there is a loss of about 10 per cent. of iron; the latter, owing to its controllability, excels in quality. This takes time and increases the cost by about 15 per cent., although there is incidentally a slight gain of iron.

Carbonization and Tests.

Owing to the loss of iron from oxidation when completely decarbonized, neither process is carried to an extreme, some C being always left in the metal and its final percentage being regulated by adding Sp or FM.

The percentage of C is judged of by the fracture; by the appearance of the nick required to produce fracture; and more carefully by a rapid *color test,* which consists in comparing the color of a solution of the metal in dilute HNO_3 with that of a standard solution. In the Bessemer process this information is applied to the next succeeding heat; and in the Open Hearth, as the operation is less hurried, to the heat itself.

Temperature.

The high temperature attained permits re-melting on the spot of the scrap accumulating in all steel works, which would

otherwise be of little value. In the Bessemer process this is due to the oxidation of the Si in the pig; in the Open Hearth to the Siemens regenerator, which increases the temperature cumulatively to a degree limited only by the refractoriness of the furnace linings and the tendency of the gases to dissociate. Thus, like many other inventions, the Open Hearth process had to wait for the parallel development of some insignificant art, *i. e.* that of the brickmaker.

Cranes.

The production of metal by both processes depends upon the facility of manœuvring large masses. Of the various patterns of cranes used for this purpose, Sir Wm. Armstrong's hydraulic crane, or some modification of it, is especially valuable in Bessemer practice. Its efficiency depends upon the arrangement of peculiar valves which unite at a central point called the "pulpit" and which place the control of the whole plant in the hands of one man.

For an Open Hearth plant, where frequently very heavy masses must be moved and where the operations need not be so rapidly performed, these cranes may be supplemented by power swinging cranes or replaced by a traveling crane covering the whole building. The traveling crane consists of a horizontal beam the ends of which roll on raised parallel tracks. The weight hangs from a truck rolling on the beam and may thus be transported to any point of the included volume. This crane presents many advantages and is used when the construction of the plant permits.

Casting Ingots.

In both processes, the melted steel is run from the furnace into a ladle from which it is distributed by a crane into cast-iron ingot molds.

Casting is sometimes done through an independent iron gate entering the mold from below (Figure 24). The fluid metal should enter in a quiet, solid stream so as to avoid

entangling air. This is best done by emptying the ladle into the pool, from which it issues, under a constant head, through a cylindrical nozzle.

For gun work, the ingots are like Figure 25. The tong-hold serves to attach the porter-bar in forging; and the drum, being girt with a sling chain, permits the mass to be moved about and turned axially under the hammer. The dotted lines in Figure 25 indicate the form of the corresponding sections of the ingot.

Ingots are cast at as low a temperature as possible consistent with fluidity in order to diminish internal strain and to save the inner surface of the mold, injuries to which may imprison the ingot.

In order to fill the voids resulting from the shrinkage due to internal strains, castings of all kinds are generally surmounted by a *sinking head*. This is a reservoir of the melted metal, the cooling of which is often retarded by containing it in a relatively non-conducting mold.

For economy of fuel it is generally sought to forge the ingots as soon as possible after they have solidified throughout; but, owing to interruptions in the work, the sequence cannot always be maintained. Ingots may thus require re-heating; this should be gradual so as to avoid internal strain.

Fluid Compression.

Whitworth's method of fluid compression tends to obliterate cavities by an hydraulic pressure of about 40000 lbs. per square inch. A very strong steel mold provided with a porous lining is employed. The pressure crushes down the vesiculated shell first formed next to the walls of the mold, and drives the fluid metal throughout the interstices. The lining allows the escape of gas. By this means the ingot is reduced about one eighth in length while cooling after casting. The best results, however, are thought to be obtained by careful melting and after-treatment of the steel while in a fluid state.

Bessemer Process.

(Figures 27—30.)

Varieties.

There are two general processes depending on the nature of the pig-iron converted. If free from P, silicious or *acid* linings may be used ; but if it contains much P, *basic* linings are required. The former process, which is the more common, is here described.

Metal.

The iron must contain Si as a fuel, and hence gray pig, the color of which is due to the carbon displaced, page 18, is used. It should be free from P and S, as they are not removed but, owing to the inevitable loss of iron, their proportion is increased.

Main Operation.

The pigs are usually melted in a cupola and the fluid charge, after weighing, run into the converter. A blast of air is then blown down through one trunnion and up through the perforated bottom and the fluid metal. The reactions resemble those of puddling* and are principally due to the heat evolved by the burning Si. This burns out the Mn and C in the metal and, by forming ferreous slags, removes part of the iron also. The fluidity of the metal is due to the intensity of the heat ; the latter is due to the rapidity of the reaction consequent upon the state of subdivision of the mass. The burning Si raises the temperature and promotes the fluidity of the bath more than does the C, because the CO formed absorbs much heat by expansion and carries it off ; the slag remains and protects the bath from cooling. The small portion of Mn present also acts as a fuel.

The basic process requires the blast of the Bessemer blower

* It has been said as an example of mechanical progress, that we have replaced the laborious operation of the puddler's *rabble* by piercing the molten metal by invisible bars of air.

to be prolonged after the C and Si in the pig have been removed, the burning P maintaining the fluidity of the metal.

Periods.

Three periods are recognized, lasting as follows:

I. Three to five minutes, Si burning. The free C in the pig becomes combined, in which state it is most easily oxidized. The flame is feeble, with a hissing noise.

II. Six to ten minutes. The oxidation of C, principally to CO, makes the mass boil with a thundering noise. A yellow flame of incandescent particles is emitted at the nozzle.

III. Four to five minutes. The flame, principally of N, is smaller, and of a pale bluish tint. In about 15 or 18 minutes from the beginning, the flame suddenly *drops*, showing that the C is almost entirely gone. To save loss of iron by further oxidation, the blow is then stopped as the converter is turned down; the carbonizer is then added by weight. If Spiegel is used, it is melted in a separate cupola.

Final Operations.

The carbonizer preferred for low steel is FM, which, although more costly than Sp, contains less C in proportion to the Mn, so that enough Mn may be added to reduce the iron oxide, combine with free O, and impart to the steel its characteristic qualities without introducing enough C to make it unduly hard. The production of FM is one of the improvements for which this application of the process had to wait.

After standing for a few minutes, the contents of the converter are poured into a ladle, the slag remaining in the vessel; the slag is then emptied and the vessel turned up for a fresh charge.

Remarks.

The melted pig may be conveyed directly from the blast furnace; but this is not often done, as it prevents the preliminary grading of the pigs by fracture.

The process is principally applied to the manufacture of

rails, for which it is sufficiently exact. The quality of the product may be improved if time and waste are neglected and the process carefully watched through the spectroscope.

The steps of the operation in the acid and the basic processes, showing the rates at which the solid products are oxidized and the proportions of the different gases successively formed, are represented in figures 28, 29, 30. In each figure one scale is that of time in minutes from the beginning of the blow, and the other represents the corresponding percentage of the special product in question. These diagrams are the result of experiment.

Open-Hearth Process.

Varieties.

The hearth may be either of the stationary or of the rotary type. In both cases the advantages of the process depend upon the Siemens regenerative apparatus, which requires a gaseous fuel.

The rotary hearth has the advantage of steam power and of facility in making the repairs which the intense heat due to the regenerative apparatus frequently requires. It is also better able to dephosphorize pig-iron. The principal objection to it is the liability of derangement of the rotating machinery; but this can be overcome. Its process is hereafter described.

Distinctions were formerly made between the "pig and ore" and the "pig and scrap" processes, depending upon whether the melted pig-iron is decarbonized by the iron oxide or diluted by the addition of scrap steel low in carbon. Such distinctions are no longer important, as the former process is generally employed.

The Siemens furnace with either the stationary or the revolving hearth is a mighty instrument for achieving various metallurgical ends. Accordingly, many combinations are made in

its employment, pig-iron, washed pig, ore, fluxes, and, particularly for commercial products, scrap being added as required or convenient.

Gaseous Fuel.

Advantages: 1st. Controllability, by which either an oxidizing, reducing, or neutral flame can be uniformly obtained. 2d. Economy. 3d. Cleanliness. 4th. The accuracy with which the low temperatures used in annealing ovens may be estimated by the eye, the gas having been temporarily cut off so as to obtain a background against which the true color of the heated piece will appear.

The gas may be natural or artificial.

Crude petroleum is becoming largely used as a fuel. Being thrown into the furnace as a spray, it has many of the advantages of a gas. It is also converted into gas by the action of steam at a high temperature.*

Siemens Gas Producer.

This consists of a number of chambers united in groups of four around a common stack *E*, figure 31. The stacks unite in a common trunk which leads with a slightly downward inclination to the valve box *B* of the furnace, figures 33, 34. Each chamber is essentially a wedge-shaped funnel with one inclined side terminating at the bottom in a grate *B* on which the fuel is slowly burned. The CO_2 formed, ascending through the incandescent mass, becomes $2CO$, and, with other gases due to the partial distillation of the superincumbent fuel, passes through the flue *D* to the stack *E* and thence to the trunk, having in the trunk a slight excess over atmospheric pressure to prevent leakage inward. The increase

* The oxygen in the H_2O combines with the carbon in the oil, forming CO, and decomposing the hydro-carbons into new compounds richer in H. The H derived from the steam combines with the new compounds, and makes them still lower in the paraffine series. (Bloxam, Art. 320.)

of density due to cooling causes a gradual flow along the trunk. The same effect can be obtained by using a blast which gives more, better and hotter gas from fewer producers burning poorer fuel than does the natural draft described.

Almost any kind of fuel from gas coal to sawdust may be used, depending on the purpose in view.

The charging hopper *A* and the poker hole *C* are stopped to prevent the escape of gas.

Siemens Stationary Furnace.

(Figures 32, 33, 34.)

Hearth.

This rests in a cast-iron basin *T*, beneath and around which air circulates. It is enclosed in a rectangular box-like furnace about 30 feet long, standing above the floor-line *W*, and provided with the charging door *U*, and the spout *V* for tapping out the fluid charge.

Regenerators.

These are the essential parts of the apparatus and are applied to many purposes in which high temperatures are required.

The regenerator consists of four fire-brick chambers of varying section, *K; L; M; N*, arranged in pairs. They are filled with a crib work of loosely stacked fire-brick. From each of the end chambers *K*, *N*, gas-flues *S* lead up into the furnace; and from each chamber *L*, *M*, three air-flues *P* and *R* lead up alongside the gas-flues to a point above their exit in the furnace. This arrangement protects the metal from oxidation; and the roof, made higher than where reverberation is sought, from erosion by the flame.

Valves.

The gas, air, and reversing valves are shown in vertical section (laid over a longitudinal section of the regenerators) in Figure 32; in plan (laid over a horizontal section of the main

flues) *F*, *G*; *J*, *H* in Figure 34; and in cross section in Figure 33.

Operation.

Gas from the producers, regulated by the valve *B*, passes down over the reversing valve *C*; this is set so as to direct the gas into the main flue *F* and the regenerator *K*, where it percolates through the mass of hot brickwork and thence passes at a high temperature into the furnace. Air is drawn through the regulating valve *E* over the reversing valve *C′*, through the main flue *G* into the hot regenerator *L* and, passing up the flue *P*, meets the hot gas as above described, affording progressive combustion with intense heat.

After burning, the flame passes down the flues *R*, *S* into the other pair of regenerators *M*, *N*, which absorb most of its heat. It then escapes through the main flues *J*, *H* under the two reversing valves, and into the chimney flue *AA′*.

After about twenty minutes, *K*, *L* becoming cooler and *M*, *N* heated, *C*, *C′* are reversed by the handles *D*, when the currents of gas and air are also reversed. The effect of reversal is cumulative, since to the heat of combustion is added that which the gases absorb from the brickwork before combustion. As the brickwork becomes progressively hotter, the ultimate temperature attainable is independent of blast or draught and is limited only by the refractoriness of the furnace linings and the tendency of the gas to dissociate at high temperatures.

Advantages.

The principal advantages are the high and uniform temperatures attainable, with the other advantages due to the use of gaseous fuel.

Employment.

When the furnace has been brought up to a melting heat, the bottom is repaired with fire-sand and the charge thrown

in by hand. After melting, it is stirred with iron bars and treated as hereafter described in the Rotary Hearth.

Pernot Rotary Hearth.

(Figure 35.)

Hearth.

This consists essentially of a circular wrought-iron "*pan*" lined with refractory material and mounted on conical rollers. These run on a circular trough-shaped track mounted on a carriage; the latter rolls on two parallel rails on which it may be run into and out of the stationary furnace chamber. The pan is rotated by a circular-toothed rack beneath it gearing into a toothed wheel or by an endless screw driven by steam power. The pintle, which is hollow and contains a stream of water, is inclined at about 6°, so as to bring the highest portion of the hearth next to the door. In case of accident to the tapping hole, more than one is provided.

The lining of the pan varies with the kind of work. For ordinary melting it is of refractory silicious material; but where dephosphorization is sought by the Krupp process, the lining is basic, preferably of lumps of refractory magnetite set in a paste made of powdered hæmatite and iron scale. The lower courses of the roof are then of dolomite brick.

Operation

For steel making, the charge, consisting of about 15 tons of pig-iron free from P and S, is thrown in through the charging door while the pan is revolving; this distributes it automatically. Further revolution of the pan then causes the unmelted metal to dip into and out of the bath as previously described for wrought iron. When the pig-iron is thoroughly melted, rotation is stopped and ore is added at intervals, each addition being followed by a violent ebullition of the bath. Samples of metal or "spoon tests" are taken from time to time and examined by the color test, the fracture, and

by the appearance of the nick made by the chisel at the fracture. When the *C* in the bath is low enough, *Si* and *Mn* are added to prevent vesiculation and to make the steel malleable. The process is continuous, taking about eight hours for a heat, with a variable interval for repairs.

Repairs.

The hearth is repaired between heats by revolving it so as to bring the portions most cut by the flame under a hole in the roof through which material is thrown. The stationary portion is repaired at about every twenty heats, the pan being run out bodily on its carriage. This affords a considerable advantage, since in repairing the stationary furnace, time must be taken to allow the mass of brickwork to cool down to an endurable temperature; owing to the lack of ventilation this time may be very great.

V. MECHANICAL TREATMENT OF STEEL.

CASTING.

The successful casting of steel into final forms is still uncertain. The principal difficulties arise from vesiculation and internal strain Steel castings frequently replace iron forgings of a low grade.

ROLLING

Rolling may be intended to produce forms either straight or circular, and may be performed either hot or cold. The latter has the special object of producing hard, polished surfaces of exact dimensions and is applied to iron or steel of small sections only. The reduction is small.

Hot Rolling—Straight.

The following description of the rolling of armor plate or of structural steel is taken as a type.

The interior of a newly cast ingot is too liquid for safe

working, and by the time that this has sufficiently cooled in the air, the exterior has become too hard. Consequently, the cooling is often retarded in non-conducting *soaking pits*, in which the initial heat of the interior and that which becomes sensible during solidification become uniformly distributed throughout the mass.

Or, if the ingot has become cold, it is brought slowly to the proper temperature in a heating furnace. If this is done too rapidly, the exterior may be over-heated before the interior is at the proper temperature. The principle involved is of wide application in the treatment of steel.

The universal mill consists of two pairs of massive rolls at right angles to each other, so that one pair will roll the sides of the ingot while the other pair rolls its top and bottom. Each pair is driven by an independent steam engine. The direction of the rotation may be rapidly reversed, and the space between the members of each pair of rolls be rapidly adjusted to suit the varying dimensions of the work.

A series of horizontal parallel rollers of small diameter, independently driven, convey the ingot to and from the rolls, and after rolling take it to the shears where it is trimmed and cut into lengths.

These lengths, or *blooms*, are often re-heated and re-rolled by a *mill train* into various structural shapes. For small work the mill train usually consists of a series of rolls arranged in sets of three, one above the other, or *three high*. They contain grooves of appropriately decreasing section so that successive *passes* reduce the bloom to the shape required. The rotation of each roll is continuous, so that the piece passes in one direction above the middle roll, and in the opposite direction beneath it.

In rolling large sections the *two-high* system is generally employed; the rotation being reversed and the space adjusted at every pass.

In all large forgings great care is taken to cut out all visible imperfections such as *pulls*, which arise from deficient ductility in the metal, and *cold shuts*, which are due to the folding in of projecting portions at temperatures too low to admit of their union to the mass.

Hot Rolling — Circular.

A small cylindrical ingot is flattened out or "upset" into a "cheese" and punched from each side successively with a conical drift. The *punchings*, or pieces cut out, usually contain all the *pipe*. It is afterward hammered on the horn of an anvil, figure 37, until the approximate form is obtained. Then, being hung upon a fixed roller *A*, figure 38, another roller *B*, independently driven at a higher rate of speed, is raised by hydraulic pressure to the position shown by the dotted lines. The thickness of the hoop is thus diminished, and its diameter increased, since lateral spread is prevented by the flanges *a b* which come in contact, each with the end of the other roller. The process is used for making locomotive tires and short hoops for guns. It tends to give a fibrous structure to the steel, affording great tangential strength.

WIRE DRAWING.

Operation.

This resembles rolling, except that the conical aperture in the *draw plate*, figure 39, being stationary, the wire, previously pointed and lubricated, is drawn through it by power, generally by being coiled around a revolving drum. Tubing is similarly made, but large sizes, like gun-barrels, are rolled as described for rails, the sides being kept apart by an axial mandrel which is stationary.

Effects.

The effect of wire drawing at low temperatures resembles that of cold rolling in that it raises the elastic limit and tenacity

and diminishes the ductility of the metal so much that, if the original section is much reduced, frequent annealing is necessary. Steel wire has thus been given a tenacity of over 333,000 lbs. per square inch with an elastic limit half as high. These qualities are especially adapted to the construction of "wire-wound" cannon.

FORGING.

This includes the operations by which hot metal is hammered into shape. It therefore requires furnaces, hammers and anvils.

Furnaces.

For large masses modifications of the Siemens furnace called re-heating furnaces are now employed. These furnaces are frequently served by a curved crane of the form shown in figure 42. This increases the elasticity of the crane as against the shocks due to forging.

Hammers.

For light work hammers may be of the vibrating class known as *tilt* or *helve* hammers, figure 40, in which a horizontal beam, working on trunnions, carries at one end a heavy head; this is caused to rise and fall by the action of projections on a revolving wheel. Or they may be of the class known as *drop* hammers, where a weight is raised by hand or by power and allowed to fall upon the work after the manner of a pile driver, figure 41.

But for heavy work *steam* hammers are used. They are sometimes of the *Single Acting* type, figure 42, proposed by Nasmyth in 1833. The inverted cylinder is mounted on legs which spread sufficiently to allow freedom for the workmen. The cylinder is usually traversed vertically by a heavy piston-rod, to the lower end of which, sliding in guides attached to the frame, is fastened a heavy head or *tup*. Steam being admitted below the piston, it raises the hammer,

which is allowed to fall from any desired height. Its fall may be arrested by choking the exhaust by the automatic operation of the valves so that rapid rebounding blows may be struck. See figure 43 and page 54.

Anvils.

The anvil with its foundations constitutes one of the most expensive portions of a forge plant. The anvil of the 125-ton hammer at South Bethlehem, Pa., copied from that shown in figure 42, weighs about 1600 tons.

In order to avoid the effects of vibration, the foundations of the anvil should be independent from those of the hammer.

The anvil proper, like the tup, is generally flat, but both may be of various forms required by the shape of the work. Small work is thus produced with great exactness by being stamped between dies. The parts of small-arms and of other machines made in great quantities, such as those for sewing and for agricultural purposes, are thus very economically forged into very nearly their finished sizes. When of horizontally rectangular section, the anvil is generally set with one of its diagonals in the plane of the legs, so as to give room opposite all its faces for handling long forgings. Figure 43.

The energy on impact being the same, the action of a heavy weight moving with a low velocity is preferred, as the effect is more penetrating and less local. This principle is utilized in *Condie's* hammer, in which, owing to the fact that the mass of the cylinder is necessarily greater than that of the piston-rod, the cylinder is made movable, the piston-rod being stationary.

The steam may be admitted above the piston, adding its pressure to the weight of the moving mass. Such hammers are *Double Acting.*

For small work a single support, figure 43, gives sufficient steadiness and more room. The valve may then be worked by a treadle under the control of the smith so as to give him the use of both his hands. See figure 41.

The local absorption of energy at the point of impact diminishes the reaction of the anvil, so that, as the thickness of the work increases, the thoroughness of the forging diminishes. This requires frequent rotation of the work so that all sides may be equally extended.*

For this reason Ramsbottam's duplex hammer is used, the work lying between horizontal hammers moving with equal and reciprocal velocities.

Anvils for hollow work. In forging hollow work, *mandrels*, which are heavy solid cylinders passed through the forging, are used in connection with the anvil proper. If supported throughout its length by a V-shaped notch in the anvil, the forging lying in between, the mandrel is termed *fixed;* if supported only and directly at its ends, the mandrel is called *swinging.* Figures 44, 45.

The effect of forging is greatly affected by the way in which the mandrel is used. Forging on a fixed mandrel extends the work in length but does not sensibly affect the internal diameter. Forging on a swinging mandrel increases both internal and external diameters without affecting the length of the work. Hence, the swinging mandrel is used for hoops which are too wide for the tire-rolling machine.

Forging Press.

The defects in steam hammers above referred to will probably lead in time to a more general use of the hydraulic forging press designed by Whitworth, Figures 44, 45. Its principal advantage lies in the time during which the work is operated on; this permits the molecular flow desired.†

* This is also true of rolling and limits the effective thickness of armor plates.

† Opinions are divided as to the comparative merits of the hammer and the press. The advocates of the hammer prefer it on the following grounds:

1. In forging solid work the effect of the hammer is greatest on the exterior which is retained; and least on the interior, which for cannon and heavy shafting is subsequently removed. The converse of this is attributed to the press.

VI. MOLECULAR TREATMENT OF STEEL.

The quality of steel depends upon:

1. Its composition.
2. Its structure as modified by heating and cooling.

1. Composition.

Pure iron and carbon make a typical steel. But other elements are of necessity present in all the steels met with in practice.

Pure carbon steel is here discussed.

2. Structure.

Changes in structure from the effects of heating and quenching steel appear to be associated with changes in its density and also in the state of the contained carbon. What relation exists between the change in state of the carbon and the change in the structure of the steel is still uncertain.

States of Carbon.

Indeed, the precise nature of the states of the carbon, although much experimented upon and discussed, is not definitely known. As an indication of the uncertainty in this matter, and also of the idea which most theories contain, the following suppositions may be referred to.

The carbon is supposed by Professor Abel to be either in the condition of an alloy, or of a diffused carbide. Another chemist calls it diamond, or dissolved carbon. Others, and the more recent authorities, waive this issue by calling it "hardening" or "cement" carbon. See page 18.

Avoiding any specific hypothesis, we may designate these states respectively as *Fixed* or *Free*. The former name, as

2. The prolonged contact with the dies of the press chills the forging, the initial temperature of which therefore must be excessive; while the blows of the hammer are heating.

3. Hammering exposes superficial defects while pressing conceals them. The 125-ton hammer, page 46, is intended for forging armor plates, the quality of the surface of which is most important.

the preceding nomenclature would indicate, corresponds to the hard condition of steel, resembling that of white cast iron; and the latter to its softer condition, resembling gray iron. See Bloxam, middle Art. 220.

Brinell's Experiments.

Method.

The accompanying diagrams, Figure 46, are principally based upon a long series of experiments made by a Swedish engineer, J. A. Brinell, upon the changes in the structure of steel due to heating it in different temperatures and cooling it at different rates. His results appear to agree well with those of others. His method was:

I. To heat separate bars of the same steel, but of varying structure, up to certain temperatures indicated by the color of the hot metal,* and then to cool them in one of two ways:

1. Slowly in ashes, called herein *cooling*.
2. Suddenly in cold water, called herein *quenching*.

II. To examine a freshly fractured surface, the fracture being similarly produced in all cases.

III. To subject the steel after cooling or quenching to a chemical test as to the state of the carbon contained.

Classification of Fractures.

The recognition of fractures, like that of colors due to certain temperatures, requires great experience, but the principal fractures may be designated by symbols, as follows:

	Structure.	Crystalline.	Granular.
Symbols.	Coarse,	A.	D.
	Intermediate,	B.	E.
	Finest,	C.	F.
	Aspect,	glistening,	dull.

The most important is F, which may be called *amorphous*,

* The irisated colors in figure 46 are the chameleon tints of the films of iron oxide of different thickness, which result when a polished steel surface is moderately heated.

the crystals or grains being invisible to the naked eye. The intermediate and various composite fractures described by Brinell are not noted herein.

Characteristics of Fractures.

A, B, C, are relatively soft.

D, E, F, are relatively hard.

A, D, have low density (open grain) and are weak.

C, F, have high density (close grain) and are strong.

Therefore, C has softness and strength; it is extensible. This fracture is sought in annealing.

Therefore, F has hardness and strength; it is inextensible. This fracture is sought in hardening.

F has the maximum density.

DIAGRAMS.

Explanation.

Figure 46 illustrates the changes in structure and state due to heating and either cooling or quenching the steel experimented on by Brinell. The axis of each diagram intersects a common scale of temperatures which, for any particular grade of steel, are indicated by the accompanying colors.

The temperature W is critical in its effects on structure and state: *it is the only high temperature at which, without having been exceeded, if steel be quenched, the resulting fracture will be amorphous*, F. The lower the grade of steel, the higher is the temperature corresponding to F, and conversely. The corresponding color must be determined empirically for each grade, and, for important work, even for each ingot of steel.

The steel used by Brinell had about 0.50 per cent. carbon, such as is used for cannon.

Each diagram represents a group of experiments upon bars in which the same structure had been previously produced by the methods indicated above. An ordinate along the axis represents the temperature to which a piece of steel was heated; the abscissa to the left represents roughly the result-

ing coarseness of structure. The character of the structure is indicated by reference letters. The extremities of abscissæ so determined are connected by a line indicating whether, after heating to the desired extent, the bars were quenched or cooled.

Quenching is represented by a full line ________.

Cooling is represented by a wavy line ⁀‿⁀‿⁀‿.

The dotted line to the right of the axis represents roughly by its abscissæ the state of the carbon at different temperatures, its relative freedom being represented by the corresponding abscissæ of the dotted line.

Interpretation.

Study of the diagrams will show that—

At W quenching always gives F and fixes carbon.

At W cooling always gives C and frees carbon.

Below W the crystalline structure does not change.

Below W the granular structure gradually becomes finer.

Below W the amorphous structure gradually becomes coarser (the only change possible).

Above W all structures gradually become coarser, being crystalline if cooled, and granular if quenched.

The change of carbon from free to fixed is sudden and is called *hardening*.

The change of carbon from fixed to free is gradual. If partial, it is called *tempering*, and if total, it is properly termed *annealing*.

Crystalline structure is associated with free carbon.

Granular structure is associated with fixed carbon.

Conclusions as to the Treatment of Steel.

1. After forging a cutting instrument or spring, it must be hardened so as to fix the carbon, as a necessary preliminary to its gradual release by tempering.

2. In tempering hardened steel, the less it is heated the less

is its structure affected; and the less is the change in the state of the carbon.

3. The fixed state is the more stable, so that it takes time to change it throughout the mass without exceeding the desired temperature externally. Such an excess would affect the structure of the over-heated parts. The metallurgical term *soaking* aptly illustrates the manner of heating steel from which the best results are obtained.

The effects due to a given temperature may, however, be produced by exposing the steel to a lower temperature for a longer time than usual. Many of the following apparent exceptions to the general rules appear to depend upon the question of time.

4. The carbon having been freed by slow heating, the rate of the cooling below W is indifferent unless the mass of the piece be so great as to cause the structure to change from the prolonged action of its internal heat.

5. If W be exceeded the effect on structure of hardening is lost, and the steel must be cooled below W and re-heated to W to *refine* it.

Use of the Term, Temper.

Much confusion has followed the loose use of the term *temper*. Besides being applied to the grade of steel, it is also commonly used to indicate *hardening;* whereas we see that—

Hardening is produced by quenching at W and fixing the carbon.

Tempering is a mitigation of the hardness above produced which follows from subsequently heating steel to some temperature below W, the proportion of the carbon thus freed depending on the temperature attained. Whether the steel should then be cooled or quenched depends upon the mass of the piece. It is usually quenched.

Annealing properly consists in cooling at W so as to free all the carbon possible and to destroy the effects of harden-

ing. But it is also a term commonly applied to the cooling below W of steel, whether previously fully hardened or not. According to the temperature attained and to the time taken to cool the piece, it is softened and freed from internal strain.

Rate of Cooling.

The brittleness and the hardness of steel will be increased by increasing the rate of cooling from W, either by quenching in mercury, or in water the conductivity of which has been increased by acidulation or by the solution of a salt. The same effect is obtained by using water at a low temperature, or by frequent changes of the particles in contact, by motion either of the metal or of the water.

By reducing the rate of cooling as by the use of oil or tallow, the effect known as *oil hardening* is produced. Its effect is intermediate between C and F, and is probably largely mechanical, the sudden cooling of the external layers preventing the expansion of the internal mass during subsequent attempts at crystallization. This limits the size of the crystals formed and increases the strength of the metal; but it produces some internal strain which may be relieved by tempering at a low heat. The charred oil next to the surface, like the scale, tends to delay cooling.

EFFECTS OF FORGING.

Above W.

Except when in small masses steel is generally heated above W in order to give it the plasticity required for forging. In this case the crystals are not supposed to be destroyed but to be softened and expanded by the heat. Having been further disturbed by the hammer, they are supposed on cooling to assume the sizes and shapes due to the temperature at which they have been worked, with intervals between the crystals depending on the treatment received. Free crystallization thus implies porosity and a diminished density, which is further diminished by heavy forging at a high heat.

Owing to the tendency of the crystals to slide over their adjacent surfaces, a heavy blow may cause the fracture of overheated steel. It may indeed fall to pieces in the fire. But, if such steel be lightly and rapidly hammered over its entire surface, the effect will resemble that due to agitating a crystallizing solution (Bloxam, Art. 38), in the reduction of the size of the crystals and in the increased strength of the material.

This effect having been attained, further forging at a lower temperature tends to form the piece and to distribute locally any cavities which may exist.

In forging gun work the ingot is reduced in thickness about one-half; the reduction being greatest for those portions of the gun that lie nearest to the bore.

Water Annealing. Owing to the difficulty of penetrating large masses of metal by the vibrations of the hammer, the greater part of the metal treated as above will be, when cooled, like A or B or a combination of both. When the size of the piece permits, one remedy proposed is to re-heat slowly to W, to quench so as to prevent free crystallization, and, as soon as the temperature falls sufficiently below W, to remove the steel from the water and allow it to cool slowly in air. See Diagrams IV and V. The internal heat removes internal strain. Railway axles are thus treated, the process being called "water annealing."

The structure of a steel casting may be improved by heating it to *W* and cooling it slowly.

Forging Below W.

When the hammer is of sufficient power, the best effect will be attained by forging just below W. The crystals are supposed not to be much expanded by this heat; but, being softened, they may be compacted so as to destroy the porosity due to free crystallization. This treatment gives the highest density attainable, viz., 8.0; the steel resists the file, has a

waxy fracture, and yields a beautifully veined surface when etched.* This work requires hammers of great power when large masses are thus forged.

The experience of all steel makers tends to show the advantage of forging at as low a temperature as possible. Workmen incline to over-heat the steel so as to diminish their labor, —but this should be avoided.

A very fine quality of steel wire made in England by Stubbs and much used for making drills and fine tools is said to be made by being forged between semi-cylindrical dies by a light "pony" trip-hammer running with very great rapidity. The temperature required is attained by the hammering.

INTERNAL STRAINS.

These arise from differences in the rate of cooling throughout the mass, being increased in large masses by the deficient conductivity of hot metals. It thus requires much experience to judge of the internal temperature from the appearance of the outside of the mass.

These strains increase with the maximum temperature attained, being greatest in the ingot.

They also increase with the area of cross section of the mass, so that it is well to defer the hardening until the pieces are as nearly as possible of their finished dimensions.

Uneven forging produces "hammer strain" which is relieved by annealing.

Difference in section causes differences in rate of cooling, so that it is well to quench the thicker portions of irregular masses first.

Other things being equal, internal strain increases with the *grade* of steel.

* This is probably the original Damascus steel, which has been imitated by the moderns by twisting together and welding wrought iron and steel as in shot-guns.

VII. GUN CONSTRUCTION.

I. BUILT-UP GUNS.

The operations are substantially as follows:

Casting Ingot.

For forgings such as tubes and jackets the ingot is often cast square, as shown in Figure 25. For short pieces like hoops, it is sometimes cylindrical. In order to obtain solid metal and to free it from slag, sand and other impurities a given amount of the top and bottom of each ingot is cut off and discarded during the process of forging.

The ingot is sometimes cast hollow. But this is objectionable, for it transfers the unsoundness found in the centre of a solid casting to the middle of the walls of the gun.

Coring.

The ingot may then be trepanned by a sort of cylindrical saw by which a solid core is removed. This rapidly removes the more porous portions of the ingot, which are in a more valuable form for minor purposes than the shavings from ordinary boring. This operation sometimes precedes and sometimes follows the forging, depending upon what tools the plant affords and also upon the size of the gun.

Forging.

For solid ingots the work is constantly rotated during forging by means of the *porter-bar*, which is a long handle clamped to the tong-hold. A sling chain around the drum forms a fulcrum. Man or steam power is used according to the size of the work.

Cored tubes and jackets are forged on a fixed mandrel to approximately their finished dimensions.

Blanks for hoops are cut off the ingot and *upset*, or hammered lengthwise into a cheese-like form. After punching they are treated as described page 44, or, instead of rolling, they are

forged on a mandrel. The choice of operations depends on the length of the hoop and the facilities available.

After every operation the piece is carefully chipped by hand to remove *pulls*, *seams* and *cold shuts*.

Treatment and Tests.

The term *treatment* applies to the methods employed to affect the structure of steel, viz., *annealing*, *hardening* and *re-annealing*. The sequence of the tests is important.

The "hammer strain" is relieved by annealing. Annealing also facilitates the reduction by cutting tools to the rough-finished sizes required for oil-hardening.

After annealing, tests of the metal are made to discover its characteristics, and thus, to a certain extent, to regulate its subsequent treatment.

The pieces are then rough-bored and turned to nearly their finished size.

They are afterwards oil-hardened (generally called "oil-tempered") by being uniformly heated to W in a furnace constructed with reference to the shape of the heated piece, *e.g.* tubes in a vertical flue, through many ports in the sides of which flame enters tangentially and hoops in an ordinary low furnace. Each piece is then immersed with its axis vertical in a large tank of oil, holding many tons.

The tank is surrounded by a jacket through which a stream of water flows with required velocity. The oil is also caused to circulate by suitable arrangements.

The pieces are *re annealed** to remove the internal strain due to hardening. For this they are slowly heated to a low, red heat and allowed to cool very slowly. This heat improves the structure, but may slightly reduce the strength of the metal.

* This is properly *tempering*.

Tests of the metal are again made to see if it fulfils the necessary requirements.

Assembling.

The parts are then turned and bored to finished dimensions and assembled by shrinkage, the interior diameter of the outside cylinder to be assembled being finish-bored to the diameter prescribed for the contact surface, and the exterior diameter of the surface upon which it is to be assembled being turned to the excess prescribed for the shrinkage. The effect of the shrinkage, which follows the heating of the outer cylinder so that it may pass over the inner one, is sometimes to bring the surfaces in contact within the range of molecular cohesion. This phenomenon may sometimes be seen even between cold bodies. When the steel plugs, used to gauge the calibre of small-arms, being chemically clean, enter forcibly a clean bore, they are sometimes lost through "freezing."

The hoops are secured by being screwed together, but preferably by interlocking projections that slip by each other when expanded by heat, figure 47.

The policy of our government with regard to gun construction has been to obtain from private manufacturers the forgings rough-bored and turned, and to finish and assemble the various parts in its own shops.

II. STEEL CAST GUNS.

Objections.

The economical advantages of this process, which consists in making the gun of a single steel casting after the manner formerly adopted for cast-iron guns, are offset by the following objections:

I. Mechanical.

1. The enormous increase of the masses to be handled due to the weight of the sinking head, which, unless its functions can be replaced by other means, may weigh almost as much as the ingot itself.

A steel-cast gun weighs, in the rough, about 3 times as much as the heaviest ingot required for a built-up gun of the same calibre.

2. The difficulty of making molds strong enough to retain the high columns of metal required by modern powder.

3. The loss in cutting up the sinking head, for re-melting or in disposing of failures.

II. Constitutional.

1. The vesiculation, impossible to correct by forging.

2. The effect on crystallization due to slow cooling, also impossible to correct by forging.

3. The segregation of elements of different densities in cooling.

4. The internal strains developed in cooling castings which, for the heaviest guns, cast hollow, would possibly be 60 or 80 feet high, with walls 3 or 4 feet thick. If cast solid, this thickness would be increased.

Remark. This class of objections may possibly be overcome with increased experience in the treatment of the fluid metal, and by annealing the gun after casting.

Such experience must be costly, for it can be acquired only by dealing with masses approximately as great as those of the guns themselves.

III. Structural.

1. The impossibility of making physical or chemical tests of internal specimens.

2. The impossibility of adapting the composition of concentric parts to their specific functions by the principle of "Varying Elasticity," to be hereafter discussed.

3. The neglect of the principle of "Initial Tension" by which the inner parts may, by preliminary compression, be prepared for the strain of extension on firing. This principle is illustrated when a blacksmith shrinks on a tire.

IV. Historical.

1. Krupp's original guns, which were massive forgings,

have been gradually replaced by guns of increasing complexity of structure.

2. The only recorded failures in built-up guns have occurred in the large masses constituting the tube; sometimes when unsupported, as in the chase; or when imperfectly supported, as when a steel tube was surrounded by a jacket of ductile wrought iron. This, having been expanded beyond its elastic limit, failed to support the tube, which, on further firing, cracked.

CHAPTER XVI.

PROJECTILES.

Definition.

Functionally speaking, a projectile is a vehicle for the transfer of energy to a disconnected object.

The energy transferred may be wholly kinetic, as when the projectile acts by impact only. It may be wholly potential, as when the kinetic energy of the envelope of the mass may be neglected in comparison with the potential energy of its contents. And it may be of both kinds, as when the kinetic energy of the envelope is considerable.

SECTIONAL DENSITY.

On account of the work done on the intervening resistances, the energy actually transferred to the object will always be less than that originally imparted to the projectile.

The resistance to penetration offered by the intervening medium and the object, other things being equal, varies directly with the area of cross-section, a, at right angles to the trajectory. Let us call, d, the diameter of the circle whose area is, a, and $P=\frac{p \pi d^2}{4}$ the total resistance causing retardation.

The retardation, which for a given projectile is proportional to the loss of energy per unit of path, will, for different projectiles meeting the same resistance, vary inversely with the mass of each projectile; or, calling

ρ, the retardation in the direction of the axis of X;
M, the mass of the projectile;
E, the energy in the direction of X;
we have, neglecting variations in g;—

$$\rho = \frac{dE}{dx} \cdot \frac{1}{M} = \frac{gp\pi}{4} \cdot \frac{d^2}{W} = k\,\frac{d^2}{W}, \tag{1}$$

in which k, is some function of the pressure per unit of area, p, which pressure will vary with the velocity, the meridian section and the nature of the surface of the projectile.*

The *ballistic coefficient*, or *coefficient of retardation*, as $\frac{d^2}{W}$ is called, may therefore be used to compare the inherent capacities of projectiles for retardation; and the reciprocal of this expression, or $\frac{W}{d^2}$ which is called the *sectional density*, may be used to compare their inherent capacities to overcome resistances. In English measures W is taken in pounds, and d in inches.

VARIATIONS IN SECTIONAL DENSITY.

Causes.

The sectional density of a projectile may be increased as follows:—

I. If the dimensions are constant, by increasing the mean density.

II. If its mean density is constant, by varying its dimensions, viz.:—

*Since the form and dimensions of a projectile are independent of its velocity, and since the effect upon ρ of variations in the meridian section and the nature of the surface is small compared with those which result from changes in its diameter and weight, and disappears when similar projectiles are compared; we may for the present consider, k, for any projectile as constant, so that the value of ρ may be considered to vary only with the relation between d^2 and W.

1. If its proportions are constant, by increasing its calibre; since w varies as d^3, while a varies only as d^2.

2. If the calibre is constant, by increasing the weight.

3. If the weight is constant, by decreasing the calibre.

All these changes virtually lengthen the projectile.

Effects on Flight.

Increasing the sectional density of a projectile which has a given initial velocity increases its range and penetration, since the loss of energy over a given path is diminished. It may also increase its accuracy, since the time of flight over a given path, and therefore the effect of various perturbating causes may be diminished. The penetration is still further increased by increasing the indeformability of the material of which the projectile is composed, so that the work of deformation on impact may be done rather by the projectile, than upon it.

But, owing to the non-coincidence of the centers of mass and of the area exposed to the resistance of the air, during the flight of an oblong projectile a couple is formed which tends to cause the projectile to *tumble* or revolve about a transverse axis. This diminishes its sectional density and makes it variable. Such projectiles are therefore given the rifle motion, which impresses upon them sufficient angular velocity about the longer axis to make this a stable axis of rotation, and therefore to make their sectional density constant and a maximum. The same reason applies in a less degree to spherical projectiles, in which the centres of mass and of figure can rarely be made to coincide.

Effect upon the Gun.

Increasing the length of an oblong projectile increases its tendency to tumble, and hence requires a greater energy of rotation. This diminishes the kinetic energy of translation due to the conversion of a given charge. In a certain

sea coast rifle the rotary energy amounts to about 0.01 of the total muzzle energy.

Also, since increasing the sectional density increases the mass to be moved per unit of sectional area, a given acceleration requires an increase in the intensity of the gaseous pressure per unit of area. Therefore, since $V = \int \alpha dt$, to obtain a given initial velocity with a projectile of which the sectional density has been increased, the stress upon the gun must also be increased unless special provision be made by the methods indicated in Chapter XI.

Owing to the weakness of cannon in use when the rifle principle was first applied, the increase in sectional density required a reduction in the initial velocity; this, although compensated for by greater accuracy and longer ranges, caused the initial portions of the trajectory to be more curved than with the spherical projectiles formerly employed. See Chapter I. Consequently, the general adoption of oblong projectiles was delayed until the necessary improvements in the gun and its ammunition had been perfected. See Chapter XIII.

Comparison of Forms.

Although the oblong form is universally employed in new constructions, the following comparison illustrates some of the reasons influencing and opposing the change of form.

Advantages of Oblong Projectiles.

The form, capacity and sectional density may be altered indefinitely, with the advantages noted in the text. The following incidental advantages also exist:

Projectiles of the same caliber, but of different natures, or mean densities, may be made of the same weight; so that they may be fired at the same ranges with the same angles of projection.

The oblong form facilitates the operation of fuzes which act by impact; since the point and direction of the impact can be predicted.

Disadvantages of Oblong Projectiles.

The centers of mass and of pressure do not coincide; they are more expensive; the liability to injury of the soft metal device by which they are rotated requires greater care in their transportation and may interfere in their loading; in ricocheting over land or water their rebounds are much less certain and regular, both in altitude and direction. The rotation of rifled projectiles of the explosive class tends, upon bursting, to scatter their fragments unduly beyond the plane of the trajectory. The curvature of the trajectory at short ranges is increased.

MATERIAL.

The principle of sectional density mainly determines the selection of the proper material for a projectile, with regard to its behavior in the gun, in the air, and upon the object.

Its application is so apparent that only a few of the minor properties of the materials employed will be mentioned.

Stone was employed originally in catapults and continued to be used in cannon by the Turks as late as 1807.

Lead is suitable for use against animate objects only, since in large cannon it is disfigured and even partially melted.

Wrought Iron in large masses is expensive, as it requires welding and forging; it is also too soft.

Cast Iron was until recently exclusively used for artillery projectiles on account of its fusibility and its small original cost. When cast in molds, so that the point cools in contact with a cast iron *chill*, while the body cools more slowly in sand, its local hardness, crushing strength and density are greatly increased, without causing brittleness in that

portion cooled in the sand. Against the wrought iron armor formerly employed, such projectiles are indeformable; but they are pulverized against the steel-faced and chilled iron armor of the present day. For ordinary purposes cast iron is still generally employed.

Steel possesses all the qualities required in a projectile, but is costly. It is used in two forms, both of which are usually oil-tempered.

1. *Forged;* including for special purposes, rolled or drawn steel tubes. This form of steel, especially when alloyed with chromium, is so far the best, but the most costly. A 9 inch Whitworth forged steel shell, costing $100, or 12 times as much as a similar projectile of chilled cast iron, has been fired three times through wrought iron 12 inches thick.

2. *Steel cast* projectiles have, owing to their greater cheapness, been much experimented with; but, for the reasons given in Chapter XV, have so far proved inferior to those that are forged.

SPHERICAL DENSITY.

Since the sectional density, $\frac{W}{d^2}$, of similar projectiles increases with the caliber, if we divide the sectional density by the caliber we shall obtain a constant, $\frac{W}{d^3}$, which expresses the weight per unit of volume of a cube whose weight is equal to that of the projectile and whose height is equal to the diameter of the bore. This is taken as the measure of the *spherical density* of the projectile.

Since all spherical solid shot of the same material are similar, their spherical density is constant, and may therefore be taken as the unit by which to measure the spherical density of oblong projectiles of the same material.

Expressing the spherical density by S, and the weight in

pounds of a unit of volume of the material by δ, we have for a spherical solid shot, of which the volume is V,

$$S_s = \frac{W}{d^3} = \frac{V\delta}{d^3} = \frac{4}{3}\pi\,\delta\left(\frac{r}{d}\right)^3 = \frac{4\,\pi\,\delta}{3}\cdot\frac{1}{8}.$$

For projectiles made of iron, δ may be taken as ¼ pound, and π may be taken approximately as 3.0; therefore

$$S_{si} = \frac{1}{8} = C,$$

and for an oblong iron projectile in terms of S_{si},

$$S_{oi} = \frac{W'}{d^3}\,\frac{1}{C} = \frac{8\,W'}{d^3}.$$

S_{oi}, therefore, expresses the effective increase in density that arises from elongating the projectile.

We might proceed similarly with other materials having different values of δ; but it is convenient to retain S_{si} as a common standard; so that, in general terms, S may be taken to measure the number of times that the mass of the inscribed solid iron sphere is contained in that of the projectile considered.

Unless the caliber be fixed, the spherical and sectional densities of projectiles vary independently of each other.

The spherical density of the first oblong projectiles used in cannon in 1859, was about 2.0; but recent improvements in guns, powder and projectiles have increased it from about 3.0 in 1880, to about 4.5 in 1887, the muzzle velocity not being correspondingly reduced.

If all projectiles made of the same material had the same mean density and the same form, their spherical densities would be a function of their lengths. But as such is not the case, their length is independently stated, generally in calibers. In fact, the caliber is getting to be taken as the general unit of measure of all the linear dimensions relating to the interior of the piece.

Corollary.

Referring to the discussion on page 7, we see that the weight in pounds of a solid spherical cast iron projectile is very nearly equal to the cube of its radius in inches. This affords an easy method of approximating to the weight of an oblong projectile when the type of gun from which it is to be fired is known.

RIFLING.

History.

The invention of rifling by Gaspard Zoller of Vienna is said to have been made soon after the discovery of America. The first rifle grooves were made straight, and intended only to facilitate the loading of tightly fitting bullets. The advantages of the spiral groove, which were accidentally discovered, were not applied to oblong projectiles, even in small arms, until about 100 years ago, at which time the subject was thoroughly discussed by the eminent mathematician Robins. It is worthy of remark that to Robins we owe the first practical apparatus for the measurement of the velocity of projectiles; a pendulum into which the projectile was fired, and from the momentum of which that of the projectile could be computed.

The general adoption of the rifle principle for small arms was retarded by the difficulty found in loading the rifle: this was generally accomplished by the blows of a mallet on a stout iron ramrod. For cannon, attempts were made at an early date and are frequently renewed, to impart the rifle motion by the action of the gas, or of the air upon spiral grooves or wings formed upon the projectile. Except for low velocities, all such experiments have failed to act with certainty, and the end has been attained only by the positive means mentioned in Chapter I.

Twist.

The inclination of a rifle groove at any point is determined by the angle which its tangent at that point makes with the axis of the bore. *Twist*, is the term generally employed to express this inclination.

Classification of Twists.

When the inclination of the groove to the axis of the bore is constant, the twist is called *uniform*. When it increases from the breech to the muzzle, the twist is *increasing*.

Figure 1 shows the development of the surface of a bore rifled with uniform and increasing twists. Such curves are traced for the construction of templets, by which a combined motion of rotation and translation is given to the cutting tool of the rifling machine.

Discussion.

Let φ be the inclination of the groove at any point; and ω the angular velocity imparted to the projectile from being constrained to follow in the groove while moving in the direction of the axis with a velocity of translation v. Let r be the radius of the projectile.

We may consider the velocity along the groove to be the resultant of two component velocities at right angles to each other; viz.: v and $v \tan \varphi$. The latter imparts to a point on the surface of the projectile a tangential velocity $r\,\omega = v \tan \varphi$. Hence,

$$\omega = \tan \varphi \, \frac{v}{r}. \tag{2}$$

That is to say that when the twist is uniform, the angular velocity increases only with the velocity of translation throughout the bore. When the twist is increasing, the angular velocity further increases from this cause; and other things being equal, it increases as the caliber diminishes.

Since the muzzle velocity of a given projectile is fixed by independent considerations, the angular velocity at the

muzzle is measured by the tangent of the angle made at that point by the tangent to the groove and the axis.

If t be the time required to make one revolution, and n be the length in calibers over which the projectile must pass in order to make one revolution, we have from Eq. (2),

$$\tan \varphi = \frac{\omega r t}{v t} = \frac{2 \pi r}{n.2 r} = \frac{\pi}{n}. \qquad (3)$$

The twist is accordingly generally expressed in terms of n.

It has been found that for ordinary artillery projectiles, about three calibers long, the requisite steadiness is given by imparting to the surface of the projectile a tangential velocity of about 200 f. s. at the muzzle of the gun. Hence,

$$200 = r\,\omega = \tan \varphi . V = \frac{\pi}{n} V \therefore n = \pi \frac{V}{200}. \qquad (4)$$

The value of n at the muzzle of the piece has generally been determined empirically as above indicated; a safe margin being allowed, since no objection to a moderate increase in twist exists but that pertaining to a diminished energy of translation, and to the increased stress upon the piece.

Recent analysis has determined the minimum twist at the muzzle for projectiles of varying proportions.

It appears from this analysis that n is constant for similarly proportioned projectiles of the same material, whatever be the caliber; that it increases as the radius of gyration about the axis of revolution and the density of the projectile increase, and as the radius of gyration about an equatorial axis diminishes. Also, that the above value for the surface velocity is only approximate, since for the same projectile this may safely diminish as the initial velocity diminishes.

Tangential Pressure on the Rotating Device.

Since, for the same muzzle velocity of translation, the stability of a given projectile depends only on the angular velocity which it has acquired at the muzzle; it appears that

so far as this is concerned, it matters not whether the angular velocity be acquired only through α, the acceleration of translation, or through the combination of this cause with the gradually increasing twist.

In the first case, the angular acceleration, will be greatest at first, when the gun and the rotating device are under their maximum strain, and will diminish as they become relatively stronger; thus making a disadvantageous distribution of the work of rotation, although the quantity of work done will be constant and equal to $\frac{W}{2\,g} k_{\prime}^2\, \omega^2$.

$k_{\prime}$ is taken as about 0.8 r in the linear units used for V.

In order to make this stress, particularly that upon the rotating device, constant throughout the bore, so as to avoid either excess or deficiency in strength, the angular acceleration must be made constant. Herein lies the value of the increasing twist; since at the breech the diminished value of φ will compensate for the increased value of α; and conversely toward the muzzle.

The determination of the precise form of the developed groove is difficult, both theoretically and practically, since the constancy of α depends upon the properties of the powder employed.

It was thought for some time that a groove, the twist of which uniformly increased with the length of the bore, and having as its development a parabola, would give the best results.

Recent practice indicates the advantage of employing a semi-cubic parabola, of the form $x^{\frac{3}{2}}=2py$, which, in the case illustrated in figure 2, passes from a value of $n=50$ at the breech, to $n=25$ at the muzzle. Figure 2 shows how variously may be distributed the tangential pressures. To steady the projectile on leaving the bore, it has been thought

advisable to give to a short portion of the rifling neaı the muzzle a uniform twist.

MEANS OF ROTATION.

I. MUZZLE-LOADERS.

Classification.

The first rifled pieces were muzzle-loaders, and hence the projectile was necessarily of smaller diameter than the bore. Rotation was imparted to it in two general ways:

1. By making the rotating device fit the grooves before firing, by providing the projectile with suitable ribs or flanges.

2. By making the device fit the grooves after firing by causing it to be expanded by the powder gases, after the manner of the gas check. Chapter VII.

Operation.

For this special purpose, and in all cases to avoid abrading the grooves, the rotating device was made of a softer metal than the rest of the projectile; or, if formed on the body of the projectile, had given to it a large area of contact so as to accomplish the same result.

Since the axis of such projectiles did not normally coincide with that of the bore, they could be *centered*, or made concentric with the bore, only by chamfering the edge of the groove giving rotation, or by some similar device, the operation of which was uncertain.

Comparison.

Examples of the first class are shown in figures 3 and 4. Those with studs were until recently generally employed in Europe. The Whitworth projectile, the surface of which is a twisted prism, is a type of this class. It was once distinguished, but is no longer employed in new constructions.

The principal advantage of this class is that the projectiles are certain to take up the rifle motion.

They require special adjustment to the gun, both in manufacture and in loading; the escape through the windage erodes the bore; the stud holes weaken the projectile, and their arrangement in tiers, or the use of flanges renders it difficult to adapt these projectiles to the increasing twist.

Examples of the second class are seen in figures 5, 6, 7.

Their advantages are their adaptation to any gun of the proper caliber and the facility with which they can be loaded, particularly in action. The former advantage led to their general employment during the Civil War, owing to the elasticity of the conditions then prevailing. Only the weight and the caliber of the projectile were fixed; so that inventors were free to adopt many ingenious variations of the expanding principle. This is accordingly known as the American system. It answered well the demands of the situation but was uncertain in its operation; the expansion sometimes failing and the entrance of the powder gases between the body of the projectile and the rotating device serving sometimes to tear this from its seat. See figure 7.

The expanding cup has sometimes been applied to projectiles of the first class so as to prevent the escape of gas above cited.

Examples of Class II.

The Butler and Eureka systems are the principal examples of the second class retained for the muzzle-loading cannon still in service.

The Butler System. Figure 5.

The distinctive feature is the double lip formed in the expanding ring. The outside lip is expanded into the grooves, while the inner one is pressed against the tenon on the base of the projectile with an intensity proportional to that of the gaseous pressure.

The Eureka Projectile. Figure 6.

The base of the projectile is a frustum of a cone in which the grooves, *a*, are cast. The expanding brass cup fits on the frustum and is prevented from turning by corresponding projections on its inner surface, and from falling off during transportation by the screw plug, *b*.

On firing, the cup is forced forward and expanded into the grooves.

II. BREECH-LOADERS.

In breech-loading cannon the chamber is of larger diameter than the bore, and permits the use of a projectile provided with a compressible device.

Beside the advantages named in Chapter XI, the advantages of this class are certainty of action, better centering and the absence of windage. These qualities have caused their general adoption to follow that of the cannon in which they are employed.

History.

Following the analogy of projectiles for small arms, it was at first attempted to coat them with lead, cast over the body of the projectile. But this was weak, the lead fouled the bore, was easily deformed, and added useless weight to the projectile when it was fired against armor. The length of the bearing prevented the use of the increasing twist, and the manner of applying the lead tended to alter the structure of projectiles of hardened steel. Such were the projectiles used by the Germans in the war of 1870.

To overcome these objections, narrow rings or bands of copper, which is much stronger than lead, were placed in pairs at equal distances from the centre of gravity. Their diameter was equal to the caliber measured between the bottom of the grooves, or slightly greater, while that of the body was a little less than that between the lands. Such projectiles required the uniform twist.

To this class belongs a projectile used in the small Hotchkiss cannon; figure 9. It has a thin sheet brass belt about one caliber wide, compressed radially into a shallow groove of equal width which is symmetrical with the center of gravity. The surface of the groove is circumferentially fluted, as seen in figure 9. When the piece is fired the powder gases press the band into the flutings, forming a series of rings, figure 10, which permit the metal to flow backward as the band takes the rifling. These bands are much cheaper to make and weaken the projectile less than the solid rings formerly employed in these projectiles. This ingenious method is confined to small calibers.

Present Practice.

The increasing twist now used in all large cannon requires a narrow bearing, which, to diminish the effect of the oblique action of the powder gases is situated in rear, at such a distance from the base of the projectile as to give sufficient shearing strength to that portion lying in rear of the band.

To center the projectile a second band was formerly placed in front, but this has been replaced by a very slight enlargement of the body of the projectile near the base of the head. Figure 21.

Position of the Rotating Band.

Although, for ease in loading, the difference of diameter between the front bearing and the lands is made as small as is safe; unless certain precautions are taken, the oblique action of the powder gases—in a manner not thoroughly understood—may set up a nutatory or oscillating motion as the projectile travels through the bore. This leads to inaccuracy, reduces penetration, and may even leave the marks of the rifling on the front portion of the projectile.

To diminish this effect, the front and rear bearings should be made the loci of conjugate axes of suspension and oscil-

lation. When the position of the front bearing is determined by the shape of the projectile, this can be accomplished by swinging the projectile as a pendulum on a diameter of the front bearing, and ascertaining the time, t, of one vibration.

Then the band should be placed at a distance from it,

$$l = g\left(\frac{t}{\pi}\right)^2 \quad \text{since } t = \pi\sqrt{\frac{l}{g}} \quad (\textit{Michie, Eq. 404.})$$

In order to diminish the effects of the oscillation in projectiles in which, from unavoidable differences in manufacture, the method above described does not suffice, the width of the rotating band should be made as great as the nature of the twist permits. It is usually taken as about one-tenth of the caliber.

LONGITUDINAL SECTION OF THE PROJECTILE

Profile.

The value of k in Eq. (1) depends largely upon the profile of the meridian section of the projectile and the nature of the surface. In the last respect breech-loading projectiles of the last class have a decided advantage over those of the first class of muzzle loading projectiles, since the atmospheric friction is much less. This has required revision of the ballistic tables computed for the non-centered studded projectiles for which these computations were originally made.

The resistance of the air is not affected by the form of the extreme point, which, even if flat, is supposed to carry along with it a pointed core of compressed air; but the curvature of the head of the projectile is of great importance in that it affects the passage of the stream lines of air past the *shoulder*, as is called the circle of tangency between the head and the cylindrical portion of the projectile. The curvature of the head is expressed by the length of the radius of curvature in calibers. This varies from 1.5 to 2.0 calibers.

The form of the base is also of importance, in that, if also curved, it facilitates the flowing of the compressed air into the vacuum formed in rear of the projectile and so diminishes the difference of pressure upon the two extremities of the projectile; to which difference the retardation is principally due. Examples of this may be seen in the Whitworth projectile and in the Hotchkiss projectile, already described. This advantage is not generally utilized, as it tends to diminish the sectional density, the strength of the base, and the facility of manufacture and of stowage.

Mass.

The mass of the projectile should be distributed so as to bring the center of air pressure as close as possible to the center of mass so as to diminish the overturning moment of the resistance of the air. This is a difficult matter, as the direction of the pressure is constantly changing; it is therefore adjusted empirically by firing projectiles so weighted that the position of the center of mass may be varied

INFLUENCE OF THE CALIBER.

Following the principle of similitude by which cannon of the same class vary their linear dimensions in a given ratio to their calibers, it appears:—

1. That the muzzle energy varies with the charge of powder, or as the *cube* of the caliber.

2. That the capacity to convey this energy to a distance varies as the *first* power of the caliber.

3. That the terminal energy varies as a power of the caliber which increases from about 3 at the muzzle to about 4 at the extreme range of the smaller of two pieces considered.

STRUCTURE AND MODE OF OPERATION.

Projectiles are classed according to their structure and mode of operation as follows:—

1. Solid shot, or shot.
2. Shells.
3. Case shot.

I. SHOT.

Shot are used for penetration, generally of armor and in small arms against animate objects. For cannon they are confined almost wholly to the sea coast service. In order to diminish the effects of internal strain due to differences in the rate of cooling, shot are made not wholly solid but with an empty concentric cavity or core, figure 11. In such projectiles the point is carefully preserved.

II. SHELLS.

The increase in sectional density resulting from making spherical projectiles solid, having been attained by a change of form, solid shot are now replaced by those which are hollow and which can therefore convey energy in a form unaffected by retardation.

Shells are hollow projectiles containing an explosive and generally a fuze for its ignition at any desired point of the trajectory, The fuze may operate at a distance which is a function of the time of flight, when it is called a *time fuze;* or the explosion may result more directly from the arrival of the projectile at the point of impact. Such are called *impact fuzes.* Each class of fuzes, as will be seen, has its special province.

The size of the cavity depends upon the specific function of the projectile. If this is intended to convey energy mainly in the kinetic form, the smaller the cavity, the greater is the sectional density, and the more violent is the explosive required. If the energy is to be mainly potential, the larger the cavity the better the effect, provided that the resistance of the projectile to the shock of discharge is not unduly diminished.

The number of pieces resulting from an explosion, and the facility with which the bursting charge will operate, increase with the brittleness of the material and with the completeness with which conversion occurs before rupture of the envelope occurs. Since for firing against troops fragments below about one ounce in weight are not considered dangerous, it is desirable to increase the number of fragments of about this weight as much as possible and so to compensate for the large, single mass formed by the base of the shell.

The sectional density of the fragments approaches constancy and practically increases as they approach the spherical or cubical form; therefore, many devices have been employed to regulate the rupture of such projectiles, as by making the walls double, figure 12; by giving to the cavity a polyhedral form; or by grooving it spirally so as to diminish the tendency to burst in a meridian plane.

This appears from the following elementary analysis.

Let R and r, be the exterior and interior radii of a shell, the tenacity of which is T, supposed uniform throughout the section. Then, for the meridian rupture of a unit of length the necessary pressure will result from the equation—

$$2rp = 2(R-r)\,T \therefore p = T\left(\frac{R}{r} - 1\right),$$

for an equatorial or transverse rupture we have

$$\pi r^2 p' = \pi (R^2 - r^2)\,T \therefore p' = T\left(\frac{R^2}{r^2} - 1\right) > p.$$

Operation.

The rupture of a shell will occur in one of the two ways above indicated only when the material is thin and inelastic, as in some shrapnel to be described. When, as is usually the case, the projectile has thick walls (Chap. V), the inner concentric layers are more extended than those outside, they are fissured until fracture is determined by the line of least resistance,

and the fragments are scattered by the energy remaining in the gases. The resistance of the envelope should therefore be kept within certain limits.

Shells used against armor are pointed, and are filled and fuzed from the rear. They replace shot whenever possible since their penetration can be made almost as great, and their effects after penetrating the sides of a vessel are much more destructive both to men and machinery.

Against masonry, shell serve a double purpose; first to penetrate the wall and fissure it by their explosion; and second, by throwing out the fragments to present a fresh surface for the next blow.

Shells used against earthworks should contain the largest possible bursting charges. Such are called *torpedo shells.* They are sometimes made 6, and even 8 calibers long and, owing to the vertical angle at which they strike, are fired with low velocities from mortars and howitzers.

A *grenade* is a form of shell, generally spherical, intended to be thrown by hand or to be rolled down a parapet against masses of troops making an assault.

BURSTING CHARGES.

Gunpowder.

When powder is used it is preferably of fine grain and of high gravimetric density. Such powder in firing, has a tendency to *cake*, or become compressed into a mass of such density that the removal can be accomplished only by the chisel. This increases with the length of the charge and evidently tends to defeat its object.

The caking, as indicated by the letters *a*, *b*, *c*, in figure 13, results from three causes; viz.: *a*, from the shock of discharge; *b*, from the rotation of the projectile, and *c*, from the shock of impact.

The effect is to compress the powder and diminish its inflammability. If the impact be sufficiently resisted, the solidified mass may be thrown forward with such energy as to cause its ignition. To diminish caking, the cavity is often varnished; and to delay the explosion of armor piercing shell, the bursting charge is sometimes enveloped in flannel. On the other hand, where promptness is required, the French place loosely in the cavity a small wooden prism, figure 14, containing on two of its sides a network of oblique grooves through which the gases resulting from ignition may penetrate the indurated mass. A slip of wood is tied over each grooved surface to prevent the channels from becoming choked.

The bursting charge has been advantageously made of discs of concrete powder, strong enough to resist the causes leading to *a* and *b*, figure 13, but disintegrating under the influence of ignition. Such charges require to be filled through a hole, the size of which is objectionable.

High Explosives.

Gun-cotton is not injured by caking and, when specially prepared, does not readily explode on impact. This, or some equivalent high explosive, appears to be required for shells in which the excessive fragmentation of the envelope is not objectionable. This objection may be removed by using a small charge of dry gun-cotton in a shell filled with water.

A high explosive is particularly required in armor piercing shells; these, if strong enough to penetrate the armor, may fail to burst with the utmost powder charge which they will contain or they may explode harmlessly before penetration is complete.

The comparative insensibility of explosives of the Bellite class would seem to fit them particularly to this purpose; although the great *force* of gun-cotton makes it well adapted for use against earthworks. Its explosion is said to reduce

a large spherical zone of earth to a pulverulent form, by which its removal, and the exposure of the masonry which it is intended to protect, are facilitated.

Size of Cavity.

Great advantage has been found to result from increasing the size of the cavity by making the envelope of the shell of thin steel tubing. A 12 pound shell so made was found more effective against earthworks than a 50 pound shell of cast iron. To produce their full effect, such projectiles require an independent steel base, concave on its interior surface, so that it may be expanded against the walls instead of being driven outward by the explosion before the powder is entirely converted into gas.

INCENDIARY PROJECTILES.

Although the explosion of the bursting charge may suffice to ignite the splintered fragments of wooden structures, greater certainty in the effect results from filling the shell with an incendiary composition ignited by the discharge and flaming through specially constructed apertures in its walls. Such projectiles, called *carcasses*, and also red-hot shot, were formerly employed against wooden vessels.

To this class may be referred *light balls*, thrown at short ranges to burn on the ground and illuminate the works of an enemy during a siege. To prevent their extinction, they were made to contain a loaded shell or a number of loaded pistol barrels.

A more recent form consists of a shell containing a parachute which is distended when the shell explodes, and, when carried over the enemy's works by the wind, illuminates them by the light of a mass of incendiary composition suspended beneath it.

For modern warfare such devices are superseded by the

electric light, projected from under cover by a reflecting surface.

III. CASE SHOT.

Where concentration of energy upon a given point, and, therefore, accuracy is required, shell, or preferably solid shot are used; but where, owing to the dispersion of the objects and their inferior resistance the energy should be distributed, as in fowling pieces, case shot are employed.

Case shot consist of a number of small projectiles, which we may call the *cluster*, contained in an envelope; according to the method of their liberation from which they are divided into two classes.

1. *Canister* and *grape shot*, which separate at the muzzle of the piece in consequence of the shock of discharge. The general name, *case*, is now usually reserved for this variety.
2. *Shrapnel*, which separate at a distance in consequence of the explosion of a small bursting charge, contained within the envelope.

These projectiles forcibly illustrate the principle of sectional density in regard to the behavior of the projectile as a whole, and to the operation of the component parts, including the fragments of the envelope.

Operation.

The fragments separate in what is called the *sheaf*, or *cone of dispersion*, of which the *mean trajectory* constitutes the axis. Figure 15 shows a shrapnel provided with a time fuze, bursting in air; and figure 16 one with an impact fuze, bursting, as it is called, "on graze."

The right section of this cone is circular and the horizontal section elliptical. The size and form of the ellipse for any section vary with the energies of the different component parts, horizontally in the plane of the trajectory and normally to that plane; and also with the sectional density of these parts.

The inevitable lateral dispersion being sufficient for the necessary distribution, it is sought by various means to increase, at the moment of their separation, the component energy of the parts in the direction of the tangent to the trajectory. It is to the success of such efforts that the superiority of shrapnel over case shot of Class I is due.

Case shot are generally employed against animate objects, a dangerous wound to which is taken to correspond to the energy required to pierce a pine board one inch thick. It is convenient to remember that this requires a velocity of about 500 f. s. in an ounce ball, or to an energy of about one eighth of a foot-ton. A velocity of 500 f. s. is accordingly taken as the limiting velocity for case shot.

For reasons given, the fragments of the envelope are ineffective as compared with the members of the cluster, which are generally spherical. The envelope is therefore made as light as possible.

Structure.

But for the shock of firing, which deforms the members of the cluster even to the extent of consolidation, and which may even burst the envelope from the dilatation of its contents, the cluster would always be made of lead. But lead is expensive, and requires to be alloyed with tin or antimony; or to be imbedded in a matrix as of sulphur or rosin; or to be packed in coal dust to resist this shock. Consequently, iron is used, except when the conditions require every structural advantage to be improved.

Iron has the further advantage in certain cases of making the mean density of the case shot equal to that of the shell; this permits the firing to be regulated as stated page 4.

The size of the balls depends upon the facility with which the envelope may be filled, upon the material of which they are made, and upon the distance at which, after separation, they are required to act.

CLASS I. CANISTER AND GRAPE SHOT.

These are distinguished by the lightness of the envelope, which is designed only for their transportation and loading. The balls, generally of cast iron, are now much smaller than before shrapnel attained its present importance.

Canister.

For smooth-bore cannon the envelope consisted of a tin case supported in rear by a disc which was designed to prevent the penetration of the gases into the cluster while within the bore. To avoid the rotation of the projectile in rifled cannon, the envelope is stiffened so as to prevent its upsetting or dilatation into the rifling. In the English service this is done by inserting three trough-shaped pieces of sheet iron around the cluster. In the United States service a thin tube of malleable cast iron, closed at one end, is employed. This is known as Sawyer's patent. The fragmentation of this tube is assisted by spiral cuts. See figure 17.

Since canister is retained only for the extreme simplicity of its operation at the short ranges at which it is employed by the defence, the size of the balls has been greatly diminished.

When, as in the defence of the ditches of permanent works, the desired effect is not complicated by requiring projectiles of different natures to be fired from the same gun, canister fire is replaced by that from machine guns. This is continuous and does not derange the aim so much.

Grape Shot.

These were formerly employed in smooth-bore guns against both animate objects and the masts and rigging of vessels. The iron balls were arranged in tiers of three, sustained by a central spindle, a top and bottom plate and two intermediate rings. Figure 18. In former times they were quilted into canvas bags, whence the name.

CLASS II. SHRAPNEL.

Principles of Shrapnel.

Notation.

Let V be the initial velocity and v be the remaining velocity of the shrapnel at its explosion.

Let v_y and v_x, estimated respectively at right angles to, and parallel to the tangent to the trajectory, be the mean velocities of dispersion and of translation of the N balls that form the sheaf, irrespective of the velocities in these directions that are due to the remaining velocity v.

The velocity v_y, which is taken without regard to its sign, may be due to either or both of two component velocities; viz.: 1st. The velocity v_{y_b}, due only to the bursting charge; and, 2d. In oblong projectiles, the velocity v_{y_r}, due only to their rotation.

The velocity v_x, which is an increment of the remaining velocity, is due only to the bursting charge, and its sign depends upon the position of the bursting charge within the projectile.

Let v' be the resultant velocity of translation due to $v \pm v_x$.

Let φ be the inclination to the surface of the ground of the tangent to the trajectory (the axis of the cone) at the point of explosion. It is important to remember that, as stated in Chapter I and to be proved in Chapter XX, the curvature of the trajectory, or the value of φ measured from the horizon, is a decreasing function of v.

Let θ be the angle at the vertex of the cone of dispersion, figure 24.

We will for simplicity suppose the cone to be composed of rectilinear elements, and the surface of the ground to be horizontal,* so that θ will be the angle included between

* For the effect of varying the inclination of the ground, see Chapter XXX, figure 34.

the upper and lower tangents to the sheaf at its vertex, figure 25.

Distribution. †

It is evident that the mean density of the sheaf will be a decreasing function of $\theta = 2\ tan^{-1} \frac{v_y}{v'}$. Also, from the horizontal projection in figure 24, that since $\frac{N}{2}$ balls will be found in the area *a b o d*, the smaller is θ, or the more nearly does the cone approach a cylinder, the more uniformly will the balls be distributed over the entire ellipse *a b o d*.

Also, the smaller the angle φ, the greater is the eccentricity of the ellipse; or, for a given lateral dispersion, the greater will be the dangerous space in the line of fire.

If a be the area of the ellipse, $\delta = \frac{N}{a}$ will be the measure of the *density of the section*; this varies along the ellipse; see figure 24.

The shrapnel will be most effective when $\frac{a}{N} = \frac{1}{\delta}$ is the mean area occupied by one man projected on the ground by the elements of the cone.

Besides θ, φ and N the value of a will depend on the height above the ground, and the distance in front of the target at which explosion occurs, or upon the distance $vo = h$.

† NOTE. The curvature of the axis of the sheaf causes the section to be not truly elliptical, but oval as in figure 25. The ascent of the balls in the upper half of the sheaf, and the curvature of their trajectories due to their small sectional density, reduces their striking energy, so that those that fall near the large end of the oval are comparatively ineffective. This loss will be partly compensated for by the ricochet of balls nearer to the axis, provided they strike ground that is sufficiently hard. For ricochet, the angle of incidence shonld be less than 20°: this establishes a limiting value for $\varphi + \frac{\theta}{2} < 20°$.

These differences between the actual and the assumed conditions, having been understood, may, for this discussion, be neglected.

Nature of Target.

The requisites of shrapnel vary somewhat with the disposition of the troops against which it is to be used. These may be:

I. Either in columns of manœuvre, or in deep masses which it is the object of the artillery to force to deploy at long distances. In open ground these distances may now be as great as two miles.

II. Deployed in line at shorter ranges.

In the first case, consistently with the limiting values of $\frac{1}{\delta}$, the values of θ and h should be small; and conversely in the second case. In the first case, as the enemy will generally be in motion, and therefore erect, φ also should be small; and in the second case, as the enemy will generally be lying down and seeking cover, φ also should be large.

These conflicting considerations require special treatment. In the first case guns with high velocities are needed, and in the second case the limit will probably be found in the use of field mortars. Between these limits guns may be used with reduced charges and at high elevations, and at the closest ranges, say within 300 yards, canister is most effective.

The two cases correspond to the limiting cases of the cone. First, when $\theta = \varphi = 0$. Second, when $\theta = \varphi = 90°$.

The first case, being that most comprehensive and difficult to satisfy, and since it involves by its opposites the second case, is that herein discussed.

Choice of Fuze.

Shrapnel may be exploded in the two ways shown in figure 16, viz., "on graze" by an automatic impact fuze, or in the air by an adjustable time fuze. These have different spheres of action as follows:

Impact fuzes may be used at short ranges where v is large

and θ and φ are therefore small; and where φ' the angle of reflexion (always greater than φ), is also small. Figure 16.

But they act irregularly when the ground is soft or rolling; and at long ranges when v is small and φ is large, the energy lost on impact reduces the already diminished value of v, and consequently increases the value of θ. The time fuze is therefore essential for soft or rolling ground, and for long ranges over any ground. A possible objection to it applies to the risk attending its premature discharge when firing over friendly troops.

On some accounts the time fuze is less well adapted for use at short ranges than is the impact fuze; since for a given error in the time of its burning,* the greater is v, the greater will be the resulting variation in h, and therefore, for given values of θ and φ, the greater will be the variation in a.

Although this objection is partly neutralized by the small values of θ and φ at short ranges, the conditions seem to require the use of two fuzes in each projectile. See the *combination fuze*, Chapter XVIII. Meanwhile the improvement of the time fuze is one of the most important problems in ordnance.

Computing the Value of θ for Rifled Shrapnel.

Let ρ be the mean radial distance of the balls.

This is taken instead of the radial distance of the outer ball, as is generally done, since the distribution of the balls throughout the cross section of the sheaf is more important than their extreme lateral dispersion.

Let r be the external radius of the shrapnel, taken equal to that of the bore of the gun.

Although V wil be reduced during flight, it is assumed, and experiment confirms the assumption, that at ordinary

* The *mean error* (Chapter XXX, page 24) in the time of burning may be taken as about 0.05 sec.

ranges the angular velocity of the projectile does not sensibly diminish.*

Under this assumption, from Equations (2) and (3) the tangential velocity of the mean ball will be nearly

$$\rho\,\omega = \frac{\rho\,\pi}{r\,n}\,V, \tag{5}$$

and, since we are considering only the tangential velocity due to rotation, we have $V_{ry} = \rho\,\omega$ and

$$\tan\frac{\theta}{2} = \frac{\rho\,\omega}{v'} = \frac{\rho\,\pi\,V}{r\,n\,v'}. \tag{6}$$

If we substitute in this equation the empirical value of n in Equation (4) we have,

$$\tan\frac{\theta}{2} = \frac{200}{v'}\frac{\rho}{r}, \tag{7}$$

and for a given shrapnel in which V, n, and $\frac{\rho}{r}$ are known,

$$\tan\frac{\theta}{2} = \frac{C}{v'}\,; \tag{8}$$

an equation easily remembered, and which agrees fairly well with practice.

History.

The history of the improvement of shrapnel, which is now the principal field artillery projectile, illustrates many important principles. As stated, page 24, improvements have tended to reduce the ratio $\frac{v_y}{v'}$, and to increase the sectional density of the projectile as a whole, and that of the balls it contains.

Spherical Shrapnel.

I. Shrapnel, as invented in 1803, by General Shrapnel of the British service, were simply spherical shell, loaded loosely with musket balls and a bursting charge.

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* Projectiles fired vertically upward have returned to the earth with sufficient rotation to keep them point foremost.

In transportation the powder was triturated by the balls, and on firing the piece the shock might cause a premature explosion, or might conglomerate the balls; sometimes even causing the projectile to be ruptured by the resulting dilation of the cluster. The ignition of the bursting charge at the proper time was also uncertain.

In order to make V large enough to give a large value to v, the walls of the shell were made thick enough to stand a heavy propelling charge. But this diminished the value of N, and, since the interstitial volume between the balls was large, w had to be made large in order to obtain sufficient pressure to burst the shell. The energy remaining in the powder gases after rupture of the walls being large, the balls were widely scattered, making v_y large. The resultant value of v_x was zero. At long ranges, $\tan \frac{\theta}{2} = \frac{v_{yb}}{v}$ was therefore large.

2. The next step was the invention of *spherical case*, much used during our civil war.

By imbedding the balls in melted sulphur and boring out a chamber for the bursting charge, figure 19, the value of w could be decreased, and the certainty of its ignition at the proper time be increased. The matrix supported the walls in firing, so that their thickness could be decreased and N be increased. But the matrix often retained the balls after explosion, and the value of v_x was still zero.

3. Colonel Boxer, of the English army, devised a shrapnel, figure 20, in which the balls, hardened by an alloy of antimony, and packed in coal dust, were separated from the bursting charge by a wrought iron diaphragm around which the envelope was cast. The seat of the diaphragm and several other meridional grooves served to weaken the envelope and to diminish the value of w.

While the projectile was necessarily fired with the fuze in

front, the non-coincidence of the centers of figure and of mass caused the resistance of the air to turn the lighter portion of the projectile to the rear, so that v_x was always positive.

This projectile marked the farthest advance of spherical shrapnel.

Oblong Shrapnel.

The advantages of the oblong form of shrapnel are as follows:

It permits the base of the envelope to be strengthened without increasing the thickness of the walls. This, with improvements in cannon and gunpowder, has increased the value of v, and since the sectional density has been increased so that at long ranges φ has been diminished, it has also increased the eccentricity of the section of the cone of dispersion by the ground. By placing the bursting charge in rear, v_x has become positive, and v_{yb} has become practically zero.

An example of such a projectile is seen in figure 21, in which *B*, is the cast iron body; *H*, the ogival head of wood, covered with a sheet iron cap by which it is riveted to *B*; *C*, the powder chamber, made conical to facilitate unloading; *D*, a disc by which the cluster is swept out to the front; *T*, a tube to carry the flame from the fuze, *F*, to *C*. A paper lining keeps the rosin matrix from adhering to the walls of the cavity.

The slight resistance of the attachments of the head makes of this projectile a sort of aërial gun.

The objections to this projectile indicate the nature of recent improvements. The wooden head, the tube, the disc, and the thickness of the walls required by the nature of cast iron, diminish *N* so much, that the balls form about one-quarter the weight of the whole projectile.

The bursting charge is too small to produce sufficient smoke to indicate the explosion at distant ranges, and thereby to assist in correcting the aim.

The position of the bursting charge is such that, while acting well in air, when used with an impact fuze the delay caused by the passage of the flame through the tube causes the projectile to rise too high before bursting.

Present Practice.

The most recent ideas on the subject are embodied in figures 22, 23.

Figure 22 contains a combined time and impact fuze.

The bursting charge is situated in front, occupying the room which in figure 21 is wasted. It is large enough to give the smallest volume of smoke visible at extreme ranges.

The envelope consists of a thin drawn steel tube, secured in rear to a separate base, and slit and compressed in front to an ogival form.

The cluster consists of a column of leaden balls, separated by discs of cast iron. The discs are sunken to fit the balls, and form a skeleton matrix.

When the bursting charge explodes, the slit ends of the point are thrown back, so as to diminish the sectional density of the envelope as compared with that of the cluster.

The latter moves on with $v' = v - v_x \doteq v -$ about $200 f. s.$

Figure 23 represents a more recent form, of which in 1891 a number are in process of manufacture for experimental trial.

Its construction is apparent. The tube is of thin brass, enlarging its capacity for powder, and facilitating the passage of the flame from the fuze. The walls are weakened by longitudinal grooves. It remains to be seen whether, compared with figure 22, the increase in v_{yb} resulting from this construction will not neutralize the increase in v_x.

THE SEGMENT SHELL.

An attempt was made some years ago to combine the functions of solid shot, shell and shrapnell in the *segment shell*, in which the cluster was composed of the sectors of

concentric cylinders arranged so as to form a solid mass. But such a violation of the principle of the *independence of function*, which requires that where simplicity permits, each specific function be separately provided for, necessarily failed. The importance of this principle in the design of machines of all kinds can hardly be too forcibly stated. The opposite of this idea, that of combination, by which more than one office or function is expected of any one member of the machine or organization, is seldom found to be compatible with the efficiency of the whole, as we shall have many opportunities of seeing during this course. The full development of the principle of the independence of function leads naturally to complication or the multiplication of parts; judgment is therefore required to compromise between simplicity and efficiency. The history of invention appears to indicate the pre-eminence of efficiency.

As a case in point, it is now conceded that three types of the two classes of projectiles are required for field and siege artillery; viz.: shell, to convey kinetic energy for penetration, and potential energy for demolition and moral effect; and case shot for kinetic energy only. Although shrapnel, when reversed in the gun, may in an emergency replace canister; it is better to carry a few rounds of the latter, preferably, as in the British service, on the carriage which supports the piece.

REGULATING SHRAPNEL FIRE.

Referring to figure 24 we may consider the horizontal and vertical projections of h, viz., $x = h \cos \varphi$; $y = h \sin \varphi$.

Of these quantities, which are separately discussed in the regulation of fire, x is mainly varied by changing the time of burning; and y by changing the angle of fire.

Under given conditions x varies inversely as the range. Its variations, however, are not great, since there are compensations that tend to keep it constant. It is found that

the best results follow a value of $x = 50$ yards for all distances except those very short, for which x may increase up to 100 yards. If x be taken too small, too great a proportion of the shrapnel fired will explode beyond the target and be wholly lost.

In order to utilize the small values of φ in the upper half of the sheaf, it is advisable to make y small. It is found that the best results follow a value of y varying from 2 yards at 500 yards range, to 6 yards at 2,500 yards range. Greater values of y are not used, since they are difficult to observe correctly at long distances. The reason for the increase of y is due to the increase of φ at long ranges, and the consequent decrease in the area of the section cut from the cone by the surface of the ground.

These rules are in the nature of approximations. In practice the fire is regulated by signals from observers, placed as far as possible to the front and flank.

EMPLOYMENT OF FIELD PROJECTILES.

Shells.

These projectiles when used with a time fuze would follow the principles laid down for shrapnel; but the large value of θ and the small value of N would make this unprofitable.

They accordingly use an impact fuze, which makes of them the best means of controlling elevations. See page 4, and Chapter XXX, page 20.

They are generally used against inanimate objects to be demolished, pierced or set on fire.

In the following cases they may be used against troops:

1. At distances too great for the time fuze.
2. When the enemy is hidden in a village, or in thick woods.

The violence of their explosion assists their moral effect, particularly against horses and fugitive masses.

Shrapnel.

These are exclusively used against animate objects in the open, or in thin cover. They were found very destructive in the Russo-Turkish war. In siege operations they serve to annoy parties working at night to repair the damages done by day.

PENETRATION OF ARMOR.

General Considerations.

The penetration of armor depends principally—

I. Upon the nature of the armor. In the order of resistance armor may be classed as follows:

Cast iron with a chilled face, used only for land defenses and not considered herein.

Steel, forged and tempered.

Compound, viz., a wrought-iron back with a hard steel face.

Wrought iron, now obsolete.

Roughly speaking, armor yields either by *punching* or *racking*. In the first case, as in wrought iron, the effect is local. In the second case, the energy of impact is distributed throughout a greater mass of the plate and tends to crack the plate or to wrench it from its fastenings. The effect is mainly to remove an obstacle to further penetration. Cast iron armor yields in this way, and so do steel armor and the face of compound armor if too brittle.

The object of the artillerist is to concentrate energy on a small area, so as to reach the objects which the armor is intended to protect, *i. e.*, to punch.

The object of the armor-maker is to protect these objects, by distributing the energy of impact as much as possible between the projectile and the mass of the plate, so that even at the risk of destroying the plate by racking, *the shot must be kept out.*

But if racking can be avoided without loss of resistance to punching, the quality of the plate is improved. In the early manufacture of armor, racking effects predominated; these disappeared as its manufacture was improved; while the resistance to punching was maintained or even increased. For example, the principal objection to steel, for armor as for other purposes, has been its brittleness. But at Annapolis, in 1890, carbon steel armor resisted punching, but was slightly racked. Nickel steel armor resisted both racking and punching. Compound armor failed in both respects.

The nature of the *backing* or support against which the plate rests, considerably affects its resistance. Except for compound armor, for which the backing cannot be too rigid, the backing should be somewhat elastic, so as to absorb energy, after the manner of a cushion supporting a board in which one seeks to drive a nail.

As the liability to racking increases, the number of the bolts by which the armor is held in place should also increase, so as to retain those portions which would otherwise be displaced.

II. As a consequence of the above must be considered the resistance of the projectile to permanent deformation; page 3.

III. Upon the striking energy of the projectile, measured in a direction normal to the plate.

Since the projectile acts after the manner of a punch, shearing its way through the plate, the energy is often estimated *per unit of circumference.* In earth and masonry, in which the material is soft, the projectile is supposed to compress it to the front, and the energy is taken per unit of *area of cross section.**

* As experience with plates and projectiles of varying resistance to permanent deformation, increases, such assumptions are gradually replaced by purely empirical formulæ suited to each special case.

Whereas in experimental tests normal impact is the rule, in firing at ships it will be the exception. The shape of the point of the projectile also tends to make it glance, so that for these reasons armored ships may be expected to resist more than the formulæ predict.

Maitland's Formula of 1880.*

The energy expended in other forms than in perforation, as in heating the plate and projectile, and in deforming the latter, has given rise to many empirical formulæ, some of which may be found in the Course of Permanent Fortification. A very successful formula, *Froloff's*, assumes that the energy so lost is proportional to the striking velocity, so that the penetration is proportional to the momentum of the projectile on impact. The following formula, which illustrates a principle already taught, is considered by recent writers to be one of the best Equation (15).

Let t be the thickness in *inches*, of wrought iron armor that would just be perforated by a *cast iron* projectile, whose weight is W, its normal velocity on impact v, and its diameter d.

Let e be the normal energy in foot-tons, or $e = \dfrac{W v^2}{2g\, 2240}$.

Let ε be the energy in foot-tons per inch of circumference, or $\varepsilon = \dfrac{e}{\pi d}$.

During a prolonged series of experiments made by Colonel Maitland it was found—

1st. That t varied directly with ε, or

$$t = f\left(\frac{W v^2}{d}\right). \tag{9}$$

* It is inferred that the experiments on which Maitland's formula is based were made with ordinary cast iron projectiles, and that the armor was backed.

2nd. It was also observed that when projectiles of different calibers were arranged in classes according to their spherical densities; in each class the penetration measured in *calibers* was very nearly proportional to the striking velocity.

For a particular class, known as the *standard projectiles*, of which the spherical density was 3.0, the penetration was nearly one caliber for every thousand feet of striking velocity. This is known as Captain *Orde-Brown's* "rule of thumb."

For purposes of comparison, let us assume a given gun to be fired against a given plate; d and t will then be constant, and the variation in spherical density will result from varying the weight of the projectile. The variables will then be W and v. For the standard projectiles let these be represented by $W_{,}$ and $v_{,}$.

Owing to the number of experiments made with the standard projectiles, special weight is given to the results obtained from them. These results are expressed in the following general formula, differing slightly from Orde-Brown's rule, viz.:

$$n = \frac{v_{,}}{1000} - 0.14. \tag{10}$$

To pass from standard projectiles to those not standard, we use Equation (9) in order to ascertain the relation between $v_{,}$ and v. Under the hypothesis that d and t are constant it becomes

$$v = f\left(\sqrt{\frac{1}{W}}\right) \tag{11}$$

Whence $v_{,} : v :: \sqrt{W} : \sqrt{W_{,}}$, or $v_{,} = v\sqrt{\frac{W}{W_{,}}}$. (12)

In the standard projectiles, $W = 0.375\ d^3$; whence, from Equation (11)

$$v_{,} = \frac{v}{d\sqrt{0.375}}\sqrt{\frac{W}{d}}. \tag{13}$$

Substituting in Equation (10) we have

$$n = \frac{v}{612.4\,d}\sqrt{\frac{W}{d}} - 0.14\,; \tag{14}$$

Whence, multiplying both members by d,

$$t = nd = \frac{v}{612.4}\sqrt{\frac{W}{d}} - 0.14\,d. \tag{15}$$

The value, t, thus obtained is the thickness of a plate that will just be perforated by a projectile having an ogival head with a radius of curvature of 1.5 calibers. If the radius of curvature is increased to 2 calibers, as is now customary, t will be increased by 5 or 10 per cent, and Orde-Brown's rule will increase in exactitude.

If the plate resists perforation, then the penetration may be taken as about 0.9 of the estimated perforation.

The Formulæ of De Marre.

The following formulæ result from recent experiments in France and, except for Equation (20), cover a great range in calibers, and in the ratio $\frac{t}{d}$.

In the English units previously used we have for modern projectiles, viz., chilled iron shot and steel shells.

I. For the perforation of *wooden backing* when used as such, *i. e.*, *not* unprotected

$$e_b = 0.1823\, t^{1.2}\, d^{1.8} \tag{16}$$

This is about 70 per cent greater than when the backing is unprotected.

II. For a *wrought iron* armor plate that is *backed;* the resistance of the plate alone being considered

$$e_i = 5.809\, t^{1.3}\, d^{1.5} \tag{17}$$

Owing to the improvement of projectiles since 1880 this is less than the value implied by Equation (15).

For the entire target consisting of the *plate and backing*

$$E_i = e_i + e_b. \tag{18}$$

III. For the rather *soft steel plates*, generally used for heavy armor, as made at Creusot, when *backed*

$$e_s = 7.236\, t^{1.4}\, d^{1.5} \tag{19}$$

See also Equation (18).

IV. For the *thin plates of hard steel*, *unbacked*, used for gun shields, when attacked by the comparatively small cannon known as Rapid-Fire guns and Revolving Cannon

$$e_p = 12.86\, t^{1.4}\, d^{1.5} \tag{20}$$

These formulæ, while abundantly verified in the French service, must be accepted with caution when the conditions differ from those under which they were deduced.

Very's Formula.

Mr. E. W. Very, formerly of the U. S. Navy, has recently proposed a means of comparing the resistance of steel plates that has long been desired, since it eliminates variables relating to the nature of the plate, its thickness and the caliber and velocity of the projectile, all of which may differ in experiments made at different times and places.

It assumes that the projectile is not deformed, and that no other effect is produced but that of punching, which is supposed to be complete. The effect is referred to that produced in wrought iron, since within ordinary limits all such armor is homogeneous, and is therefore, well adapted for use as a standard of comparison.

Suppose we find by trial that a certain projectile will just perforate a given steel or compound plate with a certain energy e_s. Calculate the energy e_i required for the same projectile to perforate a wrought iron plate of the same thickness, and similarly backed. Then $\frac{e_s}{e_i} = \varphi$, in which φ is a factor

expressing the relative *per cent* of energy required to perforate the steel plate, *e. g.* $\frac{2140}{2000} = 107$ per cent.

Since 1880 the improvement in projectiles has been so great that e_1 has decreased considerably. Improvements in the quality of steel armor have increased e_s, so that φ has increased from about 125 to over 150. See next topic.

Weaver's Formula.

It has long been thought that besides its thickness, the mass of the plate affects its resistance to penetration, and consequently the "*energy per ton of plate*" is often recorded in the reports of firing against armor. No use is known to have been made of this knowledge, however, until the following formula, proposed by Lieut. Weaver of the U. S. Artillery.

It is probable that the work is practically confined to a mass of some definite volume immediately surrounding the point of impact, and that the volume of this mass is a cylinder, the diameter of which is n times the diameter, d, of the projectile, and the height of which is t, the thickness of the plate.

Experiments show that a notable increase in temperature and the *bulge* are confined to a tolerably distinct ring, about 2 calibers wide, so that n is probably not less than 5. It is probable also that it is safe to allow for an exterior ring which absorbs part of the energy, although the effects in this ring are not apparent. The value of n also depends upon the relation between t, the thickness of the plate, and d, the diameter of the projectile. Lieut. Weaver expresses this relation for Creusot steel by

$$n = 6.25 + 0.22\,(t - d) \qquad (21)$$

in which t and d are in inches. This value will depend upon the rigidity of the material, and is subject to correction by experiment.

In wrought iron n will approach unity, as the effect is noticeably local and no great increase of temperature in the adjacent parts is observed.

From a general consideration of records Lieutenant Weaver finds that about 1828 foot-tons of energy per ton of plate is necessary to perforate the entire target, consisting of a steel armor plate *and* its backing. This assumes the plates to be substantially uniform in resistance, the projectile to be indeformable, and neglects secondary effects, such as racking. Calling this coefficient, C; the weight in tons of the disc in question, W_d; and the weight in tons of one cubic inch of the plate, w; he writes:

$$E_s = W_d\, C = \left(\frac{n\ d}{2}\right)^2 \pi\, t\, w\, C. \qquad (22)$$

Supposing $w = \log^{-1} \bar{4}.1028$ and substituting for π, w and C their values, we have the general formula

$$E_s = 0.1828\, n^2\, d^2\, t \qquad (23)$$

APPLICATION.

We may now compare the foregoing formulæ by reference to the experiments at Annapolis in September, 1890. The plates were 10.5 inches thick, backed by 36 inches of oak, and were fired at by steel shells, as follows:

No. of shots.	Kind.	d	W	Spher. Dens.	v	e	Effect on Projectiles.
8	Holtzer steel,	6 in.	100 lbs.	3.70	2075	2988	6 unbroken.
2	Firminy "	8 "	210 "	3.28	1850	4988	2 broken.

Confining our attention to the unbroken 6 inch projectiles, only the points of which perforated the carbon and nickel steel plates, Lieutenant Weaver's formula gives $E_s = 3621$. If from this we subtract $e_b = 77$, per Equation (16), we have $e_s = 3544$, which is 18 per cent more energy than the plates received from any one blow. As the plates were not completely perforated, this would indicate that the value of

$\dot{\varphi} = 195$ from Equations (18, 21) is more nearly correct than that of $\varphi = 158$ assigned by Mr. Very's method; viz., by dividing 2988×100 by the value of $e_1 = 1816$, given by Equation (18).

ROCKETS.

Definition.

A rocket is a projectile propelled by a source of energy which it contains; it therefore performs also the functions of a cannon.

Structure.

A rocket consists of a cylindrical case of paper or metal, containing a composition formed of the ingredients of gunpowder mixed in suitable proportions. The front end of the case is usually closed, but the other end contains one or more holes or *vents* for the escape of gas from the ignited composition. Within the rocket is a hollow space called the *bore;* this may be formed by driving the composition around a spindle which is afterwards withdrawn; or by boring out the composition after its compression to a solid state.

The case is surmounted by a pointed head, which, for signal rockets, consists of a hollow paper cone, and for military rockets of any suitable projectile Depending upon the particular system of construction, some means is also provided for guiding the rocket in its flight.

Composition.

Since the composition is required to ignite readily, and since the amount of fouling is not objectionable, the proportion of sulphur is increased; and, since the gradual evolution of a large volume of gas rather than a large amount of heat is required, the proportion of nitre is diminished, while that of charcoal is increased, so as to yield CO rather than CO_2. To further delay the combustion the ingredients are often mixed, rather than incorporated.

Bore.

The bore is necessary to provide a large surface of initial combustion. In order to maintain a uniform pressure throughout the flight and so avoid either excessive strength and weight of the case when, at first, the pressure is low; or a deficiency of strength when, by the increase of the surface, the pressure increases, the composition should burn on a surface which is nearly uniform.

To prevent its burning on a decreasing surface, the composition must be so tightly packed within the case that the flame cannot pass around it.

The conical form increases the initial surface without increasing either of the above objections. It also facilitates the withdrawal of the spindle and increases the strength of the composition at the section corresponding to the immovable layer in a gun. Chapter VII, page 1.

Vent.

The momentum of the rocket is proportional to that of the escaping gas. The velocity of the gas will increase with the pressure, and this will increase as the size of the vent diminishes. The longitudinal and the cross sections of the vent must be so chosen that the gas will escape as fast as it is formed, or nearly so, otherwise the velocity of the rocket will be diminished and it may burst. See Chapter XI, page 8.

The excess of the total pressure on the head of the bore over that on the base, and the diminishing mass of the composition accelerate the motion of the rocket until the resistance of the air is equal to the propelling pressure: the variation in velocity will then be slight. When the gas ceases to flow the rocket becomes an ordinary projectile.

Guiding Principle.

The propelling force of the gas acts always in the direction of the axis of the bore; it follows, therefore, that with-

out some means of giving stability to this axis, the path described will be very irregular; so much so at times as to fold upon itself. Instances have been known when rockets have returned to the point from which they started. Steadiness of flight is obtained either by a guide stick, or by rotation.

The guide stick is used for signal rockets. It consists of a long wooden stick affixed to the case so as to bring the center of atmospheric pressure well in rear of the center of gravity. Any tendency to deflection is resisted by the atmospheric moment.

The Hale rocket, figure 26, owes its stability to rotation produced by the escaping gas. As this expands on escaping through the vents, it presses against the concentric *fences*, *F*, partly surrounding each of the three vents, and so causes rotation.

The effect is increased in Macdonald's Hale rocket by a similiar arrangement in front. In this rocket the bore extends throughout the composition.

General Remarks.

The difficulties found in constructing rockets so as to prevent the shrinking of the composition from the walls of its envelope; their inaccuracy, and their low capacity as vehicles of kinetic energy have limited their use in recent times to incendiary purposes, particularly in savage warfare. Where transportation is difficult and the enemy dwells in huts of an inflammable nature, as in Africa, the portability of these weapons causes them still to be retained by the British service.

Rockets are also much used for transferring life-lines to the crews of wrecked vessels, and may be applied to the movement of floating torpedoes.

Rockets are fired from inclined troughs or tubes.

The 12 pounder rockets of the following named varieties,

fired at an angle of elevation of 8° 15′, gave the following mean ranges and mean lateral errors (for the definition of these terms see Chapter XXX, page 24):

	Range.	Mean lat. error.
Hale rocket..................	1312 yards.	37 yards.
McDonald-Hale rocket...	2012 "	"

CHAPTER XVII.

FABRICATION OF ARTILLERY PROJECTILES.

The fabrication of projectiles involves reference to the principles of founding, some knowledge of which is necessary to a practical education.

Founding, or as it is less properly called, *casting*, may be divided into three parts, viz.:

I. Molding: by which a cavity, or *mold*, is formed to receive the molten metal; II. Melting; III. Pouring.

I. MOLDING.

Material of Mold.

Metallic.

When the metal to be cast is fusible at a low temperature, so that it will remain liquid for some time after contact with a metallic surface, the mold may be made of a less fusible metal. This permits great exactness in the resulting *casting*, particularly if the metal does not contract much in cooling, and it allows the mold to be repeatedly employed. For such reasons the molds formerly used for making bullets, and those now employed for making fuze-cases of pewter, and for printing-type, are metallic.

Metallic molds are also used in casting ingots that are to be forged, and for chill castings, as explained in the Chemistry and hereafter.

Non-metallic.

But when the metal to be cast cools so quickly on contact with a metallic mold that it is apt to set up considerable

internal strain; when it is apt to form blow-holes; and particularly, when its temperature is so high as to be destructive to the mold, this must be made of sand.

Because of its refractoriness, the sand used is generally silicious; and to increase its porosity to the gases, and its cohesiveness, that which is angular and of moderate size is preferred. Sand also yields slightly to the change in form of a casting while cooling. For example, a dumb-bell, cast in an iron mold would probably pull in two, from longitudinal strain.

Sand possessing all the properties to be desired for molding is seldom found in a natural state. Accordingly, artificial *molding compositions* are made by mixing sand with various proportions of clay or flour to increase its cohesiveness; or with some combustible material such as coal dust, horse manure or straw, to increase its porosity at a high temperature.

The addition of water is necessary to give plasticity; but as this causes blow-holes, and even dangerous explosions to occur, as little of it as possible is employed. In some cases it is removed by drying the mold; or, when great strength is required, by baking it.

Molding Compositions.

The presence of water or of a combustible material in the molding composition exercises an important effect upon the casting. In both cases the gases resulting from contact with the molten metal act, as in the familiar example of water in the spheroidal state, to prevent close contact between the fluid metal and the particles of sand. The effect of this contact would be to make a rough, gritty surface, destructive to cutting tools. The combustible may be incorporated with the sand or applied upon the surface of the mold.

Molding compositions are divided into three classes:—

1. **Green Sand**, which is wholly or nearly in its natural condition, and slightly damp. This is principally used for low grade castings, often molded in the floor of the foundry so as to avoid the use of flasks.

2. **Dry Sand**, which is artificially dried after molding. This is used for cylindrical objects, cast vertically, as it permits a freer escape of gas than does the green sand. It is also used for castings of copper and brass on account of their greater conductivity, the object being to prevent their cooling as rapidly as in the moist green sand. For cohesion, a certain proportion of clay is mixed with the fresh sand; and to compensate for the absence of water and the incorporated carbon, sand which is of a fine grain is employed to give a smooth surface to the casting.

3. **Loam.** This consists of a plastic mixture of clay and sand, to which straw, etc., are added for porosity. It is used for forming large volumes of revolution by the operation of *sweep molding*, to be described. Such objects are cast in pits, and hence the old sand resulting is called *pit sand.*

Besides these there are employed *parting sand* and *facings.* The former is lighter in color and of a finer grain than that employed in molding, and particularly free from moisture. Facings are generally composed of carbonaceous material, such as *black wash*, a mixture of finely ground coal and water, or of dry flour, soot, etc.; though chalk is sometimes employed on account of the CO_2 it gives out when heated.

Patterns.

These are of two classes, according as they have a solid or a hollow form. The former may be called *positive*, and the latter *negative* patterns. As each kind of pattern is intended to produce its like in metal, the positive pattern is

used to form a negative mold, and the negative pattern, or *core box*, to form a positive mold or *core.**

To indicate in the mold the position which is to be occupied by the core, *core prints* are made on the surface of the pattern. These form cavities in the sand into which fit corresponding projections on the core.

Positive patterns require to be made somewhat larger than the casting; the difference being determined by the *shrinkage* of the metal in cooling from the temperature of solidification to that of the atmosphere.

To facilitate their withdrawal from the sand, patterns are given a smooth taper surface; the difference in diameter is called the *draught.* This requirement influences the number of *parts* in which a pattern shall be made.

Parting Plane.

The parting plane is that in which the main sections or *parts* of the mold unite. The number of parts depends on the choice of the parting plane. Thus, for a rod of elliptical section, figure 7, if the parting plane contains either of the principal axes, AB, CD, there will be but two parts. But if the parting plane contains an oblique axis, as EF, either the mold or the pattern must be further subdivided.

The parting plane is accordingly taken so as to include either the maximum or the minimum diameter of the pattern. Long cylindrical pieces are therefore parted on an axial plane, as this direction gives them abundant draught.

The parting plane is the plane of reference for most of the operations of molding.

Negative patterns part on an axial plane to facilitate the withdrawal of the core, which is often made truly cylindrical, or of a form not readily admitting of its withdrawal in the direction of the axis.

* This distinction is introduced only for purposes of instruction; the ordinary classification being simply, *patterns* and *cores.*

Material.

The material of which a pattern is made depends upon the number of times which it may be employed, and somewhat upon its size. If made of wood, it should be built up of pieces having the grain running in different directions so as to prevent its warping.

In some cases where large castings are made by *sweep molding*, the expense of patterns may be spared, and the necessary concave and convex surfaces of revolution formed, by templets revolving about an axial spindle. The difference of radii between the core and the mold so formed determines the thickness of the casting.

Flask.

The sand forming the mold is supported by an outer frame or box, called the *flask*. As many separate flasks are used as there are *parts* in the mold.

For ordinary molding a *two-part* flask suffices; the part uppermost in casting being called the *cope*, and the lower part the *drag*.

Lateral motion between the parts of the flask is prevented by dowels, and the cope is prevented from rising under the hydrostatic pressure of the melted metal by weights or clamps, or flanges bolted or keyed to the sides of the drag.

The flask should conform to the general shape of the casting so as to avoid great differences in the rate of cooling and to facilitate the operations of molding. It often contains cross pieces to support the sand.

For loam castings, the flask may consist of a pit, sunk beneath the surface of the foundry floor.

Together with the flask is used, as a temporary bottom, the *follow board*. This may have a plane surface next the flask, or may contain in relief one or more patterns so placed as to determine the proper position of the corresponding molds.

Molding Tools.

These consist of shovels, watering pots and sieves for mixing the sand; rammers for packing it around the pattern: trowels of various forms for repairing imperfections, porous bags containing parting sand and facings, and venting wires with which to open an escape for the occluded gases.

II. MELTING.

Metal.

The properties of the metal employed depend on the size of the casting and the nature of the projectile.

A decided advantage in tenacity follows the use of a large proportion of gun-steel scrap.

The higher the grade of iron, the stronger it is; but the less fluid it is when melted, and the greater is the shrinkage and the difficulty of subsequently reducing it to finished size. The effects of shrinkage are relatively greatest in small molds.

Consequently for field projectiles grey iron is used, and for those of larger size that which is more mottled or contains a larger proportion of white iron. For chilled shot a mixture is made of charcoal and anthracite pig irons, or of old shot and car wheels in about equal proportions. The components first named in each pair give toughness, and the latter the desired hardness to the casting. Car wheels are cast in chills surrounding the tread, while the centers are cast in sand. The chill gives hardness where abrasion is to be feared, and the sand causes the interior to cool more slowly, thus converting it into grey iron and giving it the softness and toughness required. The same principle is applied in the chill casting shown in figure 6.

Furnaces.

The cupola, or the reverberatory furnace is employed according to the quantity and quality of the projectiles to be cast.

III. POURING.

To diminish the shrinkage the iron is poured at the lowest temperature consistent with fluidity; and to make the shrinkage uniform, small ladles filled from the furnace are preferred to the large ladles used for great castings.

The melted metal is skimmed while pouring.

FABRICATION OF PROJECTILES.

To apply the preceding principles we will explain the manufacture of the 4.5 inch shot and shell of the Butler pattern, referring to figures 1 to 6.

Patterns.

The parting plane is taken at the junction of the body of the projectile and the tenon for the rotating ring; concentricity of these parts being secured by an axial dowel.

The diameter of the cylindrical portion is enlarged relatively to the maximum diameter of the head; so that, during the reduction of the body to its finished size, the curvature of the head near the front bearing shall not be distorted.

For the shot a *teat* provides a small surplus of metal near the point and ensures a full casting there. The teat is afterwards turned off to the curvature shown in the dotted lines.

The shell pattern has a projecting *spindle* as a core print. This terminates at *a* in a conical surface, so that in spite of wear and unavoidable variations in manufacture it may be accurately centered in the cross piece, *f*, of the flask. Such conical bearings are frequently used in construction.

As it is difficult for the sand to penetrate the small annular cavity above the main portion of the core, this is separately formed in the *mold box*, figure 4, before the spindle is seated in the core box. The mold box represented is of wood, constructed on the same plan as the core box.

For a double-walled shell the core is covered with a cast iron corrugated sleeve of the form desired. The resistance

which this offers to the contraction of the metal about it, explains why this ingenious form of projectile is not more largely employed.

The patterns for the *gate*, by which the melted metal is admitted to the mold, and the *riser*, by which the air and scoriæ escape, are plain conical sticks, sometimes, as in figure 6, made in two parts to facilitate their removal.

Core Box.

This is of iron, made in halves uniting on an axial plane. It is bored out when bolted together through the four holes shown, and brought into correct opposition by the four conical dowels near the holes.

To form the core, an iron tube, called the *spindle*, perforated with many holes and provided with a conical bearing as at *a*, (all figures,) is wrapped with tow and secured in the core box by a nut, *n*. Sand is then rammed around the spindle, the final form of the core being given by the cup, *b*, so shaped as to strengthen the base of the shell.

Flask.

For large projectiles this is cylindrical; but for small ones it may be of rectangular cross section so as to contain several molds. For molding shell or cored shot, a *cross piece*, *f*, figures 5 and 6, containing a conical cavity, is so fixed in the flask that the parting plane of the pattern shall fall in the parting plane of the flask. For some projectiles, admitting of complete perforation, two cross pieces are provided. In such cases the conical bearings are not required.

To give greater strength to the flask and to preserve the concentricity of the projectile the parting plane is one of right section.

Position of the Pattern in the Flask.

Shot are cast point down, so as to give density to the point.

Casting shell point down leads to porosity and weakness of the base which may cause them to fail in the gun. But when a front fuze is used, as in figure 5, if the shell were cast point up, the feeding of hot metal from the riser against the thinly protected spindle would soften it and cause it to bend. This objection does not apply when the fuze is in the bottom of the shell.

On account of the difficulty of handling heavy *chills*, chilled shot are always cast point down. To prevent the wear of the chill from the hot metal, which limits its life to about 50 casts, removable linings are employed. Following the general principle which requires a symmetrical arrangement of the parts of the mold, and to prevent its cracking from unequal expansion, the exterior of the chill should follow the profile of the mold.

Gate and Riser.

For large projectiles, figure 6, the gate enters the mold preferably from below, so as to avoid splashing, and tangentially, to give a rotary motion to the ascending column of metal, and so sweep the scoriæ away from its axis.

The riser is intended:—

1st. To allow free vent to the included air and gases. These are sometimes lighted to assist their dispersion.

2nd. To allow the melted metal to be stirred during the solidification. This liberates the gases and scoriæ; and, since fused metals are poor conductors, it facilitates simultaneous solidification, and thus diminishes internal strain.

3rd. To feed the hot metal into, and sometimes to make it flow through the mold.

By careful stirring and feeding, shot as large as 12-inch have recently been cast *solid*.

Small projectiles may be cast in groups with one gate for

several molds; but each mold should have an independent riser.

Large spherical projectiles are sometimes cast in *strings*, connected by necks which increase in diameter upward.

OPERATION OF MOLDING.

Secure the spindle of the shell pattern in its seat in the cross piece by the nut *n*. Invert the drag so that the shell shall rest upon the follow board on the parting plane. Place the main shot pattern upon the point of the follow board indicated by a dowel. Dust the follow board and the patterns with parting sand. Fill the mold, ramming it sufficiently to make it solid, but not so much so as to unduly diminish its porosity; this requires much experience.

Invert the drag, holding it between the follow board and another board.

Remove the follow board; place the base patterns on the corresponding bodies and secure the cope by the dowels to the drag. Dust as before, and fill the mold; inserting the patterns for the gate and riser at the proper time.

Place the follow board on the cope; lift off the cope and reverse it; remove the patterns which it contains: they may require to be slightly jarred, so as to loosen them in the sand. Do the same for the patterns in the drag.

After repairing, drying, and facing the mold with black wash or its equivalent, place and secure the core which has been similarly treated.

Replace the cope and secure it to the drag by such means as shown in figure 6. The mold is then ready for pouring.

FINISHING.

Preliminary Operations.

As soon as the metal has become sufficiently solid, and while still hot, and therefore weak, the flask is opened and the excrescences left by the gate and riser broken

off. To facilitate contraction about the core the spindle is withdrawn. This may be easily done, since the tow with which it was surrounded has been consumed. To retard the cooling of the casting it is then covered with the loose sand which formed the mold. When cool, the cavity is carefully cleaned from sand.

The proper cylindrical form is given by the lathe, the most important of all machine tools.

Description of the Lathe.

A lathe is intended to form surfaces of revolution by causing an object to revolve on one of its axes while it is acted on by a cutting tool to which motion either along the axis of revolution, at right angles to it, or in both component directions may be given, either automatically or by hand.

Figure 8 shows a lathe, in which *A* is the *frame*, the upper surface of which is formed in parallel rectilinear *ways* or guides. *M*, is the fixed *head stock*, in which revolves the *cone pulley P*. This may be made to carry with it the concentric *live spindle*, *S*, and the *face-plate*, *F*, or may revolve independently of these parts. The spindle is hollow and carries on its interior the conical *center*, *C*. Its exterior is threaded for the face-plate.

T, is the movable *tail stock;* it contains the *dead spindle*, *S*, provided with a conical center like that in the live spindle. The tail stock may be clamped on the ways at any desired distance from *F;* a close adjustment of *S* may be made by the screw *D*, which is clamped by the *set screw E*.

The *slide rest G*, the invention of the great English mechanician, General Samuel Bentham, has been used for only about a century. To its invention is attributed the practical success of the steam engine; it having been previously found impossible to produce truly cylindrical

surfaces of large diameter. The slide rest, carrying the cutting tool, derives its motion from the rotation of the live spindle by means of a *change gear*, *H*, which connects the outer end of the live spindle with the *feed screw*, *J*. The feed screw passes through a nut on the lower side of the slide rest, with which it may be thrown into and out of gear.

Variations in Speed.

The necessary *cutting speed*, or the velocity of the surface in contact with the tool, varies with the nature and diameter of the material to be turned. The angular velocity of the work may accordingly be varied by means of the *steps* on the cone pulley. A similar pulley above the lathe, with its axis reversed, receives the power from the main line of shafting by the *driving belt*, *d*, and transmits it to the lathe by means of the *working belt*, *w*. The upper pulley is mounted on an axis provided with a *fast* and a *loose* pulley, *f* and *l*, so that the lathe may be set in motion or stopped by varying the position of the driving belt. This arrangement, which is indispensable to all machine tools, is called a *counter-shaft*. See figure 9.

Where great power, and therefore slow speed is required, the *back gear*, figure 10, is employed. This consists of two pinions, *a* and *b*, mounted on an axis, *c*, parallel to that of the spindle *s*, and so placed that when *a* engages with a toothed wheel, *d*, which is secured to the spindle, *b*, shall engage with one of corresponding size, *g*, upon the cone pulley.

To use this, the cone pulley is detached from *d*, and revolves freely upon the spindle. The back gear may then be engaged with *g* and *d*. The ratio of the diameters of *g*, *b*, *a*, *d*, indicates the resulting gain in power.

By varying the change gear any desired ratio can be obtained between the angular velocity of the work and that

of the translation of the tool. In this way screws of any desired *pitch* may be cut.

Support of Work.

The work may be supported by the conical centers forming the adjacent ends of the live and dead spindles. For this purpose it is provided with corresponding depressions, which are called *center marks*, at the ends of the axis of revolution. As a rule these center marks are left in finished work, as they permit pieces containing them to be reworked or repaired. The work, in turning between centers, is caused to rotate by means of a *dog*, figure 11; the *tail* of the dog fits in a radial notch in the face plate.

In certain cases when turning *between centers* is impracticable, one end of the work is secured to the face plate by means of the *chuck*. This is provided with three radial set screws capable of simultaneous operation. See figure 12.

In such cases, and to prevent the springing of long pieces in turning between centers, an intermediate *back rest*, *B*, figure 8, is sometimes employed.

Uses of the Lathe.

It is evident that the lathe may be used for boring as well as for turning external surfaces, and that by the use of a hook-shaped tool, passed through the fuze hole, such cavities as that of the shell can be turned.

Also, that plane surfaces can be formed by omitting the longitudinal translation of the tool, or that, preserving this motion and guiding the tool by means of a template, any desired surface of revolution may be exactly reproduced.

By replacing the center in the live spindle by a suitable tool, against which the work may be pressed by the back spindle, also without its center, the work may be drilled.

If the tool be made after the manner of a very thick circular saw, the edge of which may be either cylindrical

or form almost any surface of revolution, the work may be moved along a plane director at right angles to the plane of rotation, so as to form a new surface composed of parallel rectilinear elements, and having its cross section correspond to the contour of the tool. This operation is called *milling;* it is of the greatest importance in the manufacture of fire arms, sewing machines and others in which the interchangeability of the parts is required. To the general use of milling machines may be largely attributed the eminence of certain American manufactures.

The principal advantages of machines employing the principles of the lathe depend upon the continuity of the motion and the ease with which it may be varied.

Final Operations.

Projectiles of soft iron are finished externally on the lathe, or may be forced by an hydrostatic press through a circular steel die. The former method is preferred. The head is not touched, in order that the *skin*, which is the hardest part, may remain intact.

Chilled shot require special treatment by a grindstone or a peculiarly shaped prismatic tool, figure 13. This forms a scraping, instead of the paring edge generally employed; it is less apt to spring away from the work on meeting any portions which are excessively hard, and may be easily and accurately sharpened by a cylindrical grindstone.

The natural silicious sandstone is frequently replaced by an artificial stone composed of emery concreted by a cement.

FABRICATION OF STEEL PROJECTILES.

Those are intended for piercing armor. Either cored shot or shell are employed. They may be either cast or

forged, The former are the cheaper; the latter, so far, the stronger.

Steel Cast Projectiles.

A rather silicious metal is preferred. In order to fix the carbon, both head and body are cast in a chill mold; this is surmounted by a sand mold containing the sinking head. After cutting off the sinking head, the projectile is hardened, the point being heated most. It is cooled by first dipping the point in water and then immersing the whole projectile in oil. In order to further soften the base so as to permit the screw thread in the fuze hole to be cut, the base is annealed while the point is kept in running water. To avoid this operation, the base of the projectile may contain a piece of wrought iron pipe, around which it has been cast, as in chilled shot.

Forged Steel Projectiles.

These are hammered into shape from bars of suitable size, turned inside and out, and hardened and tempered as above described.

Steel shrapnel are now (1891) economically made by electro-welding. Chapter XV, page 23.

ROTATING BANDS.

Copper is preferred on account of its softness and strength and its resistance to erosion by the gases. Its uniformity is increased by adding about 5 per cent of zinc. This forms an alloy known as *gilding metal*, used in the manufacture of cartridges, cheap jewelry and the bell buttons used in the Cadet uniform.

The bands are applied in two general ways.

I. In Casting.

1. The band may be cast in place on the projectile. This is the simplest plan, but does not always make a good casting.

2. An annular band, the cross section of which is as shown

in figure 14, is placed in the bottom of the mold before the metal composing the body of the projectile is poured. To keep it from melting, it may be surrounded by a much thicker band of the same material. or by a hollow band through which runs a stream of water.

II. After Casting.

A seat for the band of the undercut section shown in figure 15, is turned in the body of the projectile and the band forced into this groove by hand or by machine.

1. By hand.

In this case the band may be either a straight rolled strip with bevelled ends, as seen in figure 16; or for large projectiles it may be cast in the form of a semi-circular hoop. In both cases the placing of the band is done gradually by the hammer.

2. By machine.

The band complete is slipped over the projectile until opposite its seat; it is then set in by powerful presses acting radially.

INSPECTION AND PROOF OF PROJECTILES.

Comparison.

It can hardly be too strongly insisted upon that the inspection, not only of projectiles; but of powder and of arms of all kinds is only preparatory for and subordinate to, the *proof*. The inspection may detect the causes of failure in proof, and often applies to many more articles than can be profitably proved; but that it can not wholly replace it, is proverbially and actually true.

INSPECTION.

Object of the Inspection.

The object of the inspection is to detect defects of workmanship and material which may affect the successful operation of the projectiles.

As it is impossible to make all projectiles of exact dimensions, certain variations are allowed in manufacture. For sake of economy, the greatest variation or *tolerance*, consistent with safety and efficiency, should be allowed; both in workmanship, as shown by the gauges, and in the material. This remark is general.

Instruments.

Maximum and minimum ring gauges, see Chapter IV, page 11; a hollow cylinder gauge, five calibers long; a profile gauge; a rolling table, and calipers for measuring the thickness of the metal at the sides and bottom of the cavity are the principal instruments required. Besides these there are various gauges to verify the dimensions of the fuze hole, and of the rotating device and its seat. Also various tools for exploring suspicious cavities or defects.

An easy method of detecting small differences in the diameter of cylindrical holes consists in the use of a slightly conical bar of steel, the diameter of different sections of which is marked upon its length after the manner of a diagonal scale of equal parts.

Except for the rolling table, the names of these instruments and their appearance as represented, figure 17, sufficiently indicate their employment.

The *rolling table* is of iron with two parallel ribs at a distance apart slightly less than the length of the cylindrical portion of the projectile. These ribs are brought truly level, so that a concentric projectile will assume a position of equilibrium of indifference.

Process.

The presence of fissures in hollow projectiles may be detected by exposing them to an internal jet of steam, or by observing whether after plunging them in water, notable differences in the rate of drying occur.

When it is possible, the quality of the material is tested

by a specimen cut from the body of a projectile. For chilled shot this is not possible; so that a cast specimen may be tested and compared with those mixtures which have given good results. A certain proportion of such projectiles are also split so as to expose the chill. The homogeneity of such shot is also tested by striking them with a hammer at the junction of the body and head: a clear sound should be produced. In spite of the inspection, such projectiles are liable to split spontaneously from internal strain.

In order to stimulate the contractor to greater care, projectiles are inspected in *lots*, the failure of a certain proportion of which for defects of material suffices to condemn the entire lot. This is then permanently marked so as to prevent its being again presented for inspection.

This rule is applied also to defects in workmanship when the number of objects is too great to permit of the inspection of every one, as in the ammunition for small arms.

PROOF OF PROJECTILES.

Careful inspection generally suffices for all but those intended for use against armor. But in all cases it is more conclusive to supplement this by a proof, as by firing for accuracy.

Armor piercing projectiles are proved by firing about one per cent of a lot against wrought iron armor about one caliber thick; the chilled iron striking normally and the steel at about 20 degrees to the normal. Upon the performance and endurance of the proof projectiles, fired with penetrating charges, depends the acceptance of the lot.

CHAPTER XVIII.

MEANS OF COMMUNICATING FIRE.

These may be divided into two classes, viz.:

1. Those intended for igniting stationary charges in guns and mines. It includes various forms of *matches* and *primers*.

2. *Fuzes*, which are intended to be used in moving objects, such as explosive projectiles, torpedoes, etc.

CLASS I.

MATCHES AND PRIMERS.

According to the time elapsing between their own ignition and that of the charge, these may be considered as relatively slow or rapid.

IGNITERS COMPARATIVELY SLOW.

Slow-Match.

This was formerly employed for igniting the port-fire, by which the loose gunpowder priming laid around the upper orifice of the vent was fired. It is now employed only for preserving fire. If made of hemp rope, combustion is retarded by saturating it with lead acetate, or the lye of wood ashes. If of cotton it is only necessary that the strands be well twisted. Slow match burns from 4 to 5 inches per hour.

Quick-Match is used to communicate fire, as in fire-works and in experimental work of a dangerous character. It is made of candle wick, steeped in a mixture of mealed

powder and gummed spirits, wound on a reel, dredged with mealed powder and left to dry. It burns at the rate of about 3 inches per second.

Varieties of Quick-Match.

The rate of burning may be much increased by enclosing the quick-match in a paper tube; see Chapter VIII.

If, instead of paper, the envelope be made more pliable and strong, as by a spiral wrapping of cloth around a central core of fine powder, the ordinary *blasting*, or *Bickford fuze* results. This inflames at a less rapid rate than the kind just named.

A tube of lead or one of its alloys may replace the weaker envelopes above described and instead of simply fitting it closely, the tube, enclosing the core, may be drawn as one mass after the manner of wire.

If gun-cotton be used for the core, a most convenient and rapid form of detonator results.

IGNITERS COMPARATIVELY RAPID.

Caps and Detonators.

These consist of cups or tubes made by means of a double punch, figure 1, the inner member of which, p, passes through a conical hole, h, of somewhat larger diameter in a stationary piece, d, called a die. The outer punch, p', which is concentric with the inner and fits closely to it, as it descends into a shallow cylindrical depression at the mouth of the die, shears from a thin copper ribbon a disc which it holds by the edges while the inner punch forms it into a cup. The elasticity of the cup causes its open end to expand as soon as it has passed through the die: this *strips* it from the punch as the latter rises for another stroke. The cup is elongated into a tube by the successive operation of a series of single punches and dies of gradually decreasing diameter. See plates Chapter XXVII. This operation, which

resembles closely that of *rolling*, in chapter XV, is of great utility in the arts. For military purposes it is principally used in the manufacture of metallic cartridges.

For percussion caps for small arms, the tube receives a charge of moist fulminating composition. This is prevented from falling out, when dry, by a disc of tin foil, held in by varnish.

The construction of the detonator has already been described in chapter XIV.

Cannon Primers.

These are of two classes, according as they are fired by friction or electricity.

I. Friction Primers.

The friction primer presents the following advantages over the method of firing cannon described, page 1. It is portable, certain and rapid; it affords the means of firing pieces at a distance, and does not attract the attention of the enemy's marksmen at night.

According to the direction of the vent, friction primers are divided into two classes.

1. Radial Vent.

The primer used in the military service of the United States consists of two copper tubes, soldered at right angles to each other, figure 2.

The short tube contains a charge of friction composition, (Sb_2 S_3 and K ClO_3) inserted moist and surrounding the roughened end of a wire, the outer extremity of which forms a loop for the lanyard. The long tube is filled with fine powder, retained by a wad of wax. The nib of the wire is folded over the end of the short tube, so as to prevent its accidental displacement and the firing of the composition in consequence.

For large guns, the column of fine powder may surmount

a pellet of compressed powder which will be shot, burning, into the cartridge.

In some services the cross tube is omitted and the wire, inserted axially, is withdrawn by a motion which causes it to bend continuously around the edge of the vent. See figure 3.

2. Axial Vent.

As the discharge serves to expel the empty tube with great velocity, unless it be thrown upward it may injure the bystanders. On this account, and also to prevent the erosion of the vent by the escaping gas, an *obturating primer* is screwed into a proper seat concentric with the vent. Figure 4 represents an obturating axial friction primer. When the wire is withdrawn, the conical portion, *c*, finds a corresponding seat at the end of the cavity surrounding the wire. This prevents the escape of gas through the hole, while the escape around the primer is prevented by the radial expansion of the thin edge in which the portion nearest to the charge is formed.

The stop, *b*, prevents the primer from being screwed in too far, and the enlargement, *e*, serves a similar purpose for the wire.

II. Electric Primers.

These are used for firing charges at a considerable distance, as in certain cases in modern warfare when the gun is so protected that the object is invisible from its neighborhood; so that the pointing and firing are controlled by a distant observer. By this means also, the simultaneous discharge of several cannon at a common object may greatly increase their effect. A similar advantage follows in mines.

The primers are of two general classes:

1. High tension, in which ignition results from the passage of the electric spark between the disconnected ends

of two insulated conductors. For this class the conductors require careful insulation and to be separated from adjacent circuits, so as to prevent the primers in one circuit from being accidentally exploded by currents induced from the other circuits.

2. *Low tension*, in which ignition results from the heating of a short wire of high resistance which connects the ends of the conductors. Owing to the ease with which the condition of the circuit can be tested before firing, and the comparatively low electro-motive force of the currents employed, this is the only class of electric primer used in artillery.

Figure 5 represents a common electric primer, and figure 6 an obturating electric primer. The platinum wire is coiled to facilitate its handling in manufacture. It is surrounded by a wisp of gun-cotton.

The obturating plug, *p*, of hard rubber seals the channel by being pressed against the sharp ring in rear. In other essentials these primers resemble figures 2 and 4.

MEANS OF IGNITING PRIMERS.

If quick match be used it suffices to unite the lines so that the distances $B\ C$, $B\ C'$, $B\ C''$, etc., in figure 7; or BC, $B\ D\ C'$, figure 8, be equal. If the detonating tubes, page 2 be used, these precautions are unnecessary.

For electric primers the voltaic battery is generally employed, although for experimental purposes a small portable dynamo or frictional apparatus is very convenient.

When it is desired to be able to fire without delay, a battery is preferred, which, like the Leclanché, can be kept for a long time in open circuit without sensible change and which only needs the circuit to be closed to produce the effect desired.

In using the electric current in *direct* or *continuous circuit*

as in figure 9, the number of cells of the battery required increases with the number of primers, p, p', p'', and it may happen that the most sensitive of the primers, exploding first, will cause the remainder to fail.

For the second reason a *derived*, or *parallel* circuit, as in figure 10, is preferred. The successive explosion of the more sensitive primers increases the current which passes through each of the remaining primers, since their number is diminished.

In order to employ a weaker battery, the arrangement shown in figures 11 and 12, serves, by sweeping the key, k, over the ends of the terminals, to produce a practically simultaneous discharge.

CLASS II.

FUZES.

Fuzes are employed to explode the bursting charge of a projectile at any desired point of its trajectory. They may be classified, according to their mode of operation, as *time*, *impact* and *combination* fuzes.

I. TIME FUZES.

A time fuze contains a column of composition, which, having been ignited at the discharge of the piece, after having burned for a definite time, ignites the bursting charge.

Requisites.

Such fuzes are principally employed to burst projectiles while in the air; they therefore require that the relation be known between the distance to the point of explosion and the time of flight, and that the column be taken of such a length that it will burn in the time so determined.

The first of these requisites involves the estimation of the distance by various systems of *range finding*, and the deter-

mination from *Ballistics* of the required angle of projection and the time of flight to the point desired. The second requirement demands that the rate of burning be known, and, since the time of burning is varied by varying the length of the column, that the rate be uniform throughout its length. Finally, that the column be taken of the exact length required by the rate, and that it both receive and impart fire with certainty.

The principal points to be considered in the development of time fuzes are, that as we increase the muzzle velocity and sectional density of our projectiles, the longer will be the maximum time of burning required for the fuze. As the remaining velocity increases, the greater will be the error in distance due to a given error in time; and the greater the range, the more difficult will it be to detect the error in distance. Therefore improvements in cannon require corresponding improvements in the uniformity of rate and in the exactness of the length of the burning column. The greater the rate of burning, the larger the scale and therefore the smaller the effect of a given error in cutting.

The rate is so much affected by the conditions relating to the resistance of the air during flight, that, while uniformity of rate may be indicated by the tests of manufacture, the lengths of column for given ranges should be determined by actual trial in the gun. On this account, and to avoid computation in the field, when the initial velocity and sectional density are fixed, the scale is preferably one of ranges, instead of units of time.

The great efficiency of projectiles properly exploded in air, as explained in Chapter XVI, and the experience gained with smooth-bore cannon, in which this was the only form of fuze that could be successfully used, account for the pains that have been taken to meet these requirements ever since the early days when the fuze was lighted *before* loading.

Rate of Burning.

This will depend upon the conditions named Chapter VIII, page 3.

The rate was formerly varied by varying the composition, but as any departure from the usual proportions is found to diminish the uniformity of the rate, to increase the difficulty of preservation, and to increase the amount of residue, it is now thought best to vary the rate only by varying the amount of incorporation and the density of the composition.

When the total time of burning is very great, as in some of the large mortar projectiles, which may be 40 seconds in the air, a return to the variable composition appears necessary.

Former Practice.

For spherical projectiles the column was cylindrical and was ordinarily contained in a conical case of paper, wood or metal. This was filled with small successive quantities of mealed gunpowder which were compacted by a drift upon which a given number of blows were struck by a mallet. By a repetition of the process the case was gradually filled.

The exterior of the case was divided into equal proportionate parts by which to regulate the time of burning, either by cutting off the case; or, since the entire column might then be dislodged backward into the cavity of the shell by the shock of discharge, by boring into it with a gimlet.

The fuze was ignited by a priming of mealed powder placed so as to catch fire from the flame passing through the windage of muzzle-loading guns, both smooth-bore and rifled.

The method of filling caused variation in fuzes of the same kind, and even between different sections of the same fuze.

Examples of Fuzes for Muzzle-loading Projectiles.

Figures 13 and 14 illustrate two varieties of time fuze, in one of which the composition was fixed in the case and in the other was movable.

The *Mortar fuze* case or *plug* was made of a close grained wood, like beech, bored out nearly to the bottom. The top of the cavity was enlarged to receive the *priming* of mealed powder and alcohol. This was covered by a cap of waterproof paper on which was marked the rate of burning. For economy of manufacture the exterior of all mortar fuze plugs was marked in inches and tenths, instead of with reference to the rate of burning of their contents.

The *Sea Coast fuze* consisted of a brass plug containing a separate paper case, filled with a composition of variable proportions and bearing on its exterior a scale of times. The mouth of the plug was closed by a *water-cap*, perforated by a zig-zag channel. This was also filled with mealed powder for the ignition of the fuze; but was so constructed as to prevent the composition from being extinguished in the ricochet fire over water, largely employed in former times.

These fuzes answered well for the comparatively low remaining velocities and short ranges usual when spherical projectiles were employed; but they required valuable time for their adjustment and were imperfectly protected from the effects of excessive heat or of moisture while in store.

The *Bormann fuze*, figure 15, was invented to overcome these and other objections. The case being of pewter is unaltered in size by meteorological changes, and it contains the composition in a channel, which, though air tight, can be readily cut by a proper tool. The circular form of the column and its diminished section allow the size of the case to be reduced, and the composition to be compressed in the direction of its shortest dimension. The mean density of

the successive layers estimated in the direction of the combustion is thereby made uniform. The case is screwed into the fuze hole by a screw driver, the prongs of which engage into the recesses *a*, *a*.

The graduated arc lies over the circular column of mealed powder which, after compression, is covered by the tightly fitting wedge shaped ring, *b*. The only outlet to the channel is under the zero of graduation; this outlet, *c*, and the *magazine*, *m*, are filled with fine powder which is retained by a disc of tin, *e*.

To enable the fuze to resist the shock of discharge, to which its softness, density and form render it especially weak; and also to increase the effect of a small bursting charge, the lower portion of the fuze hole is closed by a perforated disc, *f*.

The objections to the Bormann fuze are the short time of its burning; the uncertainty of its ignition unless it be carefully primed, and that, once set for firing, it is useless for any greater time of flight.

Present Time Fuse.

The use of breech-loading cannon necessarily prevents the ignition of the fuze through the windage so that a special device called an *inertia igniter* is employed for that purpose. Its operation is illustrated in figures 16 and 24.

In figure 16 the inertia igniter consists of a mass of lead containing a pellet of fulminate and supported a short distance above the sharp point, *p*, by some device which, while stable against ordinary shocks, will be surely moved by that of discharge. This device may be either a spiral spring or a transverse pin of brittle material.

The flame from the fulminate escapes through the holes, *h*, into the annular cavity, *c*, and, by a hole on the inner surface of the ring, *r*, ignites the circular column of composition which the ring contains.

The exterior surface of the ring is graduated, as in seconds, and the body of the fuze contains a mark, placed opposite to the entrance to the magazine, *m*, so that by setting the ring before firing with any division of its scale opposite to the mark, the length of the burning column is fixed. The cap, *k*, is used to clamp the ring in place.

To prevent the opposing rush of the gases from the two sections of the burning column, it is ignited at one of its ends; this permits a free escape of the gases to the outer air through a hole previously temporarily sealed against moisture.

Figure 17 shows the course taken by the escaping gases when the burning surface moves, as in the Bormann fuze, in two directions from the hole, *o*, to the magazine, *m*. Figure 18 shows the improved method.

For long ranges, since the form of the projectile permits its length to be indefinitely increased, the fuze may contain two or more rings arranged in tiers.

II. IMPACT FUZES.

Concussion Fuzes.

Until the introduction of rifled projectiles many unsuccessful attempts were made to combine the time fuze with some device which would be safe when the gun was fired; and yet, if the time fuze failed to act at the proper point, would explode the bursting charge on impact.

Owing to the uncertainty of the direction of the impact such fuzes are called *concussion* fuzes to distinguish them from the *percussion* fuzes now generally employed.

Percussion Fuzes.

Although as stated, Chapter XVI, page 21, the shock of impact may in certain cases suffice to explode the bursting charge, it is much more certain to employ a special apparatus for this purpose.

Although more complicated in structure than time fuzes, those of the percussion class act with more certainty since the conditions to be fulfilled are more easy of accomplishment. They are not as subject to deterioration in store, and, since they are usually entirely automatic, they require no preparation before firing. By the volume of smoke resulting from the explosion of shells containing percussion fuzes, the gunner is afforded one of the readiest means of correcting his aim.

Percussion fuzes are divided, according to their position on the projectile, into *front*, or *base* fuzes. The former possess the advantage that on impact, the bursting charge is thrown towards the fuze; the latter class is required for projectiles to be used against armor.

Requisites.

A good percussion fuze requires—

1. A *case* to hold and guide the movable parts, and to protect them from being clogged by the dust arising from the bursting charge in transportation, and by the earth against which they may strike.

2. A plunger, by the motion of which, on impact, the charge is fired.

3. A fulminating composition, ignited by the plunger.

4. The priming, a charge of fine powder ignited by the fulminate and serving to increase the certainty of the ignition of the bursting charge.

5. A safety device, by which the accidental dislodgement of the plunger is prevented; but which will certainly free the plunger when the piece is fired.

6. A device to prevent the plunger from moving forward in its cavity during flight. This tendency results from the greater retardation of the projectile than of the enclosed plunger by the resistance of the atmosphere. The effect of this relative motion may be to cause a premature explo-

sion if the fulminate is sensitive; or else on impact to deprive the plunger of sufficient motion to cause the explosion of a fulminate, the sensitiveness of which, for the reason above given, has been diminished.

The most difficult of these requisites to provide is the safety device. In the early percussion fuzes the plunger was a single mass sustained by a transverse pin or by lugs cast upon it. The pin was made strong enough to stand the shocks of transportation; but was shorn off by the shock of discharge. The mass to be given to the plunger was determined by conflicting considerations. If made so light as not to be liable to shear its support by accident, it might fail to explode the fulminate when impact occurred at a low velocity. The advantage of a proper distribution of functions among the parts of the apparatus appears from the following discussion:

Type of Improved Percussion Fuze.

Let V be the initial velocity of the projectile and v, and v', its velocities on impact, and after impact, as in the ricochet.

Let m, be the mass of the plunger and m' that of the safety device: this we will suppose to be a hollow cylinder as in figure 19, surrounding the upper part of the plunger, but kept from moving backward upon it by a suitable projection, as that upon the flat spring, s'. In the type selected the section of the plunger is square and fits the hole in the safety device.

When the piece is fired, m' moves relatively to the rear with an energy $\frac{m' V^2}{2}$ which, on account of the large value of V, is capable of overcoming a resistance great enough to be absolutely safe against all accidents of transportation. In so doing it becomes solidly united to m, so that when impact occurs, although $v-v'$ may be much less than V,

the energy $\frac{m+m'}{2}(v-v')^2$ may easily overcome the resistance of the spiral spring *s*, and so ignite the fulminate, *f*, and the priming, *p*.

If, during the flight of the projectile m' does not remain relatively at rest its conical form tends to make it roll rather towards the base of the cavity than away from it. It is also urged in this direction by the spring *s*.

This fuze, which is of French origin, represents one of the best existing types. It requires a value of *V* not less than about 1000 *f. s.*, and therefore is somewhat more complicated in construction when projectiles containing it are fired with low velocities.

Percussion Fuzes used in the United States.

Hotchkiss Front Percussion Fuze. Figure 20.

A, is the case, closed in front by the *screw-cap B*, and with a conical hole in rear closed by a lead *safety plug C*. *D*, represents the plunger, composed of lead cast into a brass jacket to prevent its dilatation by shock.

A continuous brass wire, *E*, the upper portion of which is bent in a semi-circle concentric with the plunger, is cast into the lead and supports the plunger in the case. The lower ends of the wire are securely held by the friction of the safety plug against the sides of its cavity. At *F*, is the fulminate, and at *P*, the priming.

When the piece is fired, *C*, is dislodged backward into the interior of the cavity either by its own inertia or by the blow received from *D*. The wires spread outward and prevent the plunger from moving forward until the projectile strikes a sufficiently resisting object.

Hotchkiss Base Percussion Fuze. Figure 21.

The case, *A*, carries the fulminate, *F*, in a large percussion cap contained in the perforated *screw-box*, which is

formed in two sections, *G* and *H*. The base of the case is provided with a projecting flange, *I*, brought to a thin edge which, when the fuze is screwed home, acts as a gas check. The plunger *D*, is made as in figure 20, but contains a central firing-pin, *L*, roughened so that it will hold well in the lead.

The rear end of the firing-pin projects beyond the bottom of the plunger, while its front end is sunk a little below the surface so that when this compound part (*D* and *L*) is in place, it is prevented from moving by the screw-box.

When the gun is fired the plunger slides back on the firing-pin so that the point projects above the plunger. The lead being soft, and being prevented from expanding by the jacket, it takes a fresh hold on the pin and supports it when it is thrown forward on impact.

This fuze has certain structural defects which render its operation less certain than that of the front fuze. For its special purpose it is probably one of the best known.

Krupp Fuze.

Figure 22 shows a Krupp fuze in a double walled shell. Safety in loading results from the transverse pin, *t*, which, with the screw-box containing the fulminate, is inserted just before loading. The rotation of the projectile expels the pin, leaving the longitudinal pin, *p*, free to be driven inward on impact so as to prevent the entrance of earth into the cavity of the fuze. The nomenclature of figure 22 is the same as that of figures 20 and 21. It is said that this pattern is to be replaced by one containing a safety device which is intended to be unscrewed by the rotation of the projectile.

III. COMBINATION FUZES.

These combine the principles of the time and impact fuzes so as;—

1. To increase the probability of explosion; since, if the probability of a failure in each of the two cases be, say, 0.01; that of the combination will be 0.0001.

2. To permit the character of the firing to be rapidly varied.

3. To increase the certainty of explosion when the projectile is fired with a low velocity.

Figure 23 illustrates one of the most recent combination fuzes used in the French service.

E is a leaden fuze tube made as described page 2. It is wrapped spirally about, and secured to the hollow cone, *C*; this is held in place by the clamp screw, *D*. The lower end of the fuze tube communicates through the priming, *P*, with the cavity in which lies the percussion fuze described page 13.

The *inertia igniter* consists of a loose, pointed piston, *H*, which, until the instant of discharge, is separated from the fulminate, *F*, by the spiral spring, *S*.

K, is a conical cap pierced with a series of numbered holes corresponding to the times of burning and provided with a vernier for interpolating a puncture between any two holes. The puncture, owing to the softness of the metal of which *C*, is composed, is made entirely through both its walls.

When the piece is discharged, the washer of compressed powder, *W*, is ignited by *H*, and through the puncture the fire extends to the composition in *E*. At the same time the percussion fuze acts as before described.

But for the union of the two fuzes in the same case, which the construction of the projectile and the operation of front percussion fuzes requires, this fuze illustrates the principle referred to Chapter XVI, page 34. The simplicity of construction, which was formerly considered of prime importance, has been entirely subordinated to efficiency of operation, notwithstanding the greatly increased cost which this involves.

The Flagler Combination Fuze. Figure 24.

This fuze, devised by Colonel Flagler of the Ordnance Department, is now, 1889, undergoing trial. It combines many of the principles just discussed and adds two new features to provide for the requirements numbered 5 and 6, page 12.

The first feature consists of a copper wire, *d*, screwed from the rear into the open end of the *screw-cap*, *A*. The lower end of this wire is bent at right angles so as to support firmly the leaden time plunger, *D*. Just below the screw thread by which it is suspended the diameter of the wire is reduced to any desired extent.

The wire is broken at the neck so formed by the stress due to the acceleration of the projectile, both in translation and in rotation. The latter stress, occurring only when the piece is fired increases the certainty of ignition without diminishing the safety of the apparatus against accidental shock.

On firing, the mass, *D*, is thrown against the fixed firing pin, *F*, and the fulminate d^2, is ignited. The flame from the fulminate escapes through the radial holes and the annular channel, *b*, *b*, to the end of the column of composition projecting into the radial groove, *Z*, formed on the lower side of the ring, or *carcass C*. The gas first formed blows off the vent cover, c^3, and allows the remaining gases to escape freely.

When the column has burned to the point, *b′*, corresponding to which there is a fixed mark on that portion of the body next to the graduation on the ring, the priming *K*, is ignited and the flame from it passes down the fluted surfaces of the members, *G*, *H*, *I*, of the percussion fuze into the bursting charge.

The advantage claimed for this safety device over those in which ears projecting from the mass *D*, are shorn off by

the shock of discharge, refers to the uniformity of copper wire and to the absence of the loose pieces, which, after shearing has occurred, may impede the action of the plunger.

The percussion fuze resembles the Hotchkiss base fuze, with the following advantages:—

1. The priming *K*, which serves for both fuzes of the combination, makes the volume of the flame on impact much greater than when fulminate only is employed.

2. In order to fulfil requirement 6, page 12, a disc of thin zinc separates the point of the firing pin, *h*, from the fulminate above it. This presents a positive and uniform resistance to premature explosion; and, since a pressure of 6 pounds is required to pierce it, the fulminate may be made as sensitive as may be desired.

3. On impact the powder is thrown toward the fuze.

The fuze works well in practice. The percussion fuze was found to operate when the projectile was fired through a 2-inch board, though it failed in penetrating a board one inch thick. It is thought that it will explode on striking animate objects or sandy or marshy ground.

GENERAL REMARKS.

Owing to their greater permanency of form in store, and their diminished volume, metallic cases are preferred to the wooden ones formerly employed.

In order to cheapen the manufacture, which at best is very expensive, the parts are, whenever possible, made of pewter, cast in metallic molds into finished forms. When strength and infusibility are required, brass or bronze are used, cast as described in Chapter XVII.

To prevent unscrewing during flight, the screw thread of base fuzes should turn in a direction contrary to that of the rifling.

It takes an appreciable time after impact for the explosion to occur; so that even when the fulminate was purposely ignited by the shock of discharge, the shell did not burst until it had gone several yards beyond the muzzle of the gun. This is of importance in understanding the effect of shrapnel fire with percussion fuzes, and serves to show that explosions within the gun generally result from defects in the construction of the projectile.

To prevent premature explosion from the plunger's being thrown violently forward by the elasticity of the bottom of the shell, on discharge, a perforated cardboard washer is often required.

A percussion shell, unexploded in experimental firing should never be tampered with: if possible, it should be exploded on the spot by a dynamite cartridge.

CHAPTER XIX.

GUN CONSTRUCTION.

Nomenclature of Stresses.

The total pressure of the powder gases in a gun may be analyzed as follows with reference to the direction of the resulting strains.

1. A radial stress, known by the special name of "pressure" (p).

2. A tangential stress, or hoop tension (t) which tends to split the piece open longitudinally, being similar in its action to the force which bursts the hoops of a barrel.

3. A longitudinal stress (q) which tends to pull the piece apart in the direction of its length.

4. Besides these, which are the principal stresses now considered, was formerly treated the transverse stress which tends to bend outward the staves of which the tube may be supposed to consist.

Its effects are so closely associated with the strains above named that it is no longer discussed.

Of the principal stresses named the most important is the tangential, since it is that from which failure most readily occurs.

BARLOW'S LAW.

Limitations.

This law, which was until recently applied to the construction of homogeneous cannon, if confined to stresses beneath the elastic limit, (Chap. XV, p. 11,) under which

limit the stresses are taken to be proportional to the strains, gives results which agree fairly well with those obtained by the more exact methods now generally employed.

But, when applied to built up guns composed of concentric cylinders assembled by shrinkages as described in Chap. XV, it is no longer generally used because it does not analyze the resultant strain into the component strains occuring in three coördinate directions. For example, if we compress a cube in the direction of the axis of Z, there will be developed along the axes of X and Y component strains corresponding to tensile stresses acting in each of these directions and conversely.

On account of the relative simplicity of Barlow's law it will be employed to illustrate the general principles of gun construction. A more extended discussion of the theory now accepted will be found in the appendix of this chapter.

DEDUCTION OF BARLOW'S LAW.

Hypotheses.

Suppose—(*a*) the piece to be a hollow cylinder of homogeneous metal, and—(*b*) that the effect of a central force be transmitted outward in such a manner as to make constant the area of cross-section by a plane perpendicular to the axis.

(*a*) The homogeneity of the metal is required, so that a constant relation may exist between stress and strain; or that the coefficient of elasticity, known herein as E, may remain constant.

(*b*) The constancy of area or cross-section resembles the assumption that the various stresses, from the effects of which the physical properties of the metal are determined in the testing machine, continue to be applied to the original

area of cross-section; although it is evident that, if the volume of the metal be constant, its area of cross-section must diminish or increase when exposed to tensile or compressive stress respectively.*

Wertheim's experiments show that the developed strains are each $-\frac{1}{3}$ of the principal strain.

Preliminary Statement.

Suppose that figure 1 represents a section of a homogeneous gun after firing: the radii R and R' having been extended to r and R'' so as to maintain the sectional area constant.

Then, the area whose limiting radii are r and R' being common to both states of the section, we have

$$\pi\,(r^2 - R^2) = \pi\,(R''^2 - R'^2), \quad \text{or}$$

$$(r + R)\,(r - R) = (R'' + R')\,(R'' - R').$$

But since R'' and R' are each greater than either R or r $R'' + R' > r + R$

$$\therefore r - R > R'' - R', \quad \text{or}$$

$$\frac{2\pi\,(r - R)}{2\pi\,R} = \frac{r - R}{R} > \frac{R'' - R'}{R'}$$

The two members of this inequality measure the strains on the interior and exterior surfaces respectively, so that it appears that the surface of the bore might be strained beyond its elastic limit before that of the outside layers was reached.

The resulting set, if slight, might destroy the accuracy of the piece from the dilatation of the bore; and, if considerable, it might lead to the formation of fissures which would

*Throughout the following discussion we will consider that we are dealing with a cylinder of but one unit of length, since, as the length of the cylinder varies, both the pressure tending to burst the cylinder and the resistance which it opposes will vary in the same ratio.

facilitate the final rupture in detail of the successive cylindrical layers of which the gun may be supposed to consist.*

The considerations explain the statement in Chapter V, as to strength *vs.* weight.

Analysis.

To determine the *law* by which the tangential stresses are distributed throughout the section of a gun:—

Let R and Z be the radius and the circumference of the bore. Let p_0 be the radial pressure per unit of area and T the tangential stress on the surface of the bore. Let r, z, p and t represent the same quantities on any exterior cylindrical surface, the area of cross-section between which and Z is A. Then by assumption, (b) above

$$\pi(r^2 - R^2) = A, \quad \text{and} \quad \therefore \quad r\,dr = R\,dR.$$

Multiplying the first member of the last equation by $\frac{r}{r}$, and the second member by $\frac{R}{R}$ we have $r^2 \frac{dr}{r} = R^2 \frac{dR}{R}$.

But, since the ratio between the circumference and the radius is constant, and since beneath the elastic limit the stresses are proportional to the strains

$$\frac{dr}{r} = \frac{dz}{z} = \frac{t}{E} \text{ and } \frac{dR}{R} = \frac{T}{E}$$

Therefore, we have

$$\frac{tr^2}{E} = \frac{TR^2}{E} \text{ or } t = \frac{TR^2}{r^2} \tag{1}$$

Or, since under given conditions of p_0, E and R, TR^2 will be constant, *the tangential stress (or strain) on each suc-*

* This last statement, though generally true, is subject to modification depending on the ductility of the material and the development of special elasticity. See *post.*

cessive concentric elementary cylinder varies inversely with the square of its radius. This is Barlow's law.

This condition may be represented by figure 2, in which the ordinates of the curve TT' represent the tangential stresses on the corresponding circumferences.

In this figure and in those succeeding it, positive hoop tensions are represented by ordinates laid off above the line representing the trace of the axial plane of least resistance, and negative hoop tensions (compressions) are laid off below this line. See figures 5 and 6.

RESISTANCE OF THE CYLINDER.

Bursting Effort.

Imagine the radial pressure on a unit of area, or p_0, to be decomposed into two components p' and $p_{\prime}$ figure 3, respectively perpendicular and parallel to the axial plane, OR', along which rupture tends to occur; and consider but one quadrant of the bore at a time.

Let φ be the variable angle made by the radial pressure with the plane OR'.

Then, $$p' = p_0 \sin \varphi,$$
and, since,
$$dZ = R\,d\varphi,$$
$$p'\,dZ = p_0 R \sin \varphi\, d\varphi = -p_0 R\, d\cos\varphi.$$

Integrating between $Z=0$ and $Z=\frac{\pi R}{2}$ and the corresponding values of φ, viz. 0 and 90°, and calling P, the total pressure on the inner surface of the quadrant perpendicular to the plane of rupture, we have

$$P = \int_0^{\frac{\pi R}{2}} p'\,dZ = p_0 R.$$

And for the force acting on both quadrants to lift the semi-cylinder from the axial plane, or for *the bursting effort*

$$2\,P = p_0\,2\,R. \tag{2}$$

This might have been inferred from the fact that the bursting effort is independent of the configuration of the surface, upon which it acts.

Resistance.

The bursting effort must be in equilibrium with the sum of the tangential stresses developed in both quadrants, or in figure 2.

$$2\,P = p_0\,2\,R = 2\,\Sigma\,t = 2 \times \text{area } R\,T\,T'\,R'$$
$$= 2\int t\,dr = 2\,T\,R^2 \int_R^{R'} \frac{dr}{r^2} = 2\,T\,R\,\frac{R'-R}{R'}$$

or

$$p_0 = T\,\frac{R'-R}{R'}. \tag{3}$$

COROLLARIES.

1. The maximum permissible value of T is the elastic limit of the material under tensile stress. Calling this θ and representing by $\bar{p}_0$ the corresponding powder pressure we have

$$\bar{p}_0 = \theta\,\frac{R'-R}{R'}. \tag{4}$$

Equation (4) gives the means of determining the maximum pressure for a gun of which the corresponding section is known, or of determining the thickness of which a gun of a given caliber should be made to resist a given pressure.

2. If guns be similarly proportioned, $R' = n\,R$, whence by substitution

$$\bar{p}_0 = \theta\,\frac{n-1}{n}. \tag{5}$$

Equation (5) shows that all similar guns of the same material can resist the same maximum pressure.

In the old cast-iron guns, in which for the reinforce, n was generally taken equal to 3, or the walls of the gun made one caliber thick, $\bar{p}_0 = \frac{2}{3}\theta$, if the metal be without internal strain, Chapter XV, page 21.

3. Since $\frac{n-1}{n} < 1, \bar{p}_0 < \theta$ unless $R = 0$ or $R' = \infty$. Or the powder pressure must always be less than the elastic limit of the material.

4. The curves of figure 4 are constructed from equation 1, using a constant value of $R' = 6$ and taking $T = \theta$ and $n = 3$; 2; $\frac{3}{2}$ respectively. It is apparent that as n diminishes the curve $T\,t$; $T_1\,t_1$; $T'\,t'$ becomes more nearly parallel to OR', and the area beneath the curve tends to increase from this cause. On the other hand, this area tends to diminish from the decrease in the thickness of the wall of the gun in consequence of the increase in the radius of the bore: there is consequently some value of n which will make this area a maximum.

To determine the value of n corresponding to a maximum area or resistance to bursting, denote this resistance by S, and since it is equal to the maximum bursting effort we have from equations (2) and (4)

$$S = \theta R \frac{R' - R}{R'} = \frac{\theta}{R'}(R' R - R^2).$$

Regarding R' as constant and differentiating we have

$$\frac{dS}{dR} = \frac{\theta}{R'}(R' - 2R)$$

Whence, placing $\frac{dS}{dR} = 0$ we find

$$R' = 2R, \qquad \text{or} \qquad n = 2.$$

That is to say, as shown by the following table, that when R' is fixed, if the thickness of the wall is one-half the caliber, the gun can withstand a greater bursting effort than with any other thickness.

Table for $R' = 6$.

$n = 3$	$R = 2$	$S' = \frac{4}{3}\,\theta,$
$n = 2$	$R = 3$	$S'' = \frac{3}{2}\,\theta = \frac{9}{8}\,S',$
$n = \frac{3}{2}$	$R = 4$	$S''' = \frac{4}{3}\,\theta = S'.$

It is to be noted however, that since the bursting effort for one quadrant, or P, is equal to $p_0\,R$; if p_0 be kept constant, P increases with R, so that, under ordinary circumstances, the thicker is the wall exposed to gas pressure, the greater by Equation (4) will be the value of $\overline{p}_0$.

In order, as R increases, to diminish the value of the radial stress, we may form the gun of two or more concentric cylinders. This has been done by boring out old cast-iron guns and lining them with a tube; since, for the same bursting effort, the pressure per unit of area on the cast-iron is diminished because the area pressed is increased. This will occur even if the material of the tube be of copper, the resistance of which may be neglected, and which may therefore be supposed to act only by transmitting the pressure to the outside walls.

Within limits the thicker is the tube and the greater its value of E, the stronger will be the composite gun, since for a given stress on the exterior of the tube the less will be the strain on the adjacent walls, and, therefore, the smaller will be the maximum stress that the exterior wall will be called upon to bear. Conversely the power of such guns may be greatly increased. See h, Chapter XI, p. 21.

5. From the preceding corollary it follows that if a gun, the dimensions of which are fixed, be composed of several

concentric cylinders, each one will be in the condition of maximum strength if its internal radius is half its external radius, or that the successive radii of contact will be in *geometrical progression*. This, which is known as *Gadolin's law*, is sometimes applied in modern gun construction.

EQUALIZATION OF STRAINS.

Preceding considerations show that owing to unequal distribution of the strains in a homogeneous gun, the strength of the gun increases much less rapidly than the thickness of the walls.

The most favorable case would be when the whole thickness of the wall was under a uniform strain, since then the maximum pressure would be

$$\bar{p}_0 = \theta\,(n-1),$$

which would be n times greater than that given by equation (5). This result can be approximated to only by the separate or combined application of two plans commonly known as the methods of Varying Elasticity and of Initial Tension. These are actually however, but variations of the former principle.

VARYING ELASTICITY.

This consists in varying the elasticity of the concentric cylinders as explained in Chap. XV, page 11. The elasticity may be measured either by its coefficient or by its limit. This divides the subject into two heads.

1. Varying Coefficient, or Rate E.

Suppose the gun to be composed of two concentric cylinders, the tube containing the bore, and the jacket. If these are of the same material the stress transmitted to the jacket

will follow Barlow's law. But if the jacket be made of a metal with $E' > E$, then the stress on its inner surface due to the strain arising from a given increase in the external diameter of the tube will be increased. For calling e the common strain, t, t', the corresponding stresses, and E, E', the coefficients of elasticity respectively of the tube and the jacket.

$$e = \frac{t}{E} = \frac{t'}{E'} \therefore \text{ if } E' > E, \;\; t' > t.$$

If the value of E increase as r^2, then the stress on the inner surface of the wall will be equal to that on the inner surface of the tube.

For, let us call e_0 the strain at the elastic limit of the tube, then

$$e_0 = \frac{\theta}{E}$$

At the outside of the tube the strain will be

$$e = \frac{e_0 R^2}{R'^2}$$

This will cause a stress on the inner wall of the jacket

$$t' = E' e = \frac{E' e_0 R^2}{R'^2} = \frac{E'}{E} \frac{R^2}{R'^2} \theta.$$

If now $E : E' :: R^2 : R'^2$, $E' R^2 = E R'^2$ and $t' = \theta$.

The thinner is the outside wall the less will the stress vary throughout its thickness, so passing to the limit we may say—*That to develop an uniform resistance throughout a cylinder the coefficients of elasticity of the elementary concentric cylinders must vary as the squares of their radii.*

This principle, though frequently referred to in gun construction, is now of little practical importance, since steel, the coefficient of elasticity of which is constant, and is

greater than that of any other cannon metal, is now generally employed for all portions of the gun.

2. Varying Limit of Elasticity.

Equation (1) shows that the stress on any cylinder is always greatest on its inner surface, and Equation (4) that for a given gun the value of $\overline{p}_0$ is limited by the elastic limit of the tube.

Consequently, if the value of E is constant, we may increase the strength of the gun by increasing the elastic limit of the tube.

This may be done in three ways.

a. By increasing the *primitive elastic limit* by varying the composition or structure of the tube. This is not practically done.

b. By giving it a *special elastic limit* before the bore is finished, viz.: 1st; by raising the elastic limit itself by preliminary tension, as by mandreling (Chap. XV, p. 22), or by firing high proof charges. 2nd; by lowering the origin of stresses by a preliminary compression, as by temporarily wrapping the tube with successive layers of wire until the surface of the bore receives a permanent set.

c. The principle of initial tension consists in subjecting the interior cylinders to a stress of compression by the reciprocal extension of the outer cylinders.

The effect is to increase the work required to deform the inner cylinders, in which the strains due to firing are the greatest, by diminishing the work required to deform the outer cylinders, in which the strains due to firing are the least.

The foregoing explains the gain in strength by mandreling, and also why the heat of firing may really tend to strengthen a gun instead of to weaken it as is generally

supposed; since in both cases, the inner concentric cylinders being expanded more than those exterior to them, stresses are developed in the exterior cylinders which resist the further extension of the inner portions.

It also explains the advantage of forming the tubes of more ductile material than the jacket and hoops, since if excessive powder pressure should expand the bore beyond its elastic limit, the initial tension developed outside would tend to prevent its further dilatation.

It accounts for the former preference for bronze, the ductility of which sometimes caused such guns to fissure first on the outside, where it was unsupported, while, on the contrary, cast-iron would crack first on the surface of the bore where the danger would less readily be seen.

Rodman's process of cooling cast-iron guns from the interior by a stream of water, while the exterior of the flask was heated by fires, was intended to utilize this principle and was the first instance of its application on a large scale. (Chap. XV, p. 21.)

But, while the cooling of the exterior portions of the casting might be retarded relatively to that of the portions next to the bore, it could not be postponed until all the interior portions had solidified.

Consequently the state of rest of such a gun could be represented by figure 5 in which the dotted lines represent what was desired and the full line what was attained. The process was besides uncertain, since guns have been known to break spontaneously from internal stresses so developed.

APPLICATION TO BUILT UP GUNS.

The results sought by Rodman may be attained much more certainly by the process of building up guns as explained in Chapter XV. In such a gun if the tube be com-

pressed until, under the law by which the stresses vary, the elastic limit of compression, ρ, be reached on the surface of the bore, then the effective value of the maximum tangential stress to which this surface may be safely exposed on firing will be $\theta + \rho$ and the maximum safe pressure for the tube will approach as a limit

$$\bar{p}_0 = (\theta + \rho)\,\frac{n-1}{n}.$$

The effect of this pressure is shown in figure 6, in which the stress will change from $-\rho$ to $+\theta$ on the surface of the bore. If we take $\rho = \theta$, as is commonly done, then the stress on the exterior of the tube will change from $-R'\,T_{,}$ to $+R'\,T_{,}$ as shown.

Now consider the jacket. The negative tension of the tube is due to a positive tension of the jacket resulting from shrinkage. Since the system is in equilibrium, the algebraic sum of the tensions on the tube and the jacket must be equal to 0; and, since from Barlow's law the tension, whether positive or negative, is always numerically greatest on the inner surface of a cylinder, we would have the condition represented by figure 6, in which the area

$$R\,\rho\,T_{,}\,R' = \text{area } R'\,T'\,T''\,R''.$$

This represents the state of the system *at rest.*

It is evident that the configuration of the stress area on the jacket, and therefore the maximum stress, $R'\,T'$, which it is called on to sustain from the shrinkage, will depend upon the thickness of the jacket. Also that $R'\,T'$ must not only not exceed the elastic limit under tension, θ', of the jacket, but must be so far beneath it as to admit of the increment due to firing.

Now suppose the system to be placed *in action* by the powder pressure and for simplicity assume that $\rho = \theta = \theta'$.

As the tension on the surface of the bore changes from $-\rho$ to $+\theta$ the strain on the surfaces of contact will increase the tension there by a quantity.

$$\tau = (\rho + \theta)\,\frac{R^2}{R'^2} = 2\,\theta\,\frac{R^2}{R'^2}.$$

For the tube this will simply change the sign of the stress from $-$ to $+$.

For the jacket the addition will be positive, the most favorable case being when the dimensions of the jacket are so chosen, as in figure 6, that the tension at rest + the tension in action $= \theta'$. The state of the system in action is shown by the dotted lines of figure 6.

The tangential resistance of the system will be proportional to the sum of the areas $\rho\,R\,\theta\,T_{\prime}\,R'\,T_{\prime}\,\rho\,(= 2s)$ $+ R'\,T_{\prime}'\,T_{\prime\prime}\,R''\,R'\,(= s') - R'\,T'\,T''\,R''\,R'\,(= s_{\prime})$ or $\Sigma = 2s + s' - s_{\prime}$ and $\bar{p}_0 = \dfrac{\Sigma}{R}$.

The dimensions of figure 6 render such a gun about twice as strong as if it were a simple tube of the same size and elasticity.

It is evident, that if, as on page 9, we suppose the gun to consist of an indefinite number of cylinders in which the initial tensions are properly applied, the thinner the cylinders are made, the less will be the difference of the tensions on their interior and exterior surfaces and the more nearly will the broken line $\theta\,T_{\prime}\,T_{\prime}'\,T_{\prime\prime}$ become parallel to $O\,R''$, or the more nearly will the resistance of the gun approach the ideal case.

The difficulties of manufacture have generally limited the number of cylinders to less than 5 but these difficulties can be overcome by making the cylinders of continous wire wrapped around a central tube.

SHRINKAGE.

It is seen that the initial tension depends primarily on the shrinkage. In built up guns this may be due to heating the exterior cylinder as described in Chap. XV, or to forcing by hydraulic pressure one cylinder within another, the contact surfaces being reciprocally conical, or by winding wire continuously over a central tube. By whatever method the result may be attained, the stress on the contact surfaces is due to the strain resulting from the compression of the inner cylinder and the extension of the exterior.

To determine the shrinkage required to produce a given initial compression without exceeding either ρ in the inner cylinders when the system is at rest or θ in the exterior when the system is in action, is one of the principal objects of the different theories of gun construction now in vogue. A full discussion of these theories is not possible in this course, but the following treatment of the subject based upon Barlow's law illustrates the methods now employed.

Let e, e' be the strains on the adjacent surfaces of the tube and jacket, for the corresponding stresses $R' \, T_{\prime} = t$, and $R' \, T' = t'$ and let $\sigma = e + e'$ be the shrinkage strain.

Then since $e = \frac{t}{E}$ and $e' = \frac{t'}{E}$, from note, page 21,

$$\sigma = \frac{\Delta R'}{R'} = \frac{1}{E}(t + t') \text{ or } \Delta R' = \frac{R'}{E}\,(t + t').$$

Consequently, as stated in Chapter XV, the tube would be turned to a diameter

$$= \frac{2R'}{E}\,(t + t') + 2R'$$

and the jacket would be bored to its finished size. The effect of shrinkage would be to vary the radii somewhat from

those assumed, and to increase the length of the tube. These variations, which, for this discussion are not taken into account, afford one of the best means of testing the accuracy of the hypotheses upon which different theories of gun construction are based.

LONGITUDINAL STRESS.

This tends to "unbreech" a gun or to produce what is known as a "ring fracture," the plane of which approaches that of a right section.

In homogeneous guns it was sufficiently resisted by the sections required to resist the tangential stress; but, in composite b. l. guns, except so far as friction due to shrinkage and powder pressure may assist, that portion which contains the breech block has to support this stress independently of the portions which give tangential strength.

The bursting effort is $\pi R^2 p_0$. It tends to pull the piece apart, generally in rear of the trunnions to which it transmits the pressure causing recoil. Consequently the piece carrying the breech block must be firmly united to the trunnions. When b. l. guns were first made the block was secured to the tube, but this arrangement, although theoretically advantageous,* is no longer generally employed. It is thought that the radial expansion of the tube diminishes the bearing of the screw threads of the breech block, and that the tube is inclined to fissure through the screw threads.

Supposing the longitudinal stress to vary through the cross-section, its resistance may be determined as follows:

* From the analysis referred to on page 2, it follows that the longitudinal stress of the tube should develop a *negative* radial stress which would neutralize a portion of the powder pressure. This has been confirmed by experiment on a small scale, and it is said that recent cannon made abroad have the block screwed into the tube.

The area of cross-section of an elementary cylinder whose radius is r and the thickness of which is dr will be $2\pi r\,dr$. This will receive a variable stress q the intensity of which will vary with its distance from the axis, so that the resistance of the section whose internal and external radii are R and R' will be $\int_R^{R'} q\,2\pi r\,dr$.

If we suppose q to vary by Barlow's law, we have

$$q = \frac{\theta R^2}{r^2}.$$

Substituting this value of q and integrating we have for the total resistance

$$L = 2\pi R^2 \theta \int_R^{R'} \frac{dr}{r} = 2\pi R^2 \theta \,\text{Nap. log}\, \frac{R'}{R}.$$

Equating this with the bursting effort, we have for the condition of equilibrium

$$\overline{p}_0 = 2\,\theta \text{ Nap. log } \frac{R'}{R}. \qquad (6)$$

In modern guns the ordinary values of θ, R' and R are such that the maximum value of p_0 allowed by this equation is considerably greater than that allowed by their tangential resistance, so that these guns are abundantly strong against longitudinal stress.

Equation (6) is useful for computing the pressures necessary to burst spherical shell for which purpose it gives results closely confirmed by practice. In such cases for θ should be substituted the tenacity of the material. This is allowable since the ductility of such castings is small.

WIRE-WOUND GUNS.

The peculiar properties of cold drawn wire described in Chapter XV; the direction assumed by the fibers in the gun, and the increased facility of construction have for

many years made this material a favorite subject of study by gun makers.

Until recently, however, the difficulty of providing sufficient longitudinal strength, and mechanical difficulties connected with the attachment of the ends of the wires have caused steel forgings to be preferred.

The following may be named as devices intended to provide the longitudinal resistance.

Dr. Woodbridge, of New Jersey, the originator of the idea, proposed, after winding his tube to immerse the entire gun in a bath of melted bronze, so as to braze or solder the spirals and the layers together. This was found mechanically impracticable and the bath, by annealing the wire, destroyed much of its elasticity. Various experimenters have tried longitudinal bars or staves connecting the trunnions with the breech, but so far as tried these are not believed to have given satisfaction; the objection appearing to consist in the difficulty of making all the bars resist equally, for otherwise they will tend to rupture in detail.

Crozier's Wire Wound Gun.

This gun, now under construction, is devised by Lieut. Crozier of the Ordnance Department.

It consists—

1. Of a thin steel tube forming a core for the winding; to contain the rifling and to prevent the erosion of the wire. It also incidentally gives longitudinal stiffness.

2. Of wire, to give tangential strength. This is preferably of rectangular cross-section. Relying upon the support of the wire wrapping, it is intended to produce an initial preliminary compression considerably in excess of ρ. See page 11. For experimental purposes it is assumed that the relatively thin tube will withstand dilatation and contraction through a considerably greater range than $\theta + \rho$.

The difficulties in attaching the wires have been successfully overcome by electro-welding. According to the method proposed by Mr. Longridge of England each coil of wire is wrapped with a tension diminishing from within outwards, so that the tension of the inner layers will be eventually less diminished than if the tension of winding were constant.

In such a gun, properly constructed, the tangential strain developed by firing will be uniform throughout the entire thickness of the walls.

3. Of a steel cast jacket carrying the breech block at one end and the trunnions at the other and so furnishing the required longitudinal strength. It also gives longitudinal stiffness and being lightly shrunk on it also affords some tangential resistance. This, the heaviest unit of construction is made, of cast steel on account of its cheapness, its radial distance and its adaptability to the present state of the arts in the United States.

Practical Corrections.

WEIGHT OF CANNON.

The dimensions of cannon are sometimes increased beyond what is required by their elastic strength so as to increase their weight and thereby diminish the destructive energy of their recoil; because, calling E this energy, and e that of the projectile at the muzzle, and the corresponding masses and velocities respectively M, m, V and v, we have from the equation of momenta, the general equation,

$$M V = m v \quad \text{or} \quad \overline{MV}^2 = \overline{mv}^2 \quad \text{or} \quad E = \frac{m}{M} e. \qquad (7)$$

Equation (7) is an important one to remember, particularly for small arms.

This equation is not exact, since it neglects the momentum of the powder gases, (Chapter XI, page 18), but it is convenient for general discussions. For a more exact formula see Chapter XXII.

LINERS.

In order to provide against the erosion of the bore, large built up cannon are sometimes lined for a short distance in front of the chamber with a thin tube which can be replaced with comparative facility. Guns which are properly designed appear more likely to fail from this cause than as the result of stress.

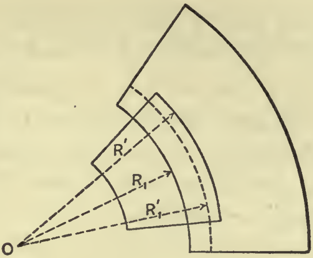
LIMITS.

It is not considered advisable to work up to the limits ρ and θ as, for the sake of illustration, has been supposed. A safe margin is allowed in both cases. Indeed the inverse method is that generally followed, the gun being designed to safely resist a certain value of $\overline{p}_0$.

It can be shown theoretically that no great advantage is gained as to tangential strength by increasing the thickness of the walls over the powder chamber, much beyond one caliber. See *General Remarks*, Appendix.

GADOLIN'S LAW.

Owing to the practical difficulty of making perfect forgings of the thickness which this law would require for the exterior layers it is not generally observed.



Note 1, Page 15.

Let R' be the exterior radius of the tube, and $R_{\prime}$ the interior radius of the jacket before shrinkage; and let $R_{\prime}'$ be their common radius after shrinkage.

The effect of the shrinkage wlll be to diminish the radius of the tube by $R' - R_{\prime}' \therefore e = \frac{R' - R_{\prime}'}{R'}$

Similarly, for the jacket $e' = \frac{R_{\prime}' - R_{\prime}}{R_{\prime}} > e$, since $t < t'$.

The total shrinkage will be

$$\sigma = \frac{R' - R_{\prime}'}{R'} + \frac{R_{\prime}' - R_{\prime}}{R_{\prime}}$$

R' and $R_{\prime}$ are so nearly equal to each other, and so large when compared with the numerators of the fractions that either R' or $R_{\prime}$ may be used as a common denominator without material error. R' it taken because it pertains to the tube on which the excess is left as described in Chapter XV,

The true shrinkage will therefore be slightly greater than

$$\sigma = \frac{(R' - R_{\prime}') + (R_{\prime}' - R_{\prime})}{R'} = \frac{R' - R_{\prime}}{R'} = \frac{\triangle R'}{R'}$$ as in the text.

THE ELASTIC STRENGTH OF GUNS.

By CAPTAIN L. L. BRUFF, U. S. Ordnance Department.

The object of this discussion is to give a general idea of the methods employed in modern gun construction, for determining the strength of guns, the strains to which they may be safely subjected, and the methods by which the requisite strength may be obtained.

Definitions.

The elastic limit of a metal is the greatest load in lbs. per square inch of section which the metal will sustain before it acquires a permanent *set.*

There are various elastic limits, such as those for tension, compression, torsion, etc., depending on the manner in which the stress is applied but the only ones of practical importance in gun construction are those for tension and compression.

The modulus of elasticity of a metal (see Michie's Mechanics, art. 22), is the ratio of the load or stress in pounds per square inch, to the elongation or strain per linear inch produced by this load within the elastic limit. It is expressed by dividing the stress by the strain. Since within the elastic limit the strain or elongation is proportional to the stress or load, it is evident that this ratio is constant for the same metal. Its value for all gun steel is taken at 30,000,000 lbs.

As in the case of the elastic limit, there are various moduli of elasticity, as for tension, compression, etc., but those for tension and compression, which are the only ones used, agree so nearly, that the uniform value given above is assumed for both.

Hooke's Law.— This law is expressed above. It is as follows: Within the elastic limit of a metal, the stress is proportional to the strain.

Stress and Strain.— In the discussion stress will be used to denote the force in pounds per square inch producing a given

extension or compression per linear inch, and strain the corresponding elongation or compression produced by the stress.

General Principles.

The construction of the modern gun is supposed to be understood. That is, that it is composed of an interior tube, surrounded by a jacket and one or more rows of hoops. That the jacket carries the breech-closing device, and that the jacket and hoops have interior diameters which are less than the exterior diameters of the parts they envelop, by a certain prescribed amount, and that the difference in diameter between the enveloping cylinder and the enveloped cylinder is called the *shrinkage*. In order to place the smaller or enveloping cylinders over the enveloped cylinders, the former are expanded by heat till they will pass over the corresponding surfaces, when they are cooled in place by the application of water.

Theory.

The principle of initial tension is employed in the modern built up gun. The interior layers which are under the greatest strain, due to the action of the powder gas, are compressed by the exterior layers, jacket and hoops. When the pressure of the gas acts upon these interior layers, it has first to overcome this initial compression, and then to extend or compress these layers until they reach their elastic limit for extension or compression, before the maximum resistance of the gun is reached. The exterior layers are subjected to initial tension, by which their capacity for resisting interior pressure is partly diminished, but owing to the law of its transmission, the strain upon them is so much less per square inch than it is upon the interior layers that they are able to resist it. Thus the interior layers are relieved of a portion of the strain, due to the action of the powder gas, and the strain transmitted to the exterior layers, by the modern process of gun construction.

The best condition for strength in a gun is when every layer of metal in its cross section is strained equally by a given stress or pressure.

The foundation of the theory of the built up gun is this. *That in whatever state the gun may be considered, whether under the pressure of the powder gas, or free from it, none of the fibers of any cylinder in the gun shall be elongated or contracted beyond the elastic limit of the metal of that cylinder, which elastic limit is determined by the test of the metal in a testing machine.*

Two states or conditions of the gun are considered in this discussion; one, called "the system in action," which means that the gun is subjected to the maximum interior pressure which it can support with safety, and the other, called "the system at rest," that is, when the gun is free from the pressure of the powder gas, although the strains due to the shrinkages still exist.

Methods of Discussion.

The general method of discussion is:

First. To assume a cube of metal the length of whose edges is unity, and which is supposed to be perfectly elastic up to a given limit; to deduce the equations of equilibrium which show the relations between the forces acting upon this cube in directions at right angles to its faces, and the corresponding elongations and contractions produced by them.

Second. To transform these equations so that they will apply to the elements of a cylinder of metal; or in other words, to deduce the equations which give the relations between the stresses at different points throughout the right section of a single cylinder.

Third. To pass from a single cylinder to a compound cylinder composed of any number of single cylinders, and to deduce for the latter the compressions and extensions pro-

duced by given pressures, and the shrinkages or differences of diameter which will produce given compressions and pressures.

FIRST.—EQUATIONS OF EQUILIBRIUM FOR A CUBE OF METAL OF CONSTANT ELASTICITY WHOSE EDGES ARE EQUAL TO UNITY.

It has been found by experiment, that when a cubical elastic solid is acted upon by a given force of extension or compression in a direction perpendicular to two of its opposite faces, this force produces an extension or a compression of the cube in the direction of the force, of a given amount, and a corresponding compression or extension in the two directions at right angles to the given force equal to one-third the first extension or compression.

The force is supposed to be within the elastic limit of the solid.

For example, suppose a force of extension P, Fig. 7, to act upon the opposite faces of a cube of metal, whose edges are each one inch long. If it extends the edges $a\ a\ a\ \frac{1}{10}$ of an inch it will shorten the edges $b\ b\ b$ and $c\ c\ c\ \frac{1}{3}$ of $\frac{1}{10} = \frac{1}{30}$ of an inch, and the same for any other force.

In figure 8

Let $E =$ the modulus of elasticity of the cube.

X, Y and $Z =$ three forces acting at right angles to the faces of the cube, being tensions in the figure.

λ; μ; ν, $=$ the extensions produced by the three forces X, Y and Z, respectively.

Then the force X, according to the preceding principle, produces an elongation in its own direction equal to

$$\frac{X}{E}.$$

But the force Y diminishes this elongation by the amount

$$\tfrac{1}{3}\frac{Y}{E}.$$

and the force Z by the amount

$$\tfrac{1}{3}\frac{Z}{E}.$$

Hence the total elongation in the direction of X is

$$\lambda = \frac{1}{E}\left(X - \frac{Y}{3} - \frac{Z}{3}\right)$$

In the same way we have for the total elongations in the directions of Y and Z

$$\mu = \frac{1}{E}\left(Y - \frac{X}{3} - \frac{Z}{3}\right)$$

$$\nu = \frac{1}{E}\left(Z - \frac{X}{3} - \frac{Y}{3}\right)$$

These three equations express the relations between the elongations of the faces of an elastic cube whose edges are unity, and the corresponding forces acting on them.

SECOND.—APPLICATION TO AN ELASTIC CYLINDER.

We have supposed the three forces to be tensions. In the case of a gun cylinder, however, two of the forces are tensions, one acting in the direction of a tangent to the cylinder, and the other parallel to the axis, while the third is a pressure and acts in the direction of the radius. In Figure 9, let $p =$ the radial pressure, $t =$ the tangential tension, q the longitudinal tension, per unit of area.

Substitute t for X, $-p$ for Y since it acts opposite to Y, and q for Z, and the above equations become

$$\left.\begin{aligned} \lambda &= \frac{1}{E}\left(t + \frac{p}{3} - \frac{q}{3}\right) \\ \mu &= -\frac{1}{E}\left(p + \frac{t}{3} + \frac{q}{3}\right) \\ \nu &= \frac{1}{E}\left(q - \frac{t}{3} + \frac{p}{3}\right) \end{aligned}\right\} \qquad (8)$$

The first of these equations expresses the total change per unit of length in the direction of the tangent of the cylinder; the second the total compression (being negative) in the direction of the radius; and the third the total change in the direction of the axis, due to the three forces p, t and q.

In order to apply these equations in practice the changes of dimensions must be expressed in terms of the radii of the cylinder and of the forces acting upon it. To express the equations in these terms we proceed as follows:

Equations of Equilibrium in Terms of the Radii of the Cylinder.

Let Figure 10 represent a section of the cylinder perpendicular to the axis.

Let $R =$ interior radius.

$R' =$ exterior radius.

$r =$ the radius of any circle of the section.

$r' =$ any other radius exterior to r.

$p =$ the radial pressure per unit of surface at the distance r from the axis.

$t =$ the tangential stress per unit of surface at r.

$q =$ the stress per unit of section parallel to the axis of the cylinder, and supposed uniform throughout the section.

$P =$ the interior radial pressure per unit of surface, being the value of p for R.

$P' =$ the exterior radial pressure per unit of surface, being the value of p for R'.

T and $T' =$ the values of t for $r = R$, and $r = R'$ respectively.

$E =$ the modulus of elasticity.

The pressure p, whether acting inward or outward, develops in the direction perpendicular to $A\ B$, Figure 10, a force equal to $2pr$.

Increase r to r', and represent by p' the new value of p. This develops a force in the direction perpendicular to $A\ B$ equal to $2\,p'\,r'$. The algebraic difference between these forces is in equilibrio with the product of twice the thickness of the ring $r' - r$ into the mean stress throughout the ring, which represent by τ. Hence

$$2\,p'\,r' - 2\,pr = -\,2\,\tau\,(r' - r)$$

dividing

$$\frac{p'\,r' - pr}{r' - r} = -\,\tau$$

passing to the limit of the ratio in the first member by making $r' = r$

$$\text{limit of}\left(\frac{p'\,r' - pr}{r' - r}\right)_{r' = r} = \frac{d\,(pr)}{dr}$$

$$\text{limit of}\left(-\,\tau\right)_{r' = r} = -\,t.$$

Hence

$$\frac{d\,(pr)}{dr} = -\,t \tag{9}$$

Taking the last of Equations (8), which expresses the strain in the direction of the axis of the cylinder, and supposing this uniform throughout the cross-section, we have

$$v = \frac{1}{E}\left(q - \frac{t}{3} + \frac{p}{3}\right)$$

From this we have

$$-\,\frac{t}{3} + \frac{p}{3} = v\,E - q$$

or

$$t - p = 3\,(q - v\,E) \tag{10}$$

But the second member of this last equation is constant, since we have supposed v uniform throughout the section; hence

$$t - p = \text{constant.}$$

From which we may unite

$$t - p = T - P \dots\dots\dots\dots\dots\dots\dots\dots (A)$$
$$t - p = T' - P' \dots\dots\dots\dots\dots\dots\dots (B)$$

From Equation (10) we have

$$t = p + 3\,(q - v\,E) \qquad (11)$$

Substituting this for t in Equation (9) we have

$$\frac{d\,(pr)}{dr} = -p - 3\,(q - v\,E)$$

performing the differentiation as indicated; p and r being variable,

$$\frac{pdr + rdp}{dr} = -\,p - 3\,(q - v\,E)$$

reducing

$$-\frac{dr}{r} = \frac{dp}{2\,p + 3\,(q - v\,E)}$$

Integrating

$$\log_e\left(\frac{1}{r}\right) = \frac{1}{2}\log_e\,(2\,p + 3\,(q - v\,E) + \log_e C$$

$$\frac{1}{r^2} = c\,(2\,p + 3\,(q - v\,E))$$

Substituting the value of $p + 3\,(q - v\,E)$ from (11) we have

$$\frac{1}{r^2} = c\,(t + p)$$

$$(t + p)\,r^2 = \frac{1}{c} = \text{constant}$$

From which we can write

$$(t + p)\,r^2 = (T + P)\,R^2 \dots\dots\dots\dots\dots\dots (C)$$
$$(t + p)\,r^2 = (T' + P')\,R'^2 \dots\dots\dots\dots\dots (D)$$

From Equations (A) and (B) we derive the following principle:

The difference between the tension and the pressure is the same at all points.

From (C) and (D) we have the following principle:

At any point whatever, the sum of the tension in the direction of the circumference, and of the pressure in the direction of the radius, varies inversely as the square of the radius.

This demonstration is given by Captain Crozier, Ordnance Department, in "Notes on the Construction of Ordnance," No. 35.

Applications.—It has been shown by Captain Birnie, Ordnance Department, that in considering the radial and tangential strains in a gun cylinder, we may, without appreciable error, omit the longitudinal strain, or the strain parallel to the axis, and afterwards consider this latter strain separately. This conclusion has been proved to be correct, by actual measurements of guns during construction. This is equivalent to making in Equation (8)

$$q = o,$$

when the equations become

$$\left.\begin{aligned} \lambda &= \frac{1}{E}\left(t + \frac{p}{3}\right) \\ \mu &= -\frac{1}{E}\left(p + \frac{t}{3}\right) \\ \nu &= \frac{1}{E}\left(-\frac{t}{3} + \frac{p}{3}\right) \end{aligned}\right\} \qquad (12)$$

In the last equation, which gives the change in the longitudinal direction, this change will be produced by p and t only.

From Equations (C) and (D) we have

$$(T' + P')\, R'^{\,2} = (T + P)\, R^2,$$

and from (A) and (B)

$$T' - P' = T - P$$

Combining these two equations and eliminating T' we have

$$T = P.\,\frac{R'^{\,2} + R^2}{R'^{\,2} - R^2} - \frac{2\, R'^{\,2}\, P'}{R'^{\,2} - R^2}$$

Substituting this value in (A) and (C), combining the resulting equations, and eliminating p we have

$$t = \frac{P R^2 - P' R'^2}{R'^2 - R^2} + \frac{R'^2 R^2 (P - P')}{R'^2 - R^2} \frac{1}{r^2} \qquad (13)$$

And by combining and eliminating t between the same equations we have.

$$p = -\frac{P R^2 - P' R'^2}{R'^2 - R^2} + \frac{R'^2 R^2 (P - P')}{R'^2 - R^2} \frac{1}{r^2} \qquad (14)$$

Substituting these values of p and t in Equations (12) we have

$$\lambda = \frac{2 (P R^2 - P' R'^2)}{3 (R'^2 - R^2) E} + \frac{4 R'^2 R^2 (P - P')}{3 (R'^2 - R^2) E} \frac{1}{r^2} \qquad (15)$$

$$\mu = \frac{2 (P R^2 - P' R'^2)}{3 (R'^2 - R^2) E} - \frac{4 R'^2 R^2 (P - P')}{3 (R'^2 - R^2) E} \frac{1}{r^2} \qquad (16)$$

$$v = -\frac{2 (P R^2 - P' R'^2)}{3 (R'^2 - R^2) E} \qquad (17)$$

These equations give the values of the elongations or contractions in terms of the pressures and radii, and the known modulus E, for any radius r.

Elastic Strength of a Simple or Single Cylinder.

Now it may be shown that the greatest elongations and compressions of the fibres of a cylinder subjected to an interior pressure P, and an exterior pressure P', take place at the inner surface of the cylinder. (See appendix, Note 35, on the Construction of Ordnance.) Assuming this, we recur now to the fundamental principle stated above "that no fibre of any cylinder in the gun shall be elongated or contracted beyond the elastic limit of the metal of that cylinder."

Let $\theta =$ the elastic limit for tension,

$\rho =$ the elastic limit for compression in pounds or tons per square inch, of the cylinder.

Then the extension and compression at the elastic limit will be respectively

$$\frac{\theta}{E} \text{ and } \frac{\rho}{E}$$

and by the above principle these must be equal to the greatest values of λ and μ respectively.

Since the greatest extensions and compressions will occur at the interior of the cylinder, we have for their greatest values by substituting R for r in (15) and (16)

$$\lambda = \frac{(4\,R'^2 + 2\,R^2)\,P - 6\,R'^2\,P'}{3\,(R'^2 - R^2)\,E} \tag{18}$$

$$\mu = -\frac{(4\,R'^2 - 2\,R^2)\,P - 2\,R'^2\,P'}{3\,(R'^2 - R^2)\,E} \tag{19}$$

Placing these equal to $\frac{\theta}{E}$ and $\frac{\rho}{E}$ respectively, we have

$$\theta = \frac{(4\,R'^2 + 2\,R^2)\,P - 6\,R'^2\,P'}{3\,(R'^2 - R^2)}$$

$$\rho = \frac{(4\,R'^2 - 2\,R^2)\,P - 2\,R'^2\,P'}{3\,(R'^2 - R^2)}$$

From which we find two values for P, viz.:

$$P^{(1)} = \frac{3\,(R'^2 - R^2)\,\theta + 6\,R'^2\,P'}{4\,R'^2 + 2\,R^2} \tag{20}$$

$$P^{(2)} = \frac{3\,(R'^2 - R^2)\,\rho + 2\,R'^2\,P'}{4\,R'^2 - 2\,R^2} \tag{21}$$

Equation (20) gives the value for the interior pressure, which will cause the layer of metal on the interior of the cylinder to reach its elastic limit by extension, and Equation (21) the value which will cause the same layer to be compressed to its elastic limit; these pressures being in pounds or tons per square inch according as θ and ρ are expressed in pounds or tons.

It must be remembered that the *less* of the two pressures, measures the elastic strength of the cylinder.

THIRD.—THE ELASTIC STRENGTH OF A COMPOUND CYLINDER, OR OF A BUILT-UP GUN.

For the sake of clearness in the nomenclature, and of simplicity in discussion, the gun will be supposed to consist of two cylinders only, shrunk one upon the other, and the resistance of this compound cylinder, and the shrinkages to be used in its construction, will be deduced.

In figure 11 let

$P_0 =$ the maximum internal pressure to which the gun can be subjected.

$P_1 =$ the normal pressure at the surface of contact of the two cylinders.

$P_2 =$ the exterior normal pressure.

$p_0, p_1, p_2 =$ variations in the pressures, P_0, P_1 and P_2 due to any cause whatever.

The above pressures and variations of pressure are those which exist with the "system in action," — that is when the maximum gas pressure is acting on the bore.

Let

$P_1' =$ the normal pressure acting at the surface of contact of the two cylinders when the system is at rest—that is, when the pressure of the gas does not act on the bore.

$p_1' =$ the variation of P_1' due to any cause whatever.

$R_0, R_1, R_2 =$ the radii of bore, of interior of second cylinder, and of exterior of second cylinder respectively.

$\theta_0, \theta_1 =$ elastic limits of inner and outer cylinders for extension.

$\rho_0, \rho_1 =$ elastic limit of same for compression.

$E_0, E_1 =$ moduli of elasticity of metal of cylinders.

Writing Equations (20) and (21) we have

$$P^{(1)} = \frac{3\,(R'^2 - R^2)\,\theta + 6\,R'^2\,P'}{4\,R'^2 + 2\,R^2} \tag{20}$$

$$P^{(2)} = \frac{3\,(R'^2 - R^2)\,\rho + 2\,R'^2\,P'}{4\,R'^2 - 2\,R^2} \tag{21}$$

Now it will be remembered that in the case of a single cylinder, Equation (20) gives the value of $P^{(1)}$, the interior pressure which will cause the layer of metal on the interior of the cylinder to reach its elastic limit by extension, and Equation (21) the value of $P^{(2)}$, the interior pressure which will cause the same layer to be compressed to its elastic limit.

Taking the outer or second cylinder of the gun, it is always under a strain of extension both in action and at rest, and hence Equation (21) will not apply to it.

Equation (20) must therefore be used. To apply it to the present case, R and R' in (20) are the inner and outer radii, which now become R_1 and R_2 respectively. P is the interior pressure, and it now becomes P_1. P' is the exterior pressure, and it becomes P_2. But this exterior pressure on the second cylinder is simply that due to the atmosphere, and it is so small in comparison with the other pressures considered that it may be neglected. Hence

$$P_2 = 0.$$

Also θ becomes θ_1. Making these substitutions in (20) we have

$$P_1 = \frac{3\,(R_2^2 - R_1^2)\,\theta_1}{4\,R_2^2 + 2\,R_1^2}$$

This gives the value of the interior pressure on the outer cylinder which will cause its inner layer to be strained to the elastic limit for tension, and as this value is expressed in

known terms, P_1 can be readily calculated. The value of θ_1 is obtained from test of the metal in a testing machine.

Now taking the inner cylinder, the pressure P_1 just found, acts not only on the interior of the outer cylinder, but also on the exterior of this inner cylinder. Hence one of the normal pressures acting on this inner cylinder is known, and we have to calculate the other.

This inner cylinder is not only extended by the action of the powder gas, but it is also compressed radially by this pressure, and it is subjected to a strain of compression by the force P_1 which we have just found. In other words the inner cylinder is subjected to both tension and compression, and hence it is necessary to calculate both strains, and to take the smaller as the limit of its elastic resistance.

Referring to Equations (20) and (21) the following changes must be made to apply them to the inner cylinder —

P	becomes	P_0
P'	"	P_1
R	"	R_0
R'	"	R_1
θ	"	θ_0
ρ	"	ρ_0

Making these substitutions we can write

$$P_0^{(1)} = \frac{3\,(R_1^2 - R_0^2)\;\theta_0 + 6\,R_1^2\,P_1}{4\,R_1^2 + 2\,R_0^2}$$

$$P_0^{(2)} = \frac{3\,(R_1^2 - R_0^2)\,\rho_0 + 2\,R_1^2\,P_1}{4\,R_1^2 - 2\,R_0^2}$$

Substituting in these equations the known values of the radii, and of θ_0 and ρ_0 together with the value of P_1 just calculated, we obtain two values for P_0, the smaller of which is the limiting value of the pressure for the compound cylinder under discussion.

For convenience of reference these equations are collected here—

$$\left.\begin{aligned} P_1 &= \frac{3\,(R_2^2 - R_1^2)\,\theta_1}{4\,R_2^2 + 2\,R_1^2} \\ P_0^{(1)} &= \frac{3\,(R_1^2 - R_0^2)\,\theta_0 + 6\,R_1^2\,P_1}{4\,R_1^2 + 2\,R_0^2} \\ P_0^{(2)} &= \frac{3\,(R_1^2 - R_0^2)\,\rho_0 + 2\,R_1^2\,P_1}{4\,R_1^2 - 2\,R_0^2} \end{aligned}\right\} \qquad (22)$$

The values of P, obtained from Equations (22), are the pressures which will cause the interior of each cylinder to reach its elastic limit for extension or compression; and since the greatest strains in a cylinder occur at its interior surface, and since also no part of any cylinder must be strained beyond its elastic limit, it is evident that the values of P, thus obtained, represent the *greatest* strains to which the cylinders can be subjected. It will be seen hereafter, that these values cannot always be used in practice, since the bore in the state of rest, may be compressed beyond its elastic limit, by the use of these values.

It is, therefore, necessary now to consider

The System at Rest.

Equations (22) give the pressures acting for the system when under the maximum pressure of the powder gas. It is evident, however, that when the system is at rest, great pressures will exist at the surface of contact of the two cylinders, due to the shrinkage of one on the other. These pressures generally increase from the exterior to the interior, and the interior of the bore is generally compressed from this cause to a greater degree than any other part of the gun. This compression of the bore may be so great as to exceed the elastic limit for compression of the metal of the inner cylinder, and thus, although the gun is properly calculated for action, the principle upon which the whole structure

is built may be violated, when the gas pressure is removed. In this case, the elasticity of the tube is destroyed as effectively as if by the powder pressure.

It is evident, also, that when the powder pressure ceases, the pressure which existed at the surface of contact of the two cylinders will change, and will assume some other value for the state of rest. The value of this variation of pressure at the surface of contact has been denoted by p_1 and at the surface of the bore by p_0. The value of the pressure at the surface of contact for the state of rest has been represented by P_1'.

Now it is evident that the difference between the pressure in action and at rest for any surface, gives the variation in the pressure at that surface. Hence, since the pressure at the interior of the bore, when the system is at rest, is zero, we have

$$0 - P_0 = p_0, \text{ or } p_0 = -P_0$$

and also

$$P_1' - P_1 = p_1.$$

When these changes of pressure occur, they are accompanied by corresponding changes of dimensions of the surfaces at which they act, and these changes of dimensions depend directly upon the variations of pressure. The greatest changes of dimensions occur in the direction of the circumference or of the tangent to the surfaces, and Equation (18) gives the value of these changes for the interior surfaces.

To Calculate these Changes of Dimensions.

The variation of pressure acting on the outer cylinder is p_1, and the exterior pressure is zero, being that of the atmosphere. Hence, substituting in Equation (18) for P its value p_1, and making

$$P' = 0$$
$$R = R_1$$
$$R' = R_2$$
$$E = E_1$$

we can write

$$\lambda = \frac{(4\,R_2^2 + 2\,R_1^2)\,p_1}{3\,(R_2^2 - R_1^2)\,E_1}$$

This represents the change of dimensions of the interior of the outer cylinder per unit of length of circumference, under the change of pressure represented by p_1.

To find the change of the exterior of the tube due to the variations of pressures p_0 and p_1 which act on it, we recur to the general Equation (15), which gives the change in the direction of the circumference, or of the tangent, of any cylinder whose exterior and interior radii are R' and R at the distance r from the axis. Replacing r by R' since the change at the exterior of the cylinder is now required, we have

$$\lambda = \frac{6\,R^2\,P - (4\,R^2 + 2\,R'^2)\,P'}{3\,(R'^2 - R^2)\,E} \qquad (23)$$

To apply this to the inner cylinder now under discussion make

$$R = R_0$$
$$R' = R_1$$
$$E = E_0$$
$$P = p_0$$
$$P' = p_1$$

and we write

$$\lambda = \frac{6\,R_0^2\,p_0 - (4\,R_0^2 + 2\,R_1^2)\,p_1}{3\,(R_1^2 - R_0^2)\,E_0}$$

for the value of the change of exterior of inner cylinder or tube.

Now since the outer surface of the tube, and the inner surface of the outer cylinder are in contact, the same change of dimensions must occur in both, at this surface of contact, and hence the two values of λ obtained above are equal.

We have therefore

$$\frac{6\,R_0^2\,p_0 - (4\,R_0^2 + 2\,R_1^2)\,p_1}{3\,(R_1^2 - R_0^2)\,E_0} = \frac{(4\,R_2^2 + 2\,R_1^2)\,p_1}{3\,(R_2^2 - R_1^2)\,E_1}$$

Solving this equation with reference to p_1 we have

$$p_1 = \frac{6\,R_0^2\,E_1\,[R_2^2 - R_1^2]\,p_0}{E_1\,(R_2^2 - R_1^2)\,(4\,R_0^2 + 2\,R_1^2) + E_0\,(R_1^2 - R_0^2)\,(4\,R_2^2 + 2\,R_1^2)} \tag{24}$$

Now in this equation p_0 is known, since it is equal to $-P_0$ as before shown, and P_0 has been already calculated by Equations (22), hence we can calculate p_1.

Limiting Value for the Exterior Pressure on the Inner Cylinder, System at Rest.

It has been stated that P_0, given by Equations (22) represents the maximum stress to which the gun can be subjected in action, the smaller of the two values of P_0 being used. It is necessary now to determine what value can be allowed for the exterior pressure upon the inner cylinder at rest, so that the interior surface of the latter will not be compressed by it beyond its elastic limit. To do this we must find the value of P_1' for the state of rest.

The value of P_1' for this state is as has been shown

$$P_1' = P_1 + p_1$$

Assuming Equation (18) and making $P = o$, since the interior pressure at rest is zero, we have

$$\lambda = \frac{-2\,R'^2\,P'}{(R'^2 - R^2)\,E}$$

which shows since it is negative, that there is tangential compression, and as this is generally greater than the radial compression, Equation (18) only is used.

This compression must not exceed that at the elastic limit

which is

$$\frac{\rho}{E}$$

hence we have

$$\lambda = -\frac{2 R'^2 P'}{(R'^2 - R^2) E} = \frac{\rho}{E}$$

for the limiting value of the compression at the interior of the inner cylinder. Changing the letters to correspond to the case of the tube under discussion; that is, making

$$P' = P_1'$$
$$R' = R_1$$
$$R = R_0$$
$$E = E_0$$
$$\rho = \rho_0$$

and omitting the negative sign, as that simply indicates compression, we write

$$\frac{2 R_1^2 P_1'}{(R_1^2 - R_0^2) E_0} = \frac{\rho_0}{E_0}$$

or

$$P_1' = \frac{(R_1^2 - R_0^2) \rho_0}{2 R_1^2}$$

but

$$P_1' = P_1 + p_1$$

hence

$$P_1' = P_1 + p_1 = \frac{(R_1^2 - R_0^2) \rho_0}{2 R^{\cdot 2}} \qquad (25)$$

and this value of P_1' must not be exceeded.

This equation gives the value of $P_1' = P_1 + p_1$ in known terms.

But we have the value of p_1 from Equation (24) by substituting for p_0 its value $- P_0$. Hence, substituting the value of p_1 from Equation (24) in (25), we obtain a new value for P_1 which will cause the interior of the inner cylinder to be compressed to its elastic limit at rest. The value thus ob-

tained for P_1 must be substituted in that one of Equations (22) which gives the least value for P_0. The new value thus obtained for P_0 will be such that the inner cylinder will not be strained beyond its elastic limit either in action or at rest, and it represents the greatest value of the stress to which the gun can be subjected without exceeding the elastic limit of the metal composing it.

THE SHRINKAGE.

In Fig. 12, let OA represent the interior, and OB the exterior radius of the inner cylinder, and OC and OD the interior and exterior radii of the outer cylinder, before they are assembled to form the gun. Then the length $CB = OB - OC$ is the shrinkage. As diameters are usually employed instead of radii in tables of shrinkages, a more usual expression for the shrinkage is

$$2\,CB = 2\,(OB - OC)$$

or, in other words, the shrinkage is the difference of diameters of the enveloping and enveloped cylinders. This is called the *absolute* or *actual* shrinkage. The *relative* shrinkage is the shrinkage per unit of diameter, or per unit of radius, and is expressed by dividing the absolute shrinkage by the interior diameter of the outer or enveloping cylinder. Thus the relative shrinkage in this case is

$$\frac{2\,CB}{2\,OC} = \frac{2\,(OB - OC)}{2\,OC} = \frac{CB}{OC}$$

To determine the shrinkage for the case under discussion. In Fig. 12, let OA, OB, OC and OD represent the same quantities as above.

Now, when the outer cylinder is heated and expanded till its interior radius OC is slightly greater than the exterior radius OB of the inner cylinder, and the exterior cylinder while hot is placed on the interior cylinder, so as to envelop it, and

is then cooled in this position, it is evident that the outer cylinder will compress the exterior of the inner one, and that their surface of contact will assume some such position as $E' E E''$, the outer radius $O B$ of the tube being compressed to $O E$, and the inner radius $O C$ of the outer cylinder being extended to $O E$. Hence the radius $O B$ has been compressed by the amount

$$OB - OE = BE$$

and the radius OC has been extended by

$$OE - OC = CE$$

and the sum of these two is equal to the original shrinkage, BC, or

$$BE + CE = BC.$$

Hence, if we can find the values of the two quantities BE and CE, we will have that of the shrinkage.

Now when the two cylinders are assembled, and the system is at rest, we have found that the pressure P_1' acts at the contact surface of the cylinders. That is, the exterior cylinder is acted upon by a force represented by P_1', and this force produces an extension per unit of radius of

$$\frac{CE}{OC}$$

CE being unknown. But Equation (18) gives the value of this extension in terms of the radii, pressures and modulus of the cylinder. Remembering that

$$\begin{aligned} P' &= P_2' = o \\ P &= P_1' \\ R' &= R_2 \\ R &= R_1 \\ E &= E_1 \end{aligned}$$

we write

$$\frac{CE}{OC} = \lambda = \frac{(4\,R_2^2 + 2\,R_1^2)\,P_1'}{3\,(R_2^2 - R_1^2)\,E_1}$$

This gives CE.

To find BE, or the compression of the exterior of the tube. The pressure acting is P_1', as before, the interior pressure being zero. This change being at the exterior of the cylinder, we use Equation (23), making the following changes,

$$\begin{aligned} P &= o \\ P' &= P_1' \\ R &= R_0 \\ R' &= R_1 \\ E &= E_0 \end{aligned}$$

Hence we have

$$\frac{BE}{OC} = \lambda = -\frac{(4\,R_0^2 + 2\,R_1^2)\,P_1'}{3\,(R_1^2 - R_0^2)\,E_0}$$

Strictly speaking, the true value is $\frac{BE}{OB}$ for the change per unit of radius, but the difference between OB and OC is so small in practice that either may be used without appreciable error.

Now it will be observed that the value of $\frac{BE}{OC} = \lambda$ just obtained, is negative, indicating compression, and this is evidently correct.

But the shrinkage sought is the sum of two positive quantities

$$\frac{BE + CE}{OC} = \frac{CB}{OC}$$

In order to avoid the negative sign, and obtain the quantity $\frac{BE}{OC}$ under a positive form, we suppose that the exterior cylinder is removed from the interior cylinder. In this case it is evident that the exterior surface of the inner cylinder will expand and regain its original diameter, and that this expansion is exactly the same in amount as the compression BE, which was produced by shrinking on the outer cylinder. This is equivalent to supposing the pressure P_1' neutralized

by an equal and opposite pressure; that is, in the value of $\frac{BE}{OC}$ we make

$$P_1' = -P_1'$$

and that value becomes accordingly—

$$\frac{BE}{OC} = \lambda = \frac{(4\,R_0^2 + 2\,R_1^2)\,P_1'}{3\,(R_1^2 - R_0^2)\,E_0}$$

a positive quantity. Now denoting by φ the shrinkage of the two cylinders, we have

$$\varphi = \frac{CE + BE}{OC} = \frac{(4\,R_2^2 + 2\,R_1^2)\,P_1'}{3\,(R_2^2 - R_1^2)\,E_1} + \frac{(4R_0^2 + 2\,R_1^2)\,P_1'}{3\,(R_1^2 - R_0^2)\,E_0} \quad (26)$$

In using this equation it must be remembered that φ is the relative shrinkage, or the shrinkage per unit of diameter. To obtain the absolute shrinkage, the relative shrinkage must be multiplied by the diameter. That is, if D represent the diameter and φ the relative shrinkage (both in inches), and S the absolute shrinkage, then,

$$S = \varphi \times D$$

and the exterior diameter of the cylinder must be made

$$D' = D + S$$

Referring to figure 12,

$$2\,OC = D$$
$$2\,CB = S = \varphi \times D$$
$$2\,OB = D'$$

GENERAL REMARKS.

It can be shown theoretically that the maximum resistance is obtained from a gun cylinder when the radii of the different cylinders composing it, vary from the interior in geometrical progression.

This, however, is never adopted in practice for various reasons, one of the principal being the objection to very thick cylinders on account of their being more difficult to forge, less uniform in quality, and more liable to imperfections in the metal.

It can also be shown that no great advantage is gained as regards tangential strength, by increasing the thickness of the walls of the gun over the powder chamber much beyond one caliber. These considerations combined with the capacity of the forging plant where hoops, tubes and jackets are made, will serve to fix the limits of thickness of the different cylinders composing the gun. Examples are given here of three modern guns:

Gun.	Diam. of powder chamber.	Thickness of tube.	Thickness of jacket.	Thickness of *A* hoops.	Thickness of *B* hoops.	Total thickness of wall.	Total thickness of wall.
	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Calibers.
8-in.	9.50	2.75	4.25	3.30	.....	10.30	1.0842
10-in.	11.80	3.20	4.90	2.525	3.10	13.725	1.1631
12-in.	14.20	3.90	5.80	2.90	3.425	16.025	1.1285

The caliber being the diameter of the powder chamber, the above table shows that the thickness of wall only slightly exceeds one calibre.

Having determined from the above considerations the radii of the different cylinders composing the gun, the values of the pressures which the gun will support in action may be calculated from Equations (22), θ and ρ being known from tests of the metal in a testing machine.

Having obtained the values of P_1 and P_0 from Equations (22), the system must be considered at rest, and the values of the pressure P_1' deduced which will be safe for that state of the system, This is given by Equation (25).

Then this value of P_1' must be used to deduce a new

value of P_1, and this value of P_1 must be substituted in that one of Equations (22) which gave the lower value for P_0. The new value of P_0 thus deduced will represent the maximum pressure to which the gun can be safely subjected.

We can now calculate the shrinkage from Equation (26), using the value of P_1' already found.

The same method can be extended to guns composed of any number of cylinders, but the subject becomes more complex as the number of cylinders increases.

After calculating the shrinkages, the same fundamental formulas may be used to calculate the compressions of the bore produced by the assembling of the cylinders. The results of these calculations are then compared with actual measurements of the bore made during the assembling of the gun, and the agreement is in every case found to be remarkably close, and furnishes a proof of the correctness of the theory.

Thickness of Gun at Different Points.

The thickness of the gun at the reinforce is determined by the considerations already given, and as stated, rarely exceeds one and a half calibres. To determine its thickness at various points along the chase, it is necessary to have the "pressure curve" of the powder at different points along the bore. Formulas have been deduced for this curve by various authors, but it is not deemed necessary to give them here. The results obtained from them do not agree, and recent experiments on a small gun show that Noble & Abel's formula, Chapter IX, agrees very well with the results of experiment. Using this formula, the pressure curve can be obtained.

The elastic resistance of the gun, or the values of P_0 for different sections of the gun, are calculated as explained, and plotted, and the curve of powder pressure for the same gun

is also plotted to the same scale. The curve of resistance should always lie above the curve of pressures.

Length of Gun.

This may be determined for a given initial velocity and given conditions of loading, charge of powder, etc., by Sarrau's formulas for velocity, but as a practical rule, it may be stated that at present the total length of modern high power guns varies between 35 and 45 calibres, and that the tendency is toward the higher limit.

CHAPTER XX.

EXTERIOR BALLISTICS.

This treats of the motion of the projectile after it has left the gun.

Definitions.

The *sights* are two projections on the upper surface of the piece, the distance between which parallel to the axis, is called the *sight radius*, or sometimes the *radius* of the gun.

Each sight contains a definite point called the *sight point*. That for the front sight, which is fixed, is generally determined by the intersection, at an acute angle, of the faces of a wedge or by the intersection of cross wires as in surveying instruments. That for the rear sight consists of a notch in a bar, or a pin hole.

The rear sight point is movable so as to vary its distance from the axis.

The difference of the distances of the sight points from the axis of the bore, divided by the radius of the gun, measures the tangent of the *angle of elevation*, or of e, figure I.

The *line of sight* is a straight line passing through the two sight points. In the act of aiming it also pierces the target. In this case, therefore, the angle of elevation is the angle included between the line of sight and the axis of the bore.

The *line of departure* is the line in which the projectile is moving when it leaves the gun. It is therefore the tangent to the trajectory at the muzzle. Owing to the incipient recoil, due to the conservatism of the system (Chapters VII,

XXI), and to necessary looseness of the joints between the trunnions and the carriage, and between the carriage and the wheels, the piece tends to revolve slightly about some point in rear, so that the projectile does not always leave the piece in the original direction of the axis. The angle included between the axis and the line of departure is called the angle of *jump*, j, figure 1. If to attain a given target the jump, which is almost always positive, were neglected, we would find $d > q$ and the computed value of e would be too great, so that the target would be overshot.

The angle made by the line of departure with the horizontal plane is called the angle of *departure*, d, figure 1. It is with this angle that we have principally to deal in ballistics, as it is the angle at which the projectile actually begins its flight.

The angle of *projection* is the angle included between the line of departure and the line of sight; it may be thought of as the angle of elevation corrected for jump.

The *quadrant angle* is the angle made by the axis of the bore and the horizontal. It is measured by the gunner's quadrant, a form of spirit level, applied to the face of the muzzle or to some cylindrical surface of the gun. Owing to the grooves in rifled guns this is preferably an exterior surface. See q, figure 1.

The quadrant angle may be measured either above or below the horizontal plane. The term *depressed* or *plunging* fire refers to a negative quadrant angle.

The *angle of sight* is the angle included between the line of sight and the horizontal plane, or s, figure 1.*

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* This depends solely upon the *altitude* of the target in the astronomical sense. It is unfortunate that the term above named should be used to designate this angle, as it has nothing whatever to do with the sights.

It would be more consistent if the terms *angle of sight.* and *of elevation* were exchanged. In the French service the angle of sight is called the angle of *site*.

It is seen from the figure and by definition that

$$q = s + e.$$

Of these quantities, s is given by the act of pointing, and e must be computed by the methods hereafter explained. The above equation is principally useful for verifying the elevation given by the sights or for guns which are not provided with sights.

To determine the Jump.

Place in front of the gun and at a distance just beyond the reach of the blast a slight screen. Mark upon the screen the point o, at which it is pierced by the axis of the bore prolonged. In breech-loaders this may be done by means of perforated discs fitting the bore, and in muzzle-loaders by making $e = 0$ and laying off on the screen the coördinates of the sight points negatively taken.

Fire the piece and determine V.

If then x and y are the horizontal and vertical coördinates of the center of the shot hole when referred to o, and d the distance of the screen from the muzzle we have approximately, from figure 2,

$$\tan j = \frac{y + ab}{d},$$

but $$ab = \frac{gt^2}{2} \quad \text{and} \quad t = \frac{d}{V} \text{ nearly},$$

hence $$\tan j = \frac{y}{d} + \frac{gd}{2V^2}.$$

If the shot strikes below the point, then y is measured negatively.

The lateral jump is evidently $\tan^{-1} = \frac{x}{d}$.

A number of such determinations should be made since the method is obviously inaccurate.

A better plan is to eliminate the effect of the perturbations near the muzzle by computing e and determining by experiment the value of p for an extended range of, say 500 yards,—then $j = p - e$

The jump is usually about 30′ which value will be taken in problems in which it is required to be assumed.

The *planes of sight and of departure* are vertical planes containing the corresponding lines.

The computed *range* is the distance from the gun to the (second) intersection of the trajectory by the line of sight. The term *range* is also applied according to circumstances to the distance of the target from the gun, and to the horizontal distance to the point of impact—in case the target be missed and the projectile strikes some horizontal surface in front or rear of the target—such as water.

Practically the dimensions of the gun may be neglected so that the front sight point may be considered in the axis of the bore and the range to be measured from either sight indifferently.

In the following discussions we will also assume that the planes of sight and of departure coincide in the vertical plane containing the axis of the piece, which is called the *plane of fire*, and that the projectile travels in the plane of departure.

This is not actually true, however, for the projectile tends to move sideways out of the plane of departure as shown by the horizontal projection of figure 1. This motion, called the *drift*, is due to the combined effect of the rotation of the projectile and the resistance of the air; combined with other causes of inaccuracy it leads at the target to lateral *deviation*, which is meaured by the distance of the point of impact from the plane of sight. See Chapter XXX.

The *deviation in range* is similarly measured.

Classification of Fire.

In this classification the sights are disregarded and the *line of fire* is the straight line from the muzzle of the piece to the point aimed at. Similarly for general discussions the quadrant angle is sometimes called the *angle of fire.*

Figure 3 illustrates the classification with reference to the vertical plane containing the target, which represents a certain face of a work.

Figure 4 illustrates the classification with reference to the horizontal plane. The limit for direct fire is imposed by the principle of the rigidity of the trajectory to be hereafter explained.

The classification is also applied, as indicated, to the angles of descent. This is more accurate since it relates to the effect produced rather than to the intention of producing a given effect.

In figure 1 it is assumed without sensible error, that the lines of sight, departure, etc., intersect at the muzzle, and the drift is very much exaggerated.

Exterior ballistics is usually divided into two parts. 1st, in vacuo; 2d, in the air.

TRAJECTORY IN VACUO.

Utility.

The first of these is sufficiently treated in the course of Mechanics. Its practical utility is confined to two cases.

1st. That of projectiles of high sectional density moving with comparatively low velocity as in mortars, since in such cases the loss of energy due to the resistance of the air may be neglected where only approximate results are required. Chapter XVI, page 1.

2d. Cases involving the flight of projectiles in the air, in which some of the data are lacking, or in which the velocity of the projectile in one of its component directions is so low that the consequent retardation may be neglected.

USEFUL FORMULÆ.

The principal equations of this kind which are used in this course may be derived from equation (167), Michie, in which we write, as is customary, y for z.

$$\frac{dy}{dt}=V\sin\theta-gt; \tag{1}$$

whence

$$y=Vt\sin\theta-\frac{gt^2}{2} \tag{2}$$

and by placing $\frac{dy}{dt}=0$

$$t=\frac{V\sin\theta}{g} \tag{3}$$

for the time to the vertex of the trajectory.

From the symmetry of the trajectory in vacuo, T, the whole time of flight is equal to $2t$ or

$$T = \frac{2\ V \sin\ \theta}{g} \tag{4}$$

and

$$V = \frac{g\ T}{2 \sin\ \theta} \tag{5}$$

Eliminating V by substituting this value in Equation (2),

$$y = \frac{g\,t}{2}(T - t) \tag{6}$$

If in this we replace t by $\frac{T}{2}$ we have $Y = \frac{g\ T^2}{8}$ (7)

in which y now becomes Y, the ordinate of the vertex.

That is, the height of the vertex in feet is nearly four times the square of the time of flight in seconds.

Equations (6) and (7) are important, and should be remembered, as they are frequently used in approximate solutions in the air.

If in Equation (169), Michie, rewritten according to the usual nomenclature, or

$$y = x\ \tan\ \theta - \frac{x^2}{4\ H \cos^2\ \theta} \tag{8}$$

in which H is the height through which the projectile must fall to acquire the velocity V, we make $y = \theta$ we may determine the range X.

$$X = \frac{2\ V^2 \sin\ \theta \cos\ \theta}{g} = \frac{V^2 \sin 2\ \theta}{g} \tag{9}$$

Therefore the range will vary less from variations in θ, as θ approaches 45^0.

Also, for the same value of θ, as in S. B. mortars,

$$X : X' :: V^2 : V'^2.$$

But, considering the powder as a reservoir of potential energy, from the equation of energy we have approximately

$$V : V' :: \sqrt{\frac{w}{W}} : \sqrt{\frac{w'}{W'}}$$

And assuming the weight of the projectile, W, to be constant for the same piece

$$w : w' :: E : E' :: V^2 : V'^2 :: X : X'.$$

Therefore, in a S. B. mortar the charges are proportional to the ranges. This is of importance in regulating charges and works well in practice.

If in equation (9) we substitute the value of V in equation (5), we find, if we call $g=32$ approximately and $\varphi=45°$, $X=16\ T^2$ or

$$T = \frac{\sqrt{X}}{4} \qquad (10)$$

which gives a rule for timing mortar fuzes.

RESISTANCE OF THE AIR.

To give an idea of the pressure exerted on projectiles in the air and consequently of the insufficiency of the preceding formulæ for practical use, except in the cases cited; it will suffice to say that a velocity of the wind of about 100 miies an hour is designated in the Ordnance Manual as a "hurricane that tears up trees, carries buildings before it, etc."

In projectiles moving with the high velocities now attained the pressure is over 80 times as great as that assigned to the hurricane.

EXPERIMENTS TO DETERMINE THE RESISTANCE OF THE AIR.

Experiments have been constantly made since Robins, called the "Father of Gunnery," began to investigate this subject about the middle of the last century. But these

gave untrustworthy results owing to the lack of suitable velocimeters.

It is upon the investigations of the Reverend Francis Bashforth, conducted under the auspices of the British Government from 1865 to 1880, that our knowledge of the effects of this resistance is based.

RESULTS OF EXPERIMENTS.

Resistance.

Bashforth's experiments demonstrate that the resistance varies with the quantities shown on the following tabular scheme.

$$\text{Resistance varies with}\begin{cases}1.\ \text{Area of cross-section or } d^2\text{, Chap. XVI.}\\ 2.\ \text{Density of air.}\\ 3.\ k\text{, Chap. XVI, page 2, viz.:}\begin{cases}1.\ \text{Meridian section}\begin{cases}a\text{ of head,}\\ b\text{ of body.}\end{cases}\\ 2.\ \text{Velocity of projectile.}\end{cases}\end{cases}$$

1. That the resistance varies with the area is recognized by all experimenters.

2. The effect of variations in the density of the air, whether due to variations in barometric pressure, in temperature or in humidity, or from the passage of the projectile through strata of varying density, is allowed for in refined computations by suitable coefficients. For this treatise the effects of such variations are neglected.

3. Variations in k due to slight variations in the meridian section are also neglected, although they may be similarly corrected, see b below.

1. *As to the Meridian Section, viz.:*

(*a*) *Form of Head.*—Bashforth in his experiments used projectiles of the same calibre and weight, and having heads of five different shapes. These were, 1st, hemispherical; 2d, hemispheroidal, with axes in the ratio of 1 to 2; 3d, ogival, radius of head 1 diameter; 4th, ogival, radius of head 2 diameters; 5th, flat.

The resistance was greatest on the flat-headed projectile, and least on the hemispheroidal and ogival of two diameters. Rashforth concludes that the resistance offered by the air to the motion of elongated projectiles is but little affected by the more or less pointed apex, but depends chiefly upon the form of the head, near its junction with the cylindrical body of the shot. At this point the forms of the hemispheroidal head, and of the ogival head of two diameters radius, are about the same, and their resistances are nearly equal.

(*b*) *Form of Body.*— Recent experiments by Krupp have shown that the resistance varies also with the shapes of the sides and rear of the projectile, and with the character of its surface.

2. *Retardation and Velocity.*

Rashforth's method was one of interpolation founded on the use of velocimeters of Class II, by which he determined by means of finite differences the retardation of the projectile at certain points of its trajectory at which the velocity was known.

Everything else being constant, the relation between the retardation and the velocity was known for each of the velocities observed at any one fire. And by varying these velocities as by varying the initial velocity or the distance of the gun from the targets, an indefinite number of velocities could be observed and their corresponding retardations computed.

Finally, the *law* connecting the velocity and the retardation could be deduced by analysis, or expressed by plotting a curve of which the retardation and velocities are the coördinate axes.

For the same velocity the retardation was found to vary with (1) the sectional density of the projectile, its (2) meridian section and (3) surface, and with the (4) density of the air as

affected by its temperature, barometric pressure and its humidity.

Accordingly, such a law must for convenience be reduced to standard conditions, that is, when (1) W (in pounds) $= d^2$ (in inches), *i. e.*, when we have the *unit projectile*, and when the (2) proportions and (3) surface of the projectile are well defined, and the (4) density of the air is at a known standard.

Variations in these four conditions are subsequently allowed for by suitable empirical coefficients of which we shall deal with only that relating to the sectional density.

It may be stated however that Bashforth used a M. L. R. gun firing studded projectiles, the points having a radius of $1\frac{1}{2}$ calibres. The more recent B. L. projectiles, having sharper points and smoother surfaces, reduce the retardation by 5 or 10 per cent. See page 10.

BASHFORTH'S METHOD.

He placed 10 targets at a constant interval of 150 feet$=l$. This gave such a number of observations at each fire that they served to correct each other by the principle of continuity, so that the final order of differences would be either 0, or would change very slowly. Examples of this are seen in the methods used in correcting tables of squares, cubes and of logarithms.

For this purpose the advantages of instruments of Class II. over those of Class I. are obvious. Such instruments ordinarily give only the velocity at some point between each pair of targets. But Bashforth sought the velocity at the target itself as follows:

Calling (v_x) the velocity at the target which is at a distance x from the gun

$$v_x = \frac{ds}{dt} = \frac{l}{dt} \tag{11}$$

In measuring velocity it is customary to express s as a function of t, in which t (one second) is the constant. But when, as in these experiments, l is constant, it is advisable to express the velocity by varying the value of t. Consequently, calling r_x the retardation at the distance x and observing that, since this is a negative acceleration, we may neglect the minus sign resulting from differentiation, we have

$$r_x = \frac{dv_x}{dt} = \frac{l\,d^2t}{dt^3} \times \left(\frac{l}{l}\right)^3 = \frac{d^2t}{l^2}\,v_x^3 \tag{12}$$

The object of presenting the retardation in this form was to make it an explicit function of the cube of the velocity since Bashforth had reason to believe that the retardation followed what is known as the *cubic law.**

In order to apply equations (11) and (12) practically, it is necessary to find by experiment such finite values for dt and d^2t that, when substituted in the preceding equations they will give proper values for v_x and r_x. Or, calling these finite values Δt_x and $\Delta^2 t_x$

$$v_x = \frac{l}{\Delta t_x} \tag{11'}$$

$$r_x = \frac{\Delta^2 t_x}{l^2}\,v_x^3 \tag{12'}$$

---

* The simplicity of such a law has always proved attractive to investigators of this subject. Sir Isaac Newton took it to vary with the square of the velocity, and others with varying powers of the velocity corresponding to certain limiting velocities.

Newton's law has recently been proved nearly true for the high velocities and smooth pointed projectiles now employed. It will be seen hereafter how Bashforth corrected the cubic law by an empirical coefficient corresponding to the velocity.

DETERMINATION OF VELOCITY AND RESISTANCE.

Referring to Bashforth's experiments, let s denote the distance from any origin to the first target. Then $s+l$ will be the distance from the same origin to the second target and $s+(n-1)l$ the distance to the n^{th} target at the distance x, and so on.

Also, let t_s denote the time from any origin until the first target is reached. Then $t_{s+(n-1)l}$ will be the time from the same origin until the n^{th} target is reached and so on.

Now, let d_1, d_2, d_3, etc., denote the 1st, 2d, 3d orders of difference and d', d'', d''', the successive terms in these orders of difference so that d_2''' will mean the third term in the second order of difference and so on.

Then $t_{s+l}-t_s=d_1'$, which will be the time of passage of the projectile between the 1st and 2d targets and

$$t_{s+2l}-t_{s+l}=d_1'',$$

and $d_1''-d_1'=d_2'$ and so on.

We may therefore form the following table which may be filled up from experiment as shown below numerically, and graphically by the diagram, figure 5.

The dotted lines in the diagram serve to indicate the successive orders of difference after the manner of the brackets in the table.

TABLE.

No. of target.	Distance.	Time of passage.	Orders of difference. d_1	d_2	d_3 . . .	$d_{(n-1)}$
1.	s	t_s				
2.	$s+l$	t_{s+l}	d_1'	d_2'		
3.	$s+2l$	t_{s+2l}	d_1''		d_3'	etc.
4.	$s+3l$	t_{s+3l}	d_1'''	d_2''	d_3''	
5.	$s+4l$	t_{s+4l}	d_1^{IV}	d_2'''		$d'_{(n-1)}$
. .	. . .	. . . . .	$d_1^{(n-1)}$			
n.	$s+(n-1)l$	$t_s+(n-1)l$				

NUMERICAL EXAMPLE.

		times.	d_1	d_2	d_3
1.	s	3.0526			
			.1090		
2.	$s+150$	3.1616		.0024	
			.1114		0
3.	$s+300$	3.2730		.0024	
			.1138		1
4.	$s+450$	3.3868		.0025	
			.1163		0
5.	$s+600$	3.5031		.0025	
			.1188		
6.	$s+750$	3.6219			

From the algebra we have

$$t_{s+nl}=t_s+nd_1'+n\frac{(n-1)}{1.2}d_2'+n\frac{(n-1)(n-2)}{1.2.3}d_3'+\text{etc.}$$

Arranging the terms of the second member with reference to n which is arbitrary, we have

$$t_{s+nl}=t_s+n\left(d_1'-\tfrac{1}{2}d_2'+\tfrac{1}{3}d_3'-\text{etc.}\right)$$

$$+\frac{n^2}{1.2}(d_2'-d_3'+\text{etc.}) \tag{13}$$

Now t_{s+nl}, being a function of the space $(s+nl)$, may be developed by Taylor's formula. Hence we have

$$t_{s+nl}=f(s+nl)=t_s+\frac{dt_s}{ds}nl+\frac{d^2t_s}{ds^2}\frac{n^2l^2}{1.2}+\text{etc.}$$

or since, $ds=l$,

$$t_{s+nl}=t_s+dt_sn+d^2t_s\frac{n^2}{2}+\text{etc.} \tag{14}$$

Equating the coefficients of the first power of n in the second members of the two identical equations (13) and (14), we have

$$dt_s=d_1'-\tfrac{1}{2}d_2'+\tfrac{1}{3}d_3'-\text{etc.}=\Delta t_s \tag{15}$$

which is the finite value of dt_s for the constant increment of s.

In other words, and as shown by figure 5, if Bashforth had taken d_1' to be the increment of time at the first target corresponding to ds, the velocity computed would have been the mean velocity between the 1st and 2d targets and would have been too small. Consequently this increment is diminished by $\frac{1}{2}\,d_2'$. This makes the velocity too great, so that $\frac{1}{3}\,d_3'$ is added, the approximation increasing with the number of targets employed; since, *under the same circumstances*, the greater the number of observations, the more truly can the law be determined; or, mathematically speaking, the greater will be the $\pm$ correction applied to d_1' since the greater will be the number of orders of difference.

Therefore, for the target at the distance s or the first target,

$$v_s = \frac{l}{dt_s} = \frac{150}{d_1' - \frac{1}{2}\,d_2' + \frac{1}{3}\,d_3' - \text{etc.}} \tag{16}$$

Similarly for the n^{th} target at the distance $x = s + (n-1)l$

$$v_x = \frac{150}{d_1^n - \frac{1}{2}\,d_2^n + \frac{1}{3}\,d_3^n - \text{etc.}} = \frac{l}{dt_{s+(n-1)l}} \tag{17}$$

The number of the targets at which velocities could be obtained is determined by the number of targets employed and by the number of orders of difference which the law of retardation permits. If n' be this number, then velocities may be determined at $(n - n')$ points.

Similarly, we have the coefficients of the second power of n.

$$d^2t_s = d_2' - d_3' + \text{etc.} = \Delta^2 t_s, \tag{18}$$

and for the n^{th} target

$$r_x = \frac{v_x^3}{l^2}\,\Delta^2 t_x = \frac{v_x^3}{(150)^2}\left(d_2^n - d_3^n + \text{etc.}\right) \tag{19}$$

Example.

The velocity at the 4th target in the preceding table is 1304 and the retardation 246.5.*

RESULTS OF THE EXPERIMENTS.

If the cubic law had held true for all velocities, the coefficient k in Equation (1), Chapter XVI, could have been replaced by some explicit function of v^3.

But while this was found to be nearly true for velocities between 1100 and 1350 feet, it failed for velocities above and below these limits, as Bashforth found by increasing his velocities progressively from 100 to 2900 feet.

He accordingly introduced an empirical constant k' by which to correct the departure from the cubic law so that

$$r = \frac{d^2}{W} k' v^3,$$

and as k' is a very small quantity, he replaced it by

$$K = (1000)^3 k',$$

so that

$$r = \frac{d^2}{W} K \left(\frac{v}{1000}\right)^3. \tag{20}$$

Table I gives the value of K for the velocities named therein and figure 6 is plotted from the indications of Table I.

Example.

A 12.5 inch shell weighing 802.25 lbs. has a velocity of 1400. The total air pressure is 1394 lbs. and the retardation is 55.96 at the instant that the velocity is 1400.

Figure 7 gives the pressure on what is called a *circular inch* (that is a circle of which $d = 1$ inch) on spherical projectiles, curve A; on studded oblong projectiles of which

*Throughout this chapter velocities will be expressed numerically, as the unit of velocity, or foot-second, may be understood.

the radius of curvature of the head is $\frac{3}{2}$ *d*, curve *B*; and on modern smooth b. l. projectiles in which the radius = 2 *d*, curve *C*, derived from recent experiments by Krupp.

In curve *B* two remarkable inflections are observed. One at about 1090, the velocity of sound, and the other at about 2413, that of air rushing into a vacuum.

The first velocity marks the passage of the projectile into a medium undisturbed by the explosion of the gun or by its own passage. The second denotes the formation of a vacuum in rear of the projectile which increases the pressure to about double that of the barometric pressure of the atmosphere.

In firing at troops, particularly in sieges, it is important to have the terminal velocity exceed that of sound, so that the projectile may precede the warning sound made by its passage through the air.

The irregularity of curves *B* and *C* shows the impossibility of expressing by any simple law the relation between velocity and pressure.

Final Velocity.

Figure 7 enables us to approximate closely to the *final velocity* of the projectile.

This term, which must be carefully distinguished from the *terminal velocity* (Chap. I), is that velocity which the projectile has acquired in falling when the resistance of the air becomes equal to the accelerating force of gravity. This velocity is necessarily uniform and a maximum.

For example, a 64 lb. projectile, 6.3 inch in diameter, has a weight per circular inch of 1.613 lbs. If it belongs to the class of projectiles used by Bashforth, an equal and contrary air pressure will result when a velocity of nearly 900 f. s. has been acquired. But for more modern projectiles a higher final velocity will result.

This velocity which formerly had only a theoretical significance, is becoming important in consequence of the great heights and high angles of fire now used in mortar firing.

The S. C. Mortar has thrown its shell over 3 miles into the air with an angle of fall of about 75°.

The result can be reached more exactly by a method of approximation based upon the fact that K enters into equation (20) in the first power while v is cubed. Consequently, the first trial values of K will not greatly affect the result, and we may finally find a velocity and corresponding value of K which will satisfy the equation

$$W = K \frac{d^2}{g}\left(\frac{v}{1000}\right)^3.$$

TRAJECTORY IN AIR.

GENERAL SOLUTION.

Notation.

Let $OVR = S$, figure 8, represent a trajectory.

Let V be the muzzle velocity in the line of departure.

Let $v = \frac{ds}{dt}$ be the velocity in the direction of the tangent at any point of which the coördinates are x and y.

Let u be the horizontal component of the velocity v.

Let v', v'', u', u'', be the corresponding tangential and horizontal velocities at the beginning and end of any arc, the coördinates of the extremities of which are (x', y'), (x'', y'').

Let φ be the variable inclination to the horizontal of the tangent to the trajectory; then $u = v \cos \varphi$.

Let θ and ω, measured as in the figure, be the particular values of φ for the angles of departure and of fall.

Let α and β be the values of φ at the beginning and end of any arc when the velocity is v' and v''.

Then the figure shows that $\delta = \alpha - \beta$ or the angle included between the tangents is the change in inclination due to a change in velocity from v' to v''.

Similarly $\Delta = \theta - (-\omega) = \theta + \omega$ is the total change in inclination.

Let $\overline{\varphi}$ (read φ "*dash*") be the inclination to the horizontal of any chord of the trajectory.

Let $\overline{v} = \dfrac{u}{\cos \overline{\varphi}} = u \sec \overline{\varphi}$ be the component velocity of v in the direction of the chord, and, as above, let $\overline{v'}$, $\overline{v''}$ be the component velocities in the direction of the chord at the beginning and end of the arc (x', y'), (x'', y'').

Let $t,'$ t'' be the times measured from any origin to the instants when the velocities are respectively

$$(v', v''), \overline{(v', v'')}, \text{ etc.}$$

Note that $t'' > t'$, $v'' < v'$.

Let $t = t'' - t'$ for any arc and T=time to vertex, T', + time from the vertex to R, $T_{\prime}$ be the whole time of flight, or

$$T = T' + T_{\prime}.$$

Similarly,

Let X', Y', be the computed cöordinates of the vertex measured from O; and $X_{\prime}$, $Y_{\prime}$, the computed cöordinates of the same point measured from R, so that the computed range, $X = O\,R = X' + X_{\prime}$.

The figure shows that the notation of v', u', $\overline{\varphi}$, changes from the ascending to the descending branch to $v_{\prime}$, $u_{\prime}$, $\underline{\varphi}$. This distinction will be observed throughout when the branches are separately considered; but, when the trajectory is considered as a whole, v', v'', &c., refer to the beginning and ending of any arc.

Let the ordinate at D represent the height of a target at the distance OD. Then DR is the dangerous space for that target. If we aim at the center of the target this evidently measures nearly twice the $\pm$ error permissible in estimating the distance, as a measure preliminary to determining the value of θ required to strike the target at some point of its height. See Chap. I., p. 3.

The dangerous space is known herein as $D.\ S.$

Resolution of Motion.

Let $A\,I\,R$, figure 9, be some arc of the trajectory to which $I\,t$ is the tangent at I, ρ is the radius of curvature, g the acceleration due to gravity, r the retardation due to the resistance of the air.

Then from the figure, for the horizontal retardation,

$$\frac{du}{dt} = r \cos \varphi \tag{21}$$

and, from Mechanics, for the normal component of the deviating force of gravity

$$\frac{v^2}{\rho} = g \cos \varphi \tag{22}$$

but $\frac{1}{\rho} = \frac{d\varphi}{ds} \times \frac{dt}{dt} = \frac{d\varphi}{dt.v}$ and, therefore,

$$v \frac{d\varphi}{dt} = g \cos \varphi \tag{23}$$

Dividing Equation (23) by (21) member by member, we have, after transposing,

$$d\varphi = \frac{g\,du}{r\,v} \tag{24}$$

Integrating between the limits $\varphi = \alpha$ and $\varphi = \beta$ and the corresponding values of u, we have

$$\int_{\beta}^{\alpha} d\varphi = \alpha - \beta = \delta = g \int_{u''}^{u'} \frac{du}{rv}, \tag{25}$$

or

$$\Delta = g \int_{u_{\prime\prime}}^{u'} \frac{du}{rv}. \tag{25'}$$

From equation (21) we have by similar means,

$$\int^{t''} dt = t'' - t' = t = \int_{u''}^{u'} \frac{du}{r \cos \varphi}, \tag{26}$$

or

$$T = \int_{u_{\prime\prime}}^{u'} \frac{du}{r \cos \varphi}. \tag{26'}$$

Similarly, from the relations between x, y, t, and u,

$$x = \int_{t'}^{t''} u\, dt = \int_{u''}^{u'} \frac{u\, du}{r \cos \varphi}, \tag{27}$$

or

$$X = \int_{u_{\prime\prime}}^{u'} \frac{u\, du}{r \cos \varphi}, \tag{27'}$$

$$y = \int_{u''}^{u'} \frac{u\, du}{r \cos \varphi} \tan \varphi, \tag{28}$$

or

$$Y = \int_{u''}^{u'} \frac{u\, du}{r \cos \varphi} \tan \varphi. \tag{28'}$$

If these general equations (25–28) could be integrated, they would give the change in the coördinates of an arc of the trajectory $(x''-x')$, $(y''-y')$ corresponding to a change of horizontal velocity $(u'-u'')$, the time t required to make this change, and the change of inclination δ, corresponding to the same change in the velocity.

But the second members contain three variables, u, φ, and r, not connected by any law, and hence the integration is impossible.

Bashforth's experiments, however, give the law connecting u and r, and in order to avoid the difficulty arising from the presence of the variable φ we assume for it a constant mean value $\overline{\varphi}$. That is, that on the same principle as that by which we have neglected the small vertical component of the resistance, we now neglect the small component velocity in a direction at right angles to the chord, and suppose the length of the arc to be that of the chord, although its curvature is retained.

COROLLARIES.

I. Equation (23) may be written

$$\frac{d\varphi}{dt} = \frac{g \cos \varphi}{v},$$

whence, by dividing member by member by

$$\frac{dx}{dt} = v \cos \varphi, \text{ we obtain}$$

$$\frac{d\varphi}{dx} = \frac{g}{v^2}. \tag{29}$$

Calling $e = \dfrac{Wv^2}{2g}$ and substituting we have

$$\frac{d\varphi}{dx} = \frac{W}{2e}. \tag{30}$$

Equations (29) and (30) express the rate of change of the direction of the tangent to the trajectory, or the rate at which the trajectory is becoming curved, as a function of the range. Equation (29) illustrates the remarks, Chap. I, top page 3, and Equation (30) explains the importance of Chap. XVI, p. 1. These equations set forth a most important property of the trajectory in air.

Figure 10, which is carefully drawn to a scale, represents in curve A the trajectory *in vacuo* of a projectile fired with $\theta = 30°$ and $V =$ about 1700 f. s.

Curves B and C represent the trajectories *in air* of spherical projectiles as follows:

B. 15 inch; $W = 450$ lbs. $d = 14.87$ inch. } $V = 1700$
C. 24 pdr.; $W = 26.92$ lbs. $d = 5.9$ inch. } $\theta = 30°$

Since from Chap. XVI, p. 2, the elements of a trajectory when θ and V are given depend on the ballistic coefficient $\frac{d^2}{W}$, it appears that the 24 pdr. projectile would describe the trajectory B if its weight were increased to 70.86 lbs., the calibre remaining constant; or, by reducing the calibre to 3.637 inches, the weight of the projectile remaining constant. The objections to this are given Chap. XVI, p. 4.

II. If in equation (29) we substitute for v^2 its value $\rho g \cos \varphi$, we find

$$\frac{d\varphi}{dx} = \frac{1}{\rho \cos \varphi} \text{ or } d\,\varphi \cos \varphi = \frac{dx}{\rho}$$

Integrating this equation between the limits $+\theta$ and $-\omega$, figure 8, corresponding to O and X, and assuming some mean value of $\rho = \rho'$ by which to measure the flatness of the trajectory, we have as a measure of its mean curvature,

$$\frac{1}{\rho'} = \frac{\sin \theta + \sin \omega}{X} \tag{31}$$

Although from equation (22) and from experience it is evident that, owing to the variable value of $g \cos \varphi$, with the sight set for a certain range it is impossible to hit any desired point of a vertical circle described about the gun with the range as a radius; yet, as shown by equation (31) and figure 11, if the *altitude* of the target or the angle of sight s, be small, the decrease of ω' tends to compensate for the increase of θ', so that $\sin \theta' + \sin \omega'$ may not differ greatly from $\sin \theta + \sin \omega$: $OX' = OX \cos s$ will also be very nearly equal to X. Under these circumstances the two values of ρ' will not differ greatly from each other.

Under this assumption we may consider the mean curvature of the trajectory to be constant or the trajectory to be practically *rigid*, so that for small altitudes the elements of the trajectory measured along the chord may be safely assumed to be independent of the inclination of the chord to the horizon.

Equation (30) shows that this assumption will increase in truth when the sectional density and the muzzle velocity increase, which is the present tendency. The principle involved is of especial importance in the rapid fire of modern small arms and field pieces, since it permits the use for inclined ranges of sights graduated for horizontal ranges, when the $\pm$ angle of sight is less than about 10°.

In such cases the change, s' figure 11, in the angle of departure, may for a first approximation be safely taken to be equal to s, and this change is automatically made by the act of pointing.

Actually however, when s is positive, ρ' decreases and conversely; so that in firing up hill the projectiles tend to fall short and in firing down hill they tend to pass over the object.

NIVEN'S METHOD.

Various expressions have been deduced for the value of $\bar{\phi}$; that of Mr. W. D. Niven, F.R.S., obtained from an expanding series, is one of the most simple and, for illustration, a particular value of $\bar{\phi}$, deduced for equation (25), is herein applied to all cases indifferently.

The appendix to this chapter contains the means of arriving at more exact values of $\bar{\varphi}$.

Under this hypothesis we shall adopt, as a sufficient approximation for small angles of θ less than about 3°,

$$\bar{\phi} = \frac{\alpha + \beta}{2}, \tag{32}$$

and for larger angles the approximation

$$\tan \bar{\phi} = \frac{\tan \alpha + \tan \beta}{2}. \qquad *(33)$$

From the notation we have

$$u = \bar{v} \cos \bar{\phi}; \quad du = d\bar{v} \cos \bar{\phi}; \quad \bar{v}' = u' \sec \bar{\phi}, \quad \text{etc.}$$

Substituting these values in equation (25), and replacing r by its new value $K\frac{d^2}{W}\left(\frac{\bar{v}}{1000}\right)^3$, we have

$$\delta = \cos \bar{\phi} g \int_{\bar{v}''}^{\bar{v}'} \frac{d\bar{v}}{K\frac{d^2}{W}\frac{\bar{v}^4}{(1000)^3}}.$$

* These values of $\bar{\phi}$ give good practical results.

In this equation δ is expressed in circular measure, that is, in terms of the ratio $\pi = 180°$. To reduce it to the corresponding number of degrees, d, we have,

$$\delta : \pi :: d : 180 \quad \text{or} \quad \delta = \frac{\pi d}{180}.$$

Substituting this value of δ in the above equation, and representing as hereafter the ballistic coefficient $\frac{d^2}{W}$ by C, we have, after reduction,

$$Cd = \frac{\cos \bar{\phi}\, 180g\, (1000)^3}{\pi} \int_{\bar{v}''}^{\bar{v}'} \frac{d\bar{v}}{K\bar{v}^4}.$$

In this K is a function of $\bar{v}$, and therefore changes between the limits of integration. Means have, however, been found for determining its mean value for limiting velocities.

The value so determined is nearly its arithmetical mean. Therefore we have, calling this mean value K',

$$Cd = \frac{\cos \bar{\phi}\, 180g\, (1000)^3}{\pi K'} \int_{\bar{v}''}^{\bar{v}'} \frac{d\bar{v}}{\bar{v}^4}. \tag{34}$$

Similarly, we have

$$Ct = \frac{(1000)^3}{K'} \int_{\bar{v}''}^{\bar{v}'} \frac{d\bar{v}}{\bar{v}^3}. \tag{35}$$

And representing by s the length of the chord, the co-ordinates of the extremities of which are $(x' \; x'')$ $(y' \; y'')$

$$Cs = \frac{(1000)^3}{K'} \int_{\bar{v}''}^{\bar{v}'} \frac{d\bar{v}}{\bar{v}^2} \tag{36}$$

We have also

$$x = x'' - x' = s \cos \bar{\phi} \text{ and } y = y'' - y' = s \sin \bar{\phi}$$

These equations are in a form to be integrated, and Tables II, III, IV have been computed for a projectile in which $G = 1$, as follows:

Assume any velocity v_0 sufficiently low as the origin of integrals, and assigning proper values for K', integrate equations (34, 35, 36) between v_0 and successive values of v'. We thus obtain what are called *angular functions* $\delta_{v'}$, *time functions*, $\tau_{v'}$, and *space functions*, $\sigma_{v'}$, which may be explained by reference to the time functions in Table II.

Explanation of the Tables.

Considering the acceleration to be positive, Table II may be considered to express by its functions the several times τ, τ', τ'', etc., required to give to a unit projectile, starting as from rest, the several corresponding velocities under the action of a variable force equal to the variable resistance of the air.

Table III may be similarly understood to express the space in feet σ, σ', σ'', etc., over which such a force would have to act in order to increase the velocity from some initial velocity as 0, to the several corresponding velocities given.

It is evident that each function in each table might be numerically diminished by the first function in its own table without affecting the value of the table or the velocities to which it applies, since it is only the differences between functions that are considered.

Conversely, considering the acceleration as negative, if any time function as τ', figure 12, measured from any origin t_0, corresponds to a change $v' - v_0$ in the velocity measured from any origin v_0, and τ'' similarly corresponds to a change $v'' - v_0$, then $\tau'' - \tau'$ can correspond only to the particular change $v' - v''$. So that knowing $\tau'' - \tau' = t$, and either v' or v'', we may determine the other velocity; and con-

versely as to t, τ', or τ'', without regard to whether the difference is positive or negative.

Similarly for the angular functions. If any angular function δ', figure 13, corresponds to a change $v' - v_0$, and δ'' to a change $v'' - v_0$, then $\delta' - \delta'' = d$ will correspond to a change $v' - v''$.

In all cases we have two pairs of unknown quantities, of which the difference between one pair and one of the other quantities is needed to determine the remaining quantity.

These data are given by the conditions of the problem, or are supplied by certain assumptions to be hereafter explained.

Example from Table II.

The change of time (or time required) for a change of velocity of 300 when the greater velocity is 1400 is 231.9883 − 230.5314 = 1.4569 sec. When the lesser velocity is 1400 it is 0.8697.

If in the first case the time from some given origin, say the firing of the piece, until the velocity was reduced to 1400 was, say, 2 seconds, then the time measured from the same origin until the velocity fell to 1100, would be 3.4569 sec., and so on.

PRACTICAL FORMULÆ.

For the ascending branch Equations (34), (35), (36) may be written.*

$$Cd = \cos \bar{\phi}(\delta_{\bar{v}'} - \delta_{\bar{v}''})\,; \tag{I}$$

$$Ct = \tau_{\bar{v}'} - \tau_{\bar{v}''}\,; \tag{II}$$

$$Cs = \sigma_{\bar{v}'} - \sigma_{\bar{v}''}. \tag{III}$$

Whence from III,

$$Cx = Cs \cos \bar{\phi} = \cos \bar{\phi}(\sigma_{\bar{v}'} - \sigma_{\bar{v}''})\,; \tag{III$'$}$$

$$Cy = Cs \sin \bar{\phi} = \sin \bar{\phi}(\sigma_{\bar{v}'} - \sigma_{\bar{v}''}). \tag{III$''$}$$

* Similar equations serve for the descending branch.

The Greek letters in the second members of the above equations are the corresponding tabular functions found respectively in Tables IV, II, III.

These tables are arranged like logarithmic tables. Except where small changes of the functions are considered, for section-room work the column of differences need not ordinarily be employed, the nearest function or velocity being taken.

It is evident that the nearer the chord is to the arc, the less will be the difference $v - v$, and the more accurate will be the result. In practice, it is considered sufficiently accurate to divide the trajectory into two arcs, at the vertex.

For simplicity, and by the principle of the rigidity of the trajectory, unless otherwise stated, the chord is taken horizontal.

It is important to note that although ranges are generally given in yards, the chords of trajectories (Chapter I) are expressed in FEET. Neglect of this frequently leads to failure in practical work.

Example.

To illustrate the use of the tables in calculating the elements of a trajectory, we will take the 100-ton Armstrong gun and consider figures 8′ and 14.

Data.

$V = 1833$; $\theta = 11° 50' = 11°.83\frac{1}{3}$; $W = 2005$ lbs.; $d^* = 17$ in.;

hence

$C = 0.14414$, $\log^{-1} = \bar{1}.15879$, $\text{co-log}^{-1} = 10.84121$.

This quantity must always be determined before any other work is attempted.

*The d above must be distinguished from the angle d elsewhere discussed.

Elements required.

1. The remaining energy at any point or : e.
2. Height of trajectory at any point or y.
3. Total range or X.
4. Angle of fall at end of range or ω.
5. The dangerous space, D.S., for any range.
6. Time of flight to any distance or t.
7. Time of flight for the whole range or T.
8. Inclination of the trajectory at any distance or ϕ.
9. Having the initial velocity to find the value of θ to attain a desired range.

etc. etc. etc.

1st. To find the remaining energy, we find the remaining velocity as follows :

From figure 8 the change of ϕ from the origin to the vertex is $\theta = \alpha = \delta$. At this point $\beta = 0$; therefore, from equation (33),

$$\tan \bar{\phi} = \frac{\tan \alpha + \tan 0}{2} = \frac{\tan 11^\circ\ 50'}{2}$$
$$= \tan 5^\circ\ 58'\ 50'' = \tan 5^\circ\ 58'.83.$$

As we shall have to use the logarithmic functions of $\bar{\phi}$, we now tabulate them as follows :

logs.	co-logs.
sin $\bar{1}.01783$	10.98317
cos $\bar{1}.99763$	$10.00237 = \log \sec \bar{\phi}$.

To find $\bar{v}'$ we project V on the horizontal or determine $u' = V \cos \theta$; thence $\bar{v}' = u' \sec \bar{\phi}$, or $u' = 1794$, $\bar{v}' = 1803.9$.

Now to find $\bar{v}''$ we transpose equation (I) to read

$$\delta_{\bar{v}''} = \delta_{\bar{v}'} - \frac{Cd}{\cos \bar{\phi}},$$

in which

$$\delta_{\bar{v}'} = \delta_{1803.9}.$$

In Table IV we find

$$\begin{array}{rr} \delta_{1803} = & 84^\circ.8199 \\ \text{p. p. for } 0.9 = & 21 \\ \hline & 84^\circ.8220 \end{array}$$

Consequently, all the quantities in the second member being known, we may write

$$\begin{aligned} \delta_{\bar{v}''} &= 84^\circ.8220 - 11^\circ.83\tfrac{1}{3} \frac{0.14414}{\log^{-1} \bar{1}.99763} \\ &= 84^\circ.8220 - \quad 1^\circ.7149 = 83^\circ.1071. \end{aligned}$$

From Table IV, again, we have for the velocity corresponding to $\delta = 83^\circ.1071$, $\bar{v}'' = 1318$ for the remaining velocity at the vertex, and hence

$$u'' = 1318 \cos \bar{\phi} = 1310.8.$$

The origin is now transferred to the vertex, and we treat the descending branch similarly to the ascending branch.

The angle α for this arc is evidently 0, and $u_{,} = u''$ just found, but the value of $\beta = \omega$ required to find $\underline{\phi}$ is unknown. It is therefore necessary to assume a value for it. Equation (29) shows that it will be greater than θ, and experience proves that it is nearly $\frac{4\theta}{3}$ or $\omega = 15^\circ.77$.

If we assume an incorrect value, as will generally be the case, the error is corrected by a subsequent operation. So let us assume an incorrect value, or $\omega = 16°$, as a first approximation. Thence

$$\tan \underline{\phi} = \frac{\tan 16°}{2} = \tan 8°\ 9'.5$$

and

$$\bar{v}_{,} = 1310.8 \times \sec 8°\ 9'.5 = 1324.1 \text{ and } \delta_{\bar{v}_{,}} = 83°.1406.$$

Using equation (I) again, we have

$$\begin{aligned} \delta_{\bar{v}_{,,}} &= 83°.1406 - \frac{0.14414}{\cos 8°\ 9'.5}\ 16° \\ &= 83°.1406 - 2°.3298 = 80°.8108; \end{aligned}$$

and $\bar{v}_{,,} = 1061$, $u_{,,} = 1050$, and $v_{,,}$, the velocity along the tangent, $= u_{,,}$ sec. $\omega = 1093$. The vertical component of v will be $= u_{,,} \tan \omega = 301$.

The component energies are generally useful for doing work against targets which are nearly vertical, as the walls of vessels or forts; or horizontal, as the decks of vessels or the roofs of magazines or casemates. We therefore find that while the projectile started with energy in the direction of the tangent or $E_{v'} = 46,700$ foot-tons, it now has $E_{v_{,,}} = 16,604$ foot-tons; only about one third as much as when it started.

Its component horizontal and vertical energies are 15,320 foot-tons and 1260 foot-tons, respectively.

The steps of the problem can be clearly followed in the first stages of Example I, which is given in the form used for written recitation.

1a.

Data: $V = 1833$; $\theta = 11° 50'$; $W = 2005$; $d = 17$ in.
Required $E_{\bar{v}''}$.

Statement of Steps.	Terms.	Quantities.	Functions.	Logs.
1. $C = \frac{d^2}{W} = 0.14414$	d^2	17^2		2.46090
	W	2005		3.30211
	C	0.14414		$\bar{1}$.15879
	$\frac{1}{C}$			10.84121
2. $\tan \bar{\phi} = \frac{\tan \theta}{2}$ $= \tan 5° 58'.83$	$\tan \theta$	$\tan 11° 50'$		9.32122
		2		.30103
	$\tan \bar{\phi}$	$\tan 5° 58'.83$		9.02019
	$\cos \bar{\phi}$	cos "		9.99763
	$\frac{1}{\cos \bar{\phi}}$	sec "		10.00237
	$\sin \bar{\phi}$	sin "		9.01782
	$\frac{1}{\sin \bar{\phi}}$	$\frac{1}{\sin}$ "		10.98317
3. $u' = V \cos \theta = 1794$	V	1833		3.26316
	$\cos \theta$	$\cos 11° 50'$		9.99067
	u'	1794		3.25383
4. $\bar{v}' = u' \sec \bar{\phi} = 1803.9$	$\sec \bar{\phi}$			10.00237
	$\bar{v}'$	1803.9		3.25620
5. $\delta_{\bar{v}''} = \delta_{\bar{v}'} - \frac{Cd}{\cos \bar{\phi}} = 1318$	$\delta_{\bar{v}'}$	$\delta_{1803.9}$	84.8220	
	C			$\bar{1}$.15879
	d	$11°.83\frac{1}{3}$		1.07300
	$\sec \bar{\phi}$			10.00237
			1.7146	0.23416
	$\delta_{\bar{v}''}$		83.1074	
	$\bar{v}''$	1318		

Statement of Steps.	Terms.	Quantities.	Functions.	Logs.
6. $u'' = u_{,} = \bar{v}'' \cos \bar{\phi}$				
$= 1310.8$	$\bar{v}''$	1318		3.11992
	$\cos \bar{\phi}$			9.99763
	$u'' = u_{,}$	1310.8		3.11755
7. $\tan \underline{\phi} = \frac{\tan \beta}{2} = 8° \; 9'.5$	$\tan \beta$	tan 16°		9.45750
				.30103
	$\tan \underline{\phi}$	tan 8° 9′.5		9.15647
	$\cos \underline{\phi}$	cos "		9.99558
	$\sec \underline{\phi}$	sec "		10.00442
	$\sin \underline{\phi}$	sin "		9.15201
	$\frac{1}{\sin \underline{\phi}}$	$\frac{1}{\sin}$ "		10.84799
8. $\bar{v}_1 = u \sec \underline{\phi} = 1324.1$	$u_{,}$	1310.8		3.11755
	$\sec \underline{\phi}$			10.00442
	$\bar{v}_{,}$	1324.2		3.12197
9. $\delta_{\bar{v}_{''}} = \delta_{\bar{v}_{'}} - Cd \sec \underline{\phi}$				
$= 1061$	$\delta_{\bar{v}'}$	$\delta_{1324.2}$	83.1411	
	C			$\bar{1}.15879$
	d	16°		1.20412
	$\sec \underline{\phi}$			10.00442
The determination of e is omitted.			2.3298	0.36733
	$\delta_{\bar{v}_{''}}$		80.8113	
	$\bar{v}_{''}$	1061		

1b.

Data as in 1a.

Required $\bar{v}$ at 1000 yards = 3000 feet.

STATEMENT OF STEPS.	Terms.	Quantities.	Functions.	Logs.
1. Determine whether to use $\bar{\phi}$ or ϕ by finding whether 3000 is < or > X' ∴	$\cos\bar{\phi}$			9.99763
	$\sigma_{\bar{v}'}$	$\sigma_{1803.9}$	44456.3	
	$\sigma_{\bar{v}''}$	σ_{1318}	42275.8	
$X' = \cos\bar{\phi}(\sigma_{\bar{v}'} - \sigma_{\bar{v}''}) \times \frac{1}{C}$			2180.5	3.33856
= 15045 ft. = 5015 yds.	$\frac{1}{C}$			10.84121
∴ use $\bar{\phi}$	X'	15045		4.17740
2. $\sigma_{\bar{v}''} = \sigma_{\bar{v}'} - Cx \sec\bar{\phi}$	$\sigma_{\bar{v}'}$	$\sigma_{1803.9}$	44456.3	
	C			$\bar{1}.15879$
$\bar{v}'' = 1697$	x	3000		3.47712
	$\sec\bar{\phi}$			10.00237
			434.8	2.63828
	$\sigma_{\bar{v}''}$		44021.5	
	$\bar{v}''$	1697		

2.

Data as in 1a.

Required Y' from data of ascending branch.

Statement of Steps.	Terms.	Quantities.	Functions.	Logs.
$y = Y'$				
$= \sin \overline{\phi}\,(\sigma_{\bar{v}'} - \sigma_{\bar{v}''})\,\frac{1}{C}$	$\sin \overline{\phi}$			9.01782
$= 1576.1$ feet.	$\sigma_{\bar{v}'}$	$\sigma_{1803.9}$	44456.3	
	$\sigma_{\bar{v}''}$	σ_{1318}	42275.8	
			2180.5	3.33856
	$\frac{1}{C}$			10.84121
	Y'	1576.1		3.19759

Similarly we may find from the value of $\underline{\phi}$ that

$$Y_{\prime} = 1713 \text{ feet.}$$

The difference, $1713 - 1576.1 = 136.9$ is evidently due to the error in our assumption of the value of ω, and therefore in our deduced value of $\underline{\phi}$; the effect is to increase the range as shown by figure 14.

This leads to the means of correcting ω to be explained.

3. To find the range.

With the assumed value of ω and $\underline{\phi}$ we find $X_{\prime}$ by the method described in 1b, or $X_{\prime} = 11949$ feet.

we also have $X' = 15045.4$ "

$$X = X' + X_{\prime} = 26994.4 \text{ "}$$

But this range is too great by the distance $B'C$, figure 14. To find this distance we assume that this short arc coincides with its tangent, which by assumption makes an angle of 16° with the horizon.

Therefore

$$B'C = \frac{136.9}{\tan 16°} = 477.4 \text{ ft. and } X_{,} = 11471.6 \text{ ft.}$$

$$\text{and } X = 26517.0 \text{ ft.} = 8839 \text{ yds.} = 5 \text{ miles} +.$$

4. To find the inclination at the end of the range or the angle of fall, ω.

We can vouch for only the elements of the trajectory in the ascending branch, but if we can determine the range as by firing or by the method just described, we may approximate closely to the angle of fall.

For

$$\tan \underline{\phi} = \frac{Y}{X_{,}} = \frac{\tan \omega}{2};$$

$$\therefore \tan \omega = \frac{2Y}{X_{,}} \text{ and } \omega = 15° \; 20' \; 28''.$$

In the example this gives a difference of but 6.8 feet in the two values of Y, which difference can be further reduced to 0 by successively approximating to the true value of ω, or $\omega = 15° \; 18' \; 12''$; and therefore $\underline{\phi} = 7° \; 47' \; \frac{4}{9}\frac{1}{4}$ and $\overline{v}_{,} = 1323$, $\overline{v}_{,,} = 1067.3$, $X_{,} = 3840$ yds.

These values will be hereafter employed, since it is most important to have a correct knowledge of the elements of the trajectory at its further end.

Practical Methods for Determining ω.

1. Fire through a screen near the point of fall, and note the height h, of the hole above the horizontal plane on which the projectile strikes, and the distance of its impact, d, beyond the screen. Then $\tan \omega = \dfrac{h}{d}$ nearly.

Or note the inclination of the shot-holes in snow or in horizontal targets composed of double layers of boards.

2. Determine the range OR, figure 15, for a given value of θ, and then increase θ by a slight increment $\Delta\theta$. This will increase OR by $\Delta R = RR'$.

Then assuming HR' to be straight and parallel to the tangent at R,

$$\tan \omega = \frac{HR}{RR'} = \frac{R \tan \Delta\theta}{\Delta R} \text{ nearly.}$$

3. Determine the range under two sets of conditions differing only in the height, h, of the gun above the horizontal plane. Then if this difference be relatively small with regard to the range from figure 16, $\tan \omega = \dfrac{h}{\Delta R}$.

4. Figure 17 shows how this would practically be done, since it would be difficult to raise a gun sufficiently without displacing it horizontally:

$$\tan \omega = \frac{h}{O'\,R' - O\,R.}$$

Application.

Range-tables are constructed to give all the principal elements of the piece, charge, and trajectory for different ranges.*

The following method and figure 18 show how, having a range-table, we may determine the co-ordinates of the vertex.

Find in the range-table two angles of departure and of fall α and β, such that their sum $= \theta$. Then by the principle of rigidity S will be the chord to the vertex, and $S \cos \beta = X'$, and $S \sin \beta = Y'$.

* See Chapter XXX, pages 9, 52.

5. To find the dangerous space at any range, or the horizontal distance over which a target of given height would be struck.

We have in this case to find the distance at which the height of the trajectory is equal to that of the target. The target will evidently be struck when situated at this point, since the trajectory passes through its summit, and it will also be struck if situated at any point intermediate between this and the end of the range. Hence if D, figure 8, be the target the dangerous space is DR.

The simplest way of determining this is as follows. Suppose the target to be 30 feet high, then from (III″)

$$\sigma_{\bar{v}_{\prime}} = \sigma_{1067.3} + \frac{C30}{\sin \underline{\phi}} = 40626.4 + 31.9 = 40658.3 = \sigma_{1070.9};$$

$$\text{or } \bar{v}_{\prime} = 1070.9.$$

Similarly

$$x = DS = 219.2 \text{ ft.} = 73 \text{ yds.}$$

If the proper value of ω has been found, the same result may be obtained by working downward from the vertex, taking $y = 1576.1 - 30 = 1546.1$, and

$$\sigma_{\bar{v}_{\prime\prime}} = \sigma_{\bar{v}_{\prime}} - \frac{Cy}{\sin \underline{\phi}} = \sigma_{1070.9}, \text{ as before.*}$$

The accordance of these methods tests the accuracy of the determination of ω; but without exacting the somewhat laborious process required for this determination, a check of the accuracy with which the dangerous space has been determined may be had by observing that the angle whose tangent is equal to the height of the target divided by the dangerous space is greater than $\underline{\phi}$ and less than ω.

* Or taking the trial values assumed for the *descending* branch, viz, $Y_{\prime} = 1713$; $\bar{v}' = 1324.2$; $\bar{v}_{\prime\prime} = 1061$ we have as an approximation $y = 1683$; $\bar{v}'_{\prime\prime} = 1064$ ∴ $x = 202.6$ ft.

The dangerous space is one of the most important properties of a trajectory, since, Chap. I, it measures the chances of striking an object at a distance which in warfare is only approximately known.

The flatter the trajectory at its further end the greater is the permissible margin of error in estimating the range before aiming.

The principles of Chap. XVI and equation (30) illustrate the importance of high velocities and high sectional densities, since if one projectile, *a*, figure 19, having less sectional density than another, projectile *b*, be projected with equal energies at the same ranges, although the trajectory of *a* may be flatter than that of *b* at the start, yet near the target the D.S. of *b* will be greater than that of *a*, if the target lies beyond the intersection of the two trajectories.

Although the method above described is generally followed, and is best suited to cases where ω is accurately known, a simpler and probably a more accurate plan is hereafter given, page 44.

6. To find the time of flight to any distance. Take the distance as 1000 yards = 3000 feet, as in 1*b*. We have from equation (II) and data previously computed

$$t = (\tau_{\bar{v}'} - \tau_{\bar{v}''})\frac{1}{C} = (\tau_{1803.9} - \tau_{1697})\frac{1}{C} = 1.7302 \text{ sec.}$$

7. To find the time of flight for the whole range.

1st. We proceed as in No. 3, using equation II and the corrected value of $\bar{v}_{,,} = 1067.3$.

$$T' = (\tau_{\bar{v}'} - \tau_{\bar{v}''})\frac{1}{C} = 9.8432 \text{ secs,}$$

and
$$T_{,} = (\tau_{\bar{v}_{,}} - \tau_{\bar{v}_{,,}})\frac{1}{C} = \underline{9.8569}$$

$$T = T' + T_{,} \qquad = 19.7001.$$

2d. Or we may pass directly to the point of fall, as follows:

$$T = (\tau_{\bar{v}'} - \tau_{\bar{v}_{,,}}) \frac{1}{C} = 19.566,$$

which is sufficiently accurate for most purposes.

3d. If the true value of ω or $\bar{v}_{,,}$ is not determined, we may still approximate to $T_{,}$ by finding the time t_c required for the projectile to pass over the correction of the range determined in No. 3, with the velocity $u_{,,}$ or $t_c = \frac{477}{1050} = 0.454$ sec.

Therefore having with the assumed value of $\omega = 16°$ found $T_{,} = 10.266$, its corrected value is 9.812, which added to T' makes $T = 19.6552$ secs.

4th. Or, if we neglect the difference in time of passage over $(Y_{,} - Y')$, due to the resistance of the air, since $t_c = \sqrt{\frac{2}{g}}(\sqrt{Y_{,}} - \sqrt{Y'})$, we obtain $t_c = 0.4186$ and $T_{,} = 9.8474$ which is a closer approximation than 9.812, since $T_{,} > T'$.

Scholium.

Equation (23), which may be written

$$dt = \frac{vd\phi}{\cos\phi} \frac{1}{g}$$

or

$$T' = \frac{1}{g}\int_0^\theta \frac{vd\phi}{\cos\phi} \quad \text{and} \quad T_{,} = \frac{1}{g}\int_\omega^0 \frac{vd\phi}{\cos\phi},$$

shows that, although for the descending branch the mean value of v is less than that for the ascending branch, the increase in the value of ϕ shown by equation (29), and the consequent decrease in $\cos\phi$, may compensate and keep the ra-

tio nearly constant; so that as far as *time* only is concerned the trajectory may be supposed to be *in vacuo.*

That this is practically so appears from the equality of T' and $T_{\prime}$ in the above problem and in those solved by other methods.

Consequently, and particularly for small values of $\triangle$, when the vertical component of the velocity is so small that it may be safely neglected, the time to the vertex may be safely taken as half the whole time of flight, and in cases of necessity Equations (6) and (7) may be employed.

For example, for this case, which is certainly an extreme one, if we substitute the value of $T = 19.70$ sec. in the equation $Y = \frac{gT^2}{8}$ we obtain for Y a value 1561, which is only 15.2 feet less than that before deduced. When the value of $\triangle$ is large, the equations of the trajectory *in vacuo* cannot be indiscriminately applied.

Principle of the Vertex.

From the above follows this important conclusion: If we represent the time to the vertex by $t_{\wedge}$ (read *t vertex*), the velocity at the vertex by $v_{\wedge}$, and the corresponding time function by $\tau_{\wedge}$, then $t_{\wedge} = \frac{T}{2}$.

Then we have from equation (II), generalized as to notation,

$$Ct_{\wedge} = \frac{CT}{2} = \frac{\tau_{v'} - \tau_{v_{\prime\prime}}}{2} = \tau_{v}' - \tau_{\wedge},$$

and

$$\tau_{\wedge} = \frac{\tau_{v'} + \tau_{v_{\prime\prime}}}{2}.$$

Or, *the time function of the velocity at the vertex is equal to the arithmetical mean of the time functions of the velocities at each end of the arc.*

This, which may be termed the *principle of the vertex*, is of great value in approximate solutions.

If we know the time interval t corresponding to two velocities, of which one is known, then the time function of the vertex of any arc may be determined as follows, from the above and Equation (II):

$$\tau_{\wedge} = \tau_{v'} - \frac{Ct}{2} = \tau_{v''} + \frac{Ct}{2}. \qquad (37)$$

8. To find the inclination at the top of the target, which we will now assume to be a rampart 30 ft. high, so that what was before the dangerous space will be the *safe* space.

From equation (I), with the corrected values given, page 37, we have

$$d = \cos \underline{\phi} \left(\delta_{\bar{v}_{,}} - \delta_{\bar{v}_{,,}}\right)\frac{1}{C} = -0^{\circ}.35674 = -0^{\circ}\ 21'\ 24''.*$$

therefore $\phi = \omega - d = 15^{\circ}\,18'\,12'' - 0^{\circ}\,21'\,24'' = 14^{\circ}\,56'\,48''$.

Or, working down from the vertex, $\phi = 14^{\circ}\ 57'$. The true safe space will, owing to the increasing curvature, be

somewhat less than $\dfrac{30}{\tan 14^{\circ}\ 57'} = 112$ ft.

The difference between this result and that before reached for the dangerous space shows the limitations of the ordinary method, and is probably due to not having found the correct value of $\underline{\phi}$ for the function δ, as explained page 25 and in the Appendix.

$$* = \cos 7^{\circ}\ 47'\ \frac{41}{94}\left(\delta_{1070.9} - \delta_{1067.3}\right)\frac{1}{C}.$$

A closer approximation to the dangerous space would probably be found from the principle of the vertex, as follows:

Assuming the rigidity of the trajectory, the tangent at the vertex of any elementary arc is parallel to the chord. So that, finding the inclination $\phi_\wedge$, at the vertex of the arc in rear of the target the dangerous space may be found, since

$$\text{D.S.} = \text{height of target} \times \cot \phi_\wedge .$$

By using the corrected values pages 37 and 39,

$$\tau_\wedge = \frac{\tau_{1070.9} + \tau_{1067.3}}{2},$$

we find

$$v_\wedge = 1069.1, \quad \delta_{1069.1} - \delta_{1067.3} = 0^\circ.0260,$$

whence

$$d = 0^\circ.1787 \text{ and } \phi_\wedge = 15.30\tfrac{1}{3} - 0.1787 = 15^\circ\ 7', 47.$$
$$\text{D.S.} = 111 \text{ feet.}$$

This method enables us to obtain the dangerous space quite closely for an approximate value of $\overline{v}_{\prime\prime}$, and to determine an important element without requiring the tedious correction mentioned, page 37.

Assuming then $\overline{v}_{\prime\prime} = 1061$, the velocity along the tangent is, since $u_{\prime\prime} = \overline{v}_{\prime\prime} \cos \underline{\phi} = v_{\prime\prime} \cos \omega$.

$$v_{\prime\prime} = \frac{\overline{v}_{\prime\prime} \cos \underline{\phi}}{\cos \omega} = 1093,$$

the vertical component of which is $v_{\prime\prime} \sin \omega = 1093 \sin 16^\circ = 301$.

The time of passage over the height of 30 ft. with this velocity will be $t = 0.09961$ sec., though it will actually be a trifle less, and $\frac{Ct}{2} = 0.0071$.

Now since $\tau_{\wedge} = \tau_{v''} + \frac{Ct}{2} = \tau_{1061} + 0.0071 = 230.2330,$

$$\therefore v_{\wedge} = 1061.8.$$

Also, since $d = \cos \phi\, (\delta_{1061.8} - \delta_{1061}) \frac{1}{C} = 0.0845,$

$$\phi_{\wedge} = 16° - 0.0845 = 15°\ 54'.9 \text{ and } \frac{30}{\tan \phi} = 105 \text{ feet.}$$

This is much nearer the true value than the result given by the method described page 39.

Very nearly the result arrived at by the method above described, viz., 105 feet, would be obtained by taking for the time of passage $t = \sqrt{\frac{2}{g}} \left[\sqrt{1576.2} - \sqrt{1546.2} \right] = 0.0947$ sec.

9. Having the initial velocity and the value of C, to find the angle of departure necessary to attain a given range, and other elements.

The conditions of this problem, which is a frequent one in practice, require (page 6) that the rigidity of the trajectory be assumed and that the principle of the vertex be applied.

Solution.

1. The piece is supposed to be fired with its axis horizontal, and we compute the elements of the trajectory as if it were the descending branch of an imaginary trajectory.

Then we revolve the trajectory upward until the chord becomes horizontal. By the principle of rigidity S is taken $= X$, which, to test the accuracy of the method, we take $= 26517.24$ feet, as previously determined.

From equation (III) we have

$$\sigma_{v_{\prime\prime}} = \sigma_{v}' - Cx, \text{ or } v_{\prime\prime} = 1081.$$

From equation (II)

$$\tau_{\wedge} = \tfrac{1}{2}(\tau_{1833} + \tau_{1081}) \text{ or } v_{\wedge} = 1348.$$

Then from equation (I), since $\theta = 0$ and $\cos \bar{\phi} = 1$,

$$d = \frac{\delta_{v'} - \delta_{\wedge}}{C} = 11^\circ\ 26'\ 13'' = \theta.$$

Compare these results with those previously deduced.

2. In such a case, to determine the angle of fall and the dangerous space, we would proceed as follows:

Find D, in *degrees*, the total change $= \theta + \omega$, by saying

$$D = (\delta_{1833} - \delta_{1081})\frac{1}{C} = 26^\circ.29 = 26^\circ\ 17'\ 24'';$$

then $\omega = D - \theta = 14^\circ\ 51'.2$ and

$$DS_{30} = \frac{30}{\tan 14^\circ 51'} = 113 \text{ feet.}$$

It is evident from the above, that, knowing the angle of fall required to strike near its foot a scarp protected by a cover at a known height and separated from it by a ditch of known width, it is only necessary to know the distance of the breaching battery from the wall, and the ballistic coefficient of the projectile, to determine approximately the value of θ and of the initial velocity or charge of powder required to strike the wall at nearly the desired spot, with a required remaining energy.

It was by some such method that the German artillery breached at hitherto unknown ranges the invisible walls of Strasburg. See problem page 51.

Thus, by the principle of rigidity,

From $CX = \sigma_V - \sigma_v$ determine V and weight of charge.
" $CD = \delta_V - \delta_v$ " D.
" conditions " ω.
" $D - \omega$ " θ.

MODIFIED FORMULÆ.

For low angles of departure and high velocities and sectional densities giving small values of Δ, the principle of rigidity permits the formulæ on page 28 to be written

$$Cd = \delta_V - \delta_v \,, \tag{A}$$
$$Ct = \tau_V - \tau_v \,; \tag{B}$$
$$Cx = \sigma_V - \sigma_v . \tag{C}$$

In these formulæ, since $\sin \overline{\phi} = 0$, we must resort to Equations (6) and (7) as explained page 7, or

$$y = \frac{gt}{2}(T - t) \,; \tag{6}$$

$$Y = \frac{gT^2}{8} \tag{7}$$

The propriety of this assumption appears from applying it to the case of the 100-ton gun, assuming the velocities to be horizontal and solving without reference to the vertex. Thus, assuming as before $\omega = 16°$, we have from (A), using whole numbers, $27°.83 = (\delta_{1804} - \delta_{u_{,,}}) \frac{1}{C} \therefore u_{,,} = 1061$.

Similarly we find $T = 19.94$ sec. and $X = 8983$ yards, leading, as seen by comparison, to but slight errors *providing that* ω has been correctly assumed.

So that we may have confidence in the results obtained by the use of Equations (A), (B), (C), when v does not differ much from u, and, when $v \sin \phi$ is so small that it may be neglected, we may use Equations (6) and (7).

Example.

The Springfield rifle and ammunition give the following data:

$$W = 500 \text{ gr.} = 0.071428 \text{ lbs.};$$
Diameter of projectile = 0.455 inches in flight
$$V = 1300 \text{ ft.}$$

By experiment we find that for a range of 500 yards θ as measured by the breech sight $= 1° \, 17' \, 18''$.

1. Find ω.
   1st. From (C) determine $v_x = 869$
   2d. " (A) " $D = 2° \, 40' \, 23''$.
   3d. " $D - \theta$ " $\omega = 1° \, 23' \, 05''$.
2. Find Y.
   1st. From (B) determine $T = 1.467$ sec.
   2d. " (7) " $Y = 8'.662$ feet.
3. Find y at 400 yards = 1200 ft.
   1st. From (C) determine v at 1200 ft. = 921.
   2d. " (B) " t to 1200 ft. = 1.132 sec.
   3d. " (6) and T above determine y = 6.105 ft.

By this means we may construct a drawing of the trajectory.

4 Find the Dangerous Space at 500 yards.

The target is a man 5 ft. 8 in. ($5\frac{2}{3}$ ft.) high $= y$. The gun is supposed to be fired lying down (from the ground) and to be aimed at the feet of the man.

1st. Reckoning from the summit of the trajectory we have

$$t_, = \sqrt{\frac{2(Y-y)}{g}} = 0.4313$$ for the time from the vertex to the top of the man's head, and $\frac{T}{2} - t_, =$ time over D. S. $= 0.3022 = t'$.

2d From t' and (B) determine v at target = 915.

3d. From (C) determine D. S. = 267 ft. = 89 yds.

It is generally better to work backward from the point of fall than forward from the gun, as the results are more consistent if the data are supplied from only one branch of the trajectory. See page 39.

However, this does not apply in the above case, in which the vertical resistance of the air is wholly neglected, so that the same results would follow from either course of procedure. See page 41.

Alternate Solution.

If in Equation (6) we supply the value of T previously deduced, and solve the resulting quadratic equation, we shall have two values of t, one of which gives the time for the projectile to rise to the height of y, and the other which gives the time for the projectile to rise to the vertex and to fall to this height above the horizontal plane, so that there will be two dangerous spaces, the interval between them being the *safe space*.

It is evident that as the range decreases, the other conditions remaining constant, the safe space finally becomes 0. The resulting dangerous space will then be continuous and a maximum.

The *maximum dangerous space* for a given small-arm thus depends upon a physical constant,—the height of a man; and assuming, as above, the mean height of a man to be $5\frac{2}{3}$ feet, the maximum dangerous space will be a function of ρ', page 23, and will be a convenient measure of the joint power of the gun and ammunition. The height Y will $= 5\frac{2}{3}$ feet.

5. Find the maximum Dangerous Space for the preceding ballistic conditions.

1st. From (7) determine $T = \overset{\text{Sec.}}{1.187}$.
2d. " (B) " $v = 912$
3d. " (C) " $X = 416.6 +$ yds. $=$ max. D. S.

APPENDIX A.

The value of $\bar{\phi}$ was obtained approximately by Mr. Niven from an expanding series. (See Proceedings of the Royal Society of England, 1887; No. 181.)

The value of $\bar{\phi}$ to be used with Table IV for changes of inclination is that given in the text for high angles of departure, say $\theta > 5°$.

For $\theta < 5°$ $\bar{\phi}$ may be taken $= \frac{\alpha + \beta}{2}$.

For the other tables he takes

$$\bar{\phi} = \frac{1}{3} \frac{u' - u''}{u' + u''} (\alpha - \beta);$$

though, where greater approximation is required, for changes of time he uses

$$\bar{\phi} = \frac{1}{6} \frac{u' - u''}{u' + u''} (\alpha - \beta).$$

APPENDIX B.

Problems.

The answers given below result from the use of the modified formulæ.

1. The 3.2-inch steel b. l. rifle. Weight of shell or shrapnel = 13 lbs. I. V. = 1634.

Determine: (1) The distance at which v of shrapnel will be 500.
(2) Time of flight for this distance.
(3) Angle of departure for this range, supposing the shrapnel to explode 40 ft. above the object, or the angle of sight = 7′.

Answers: (1) 19,106 ft. or 3.62 miles.
(2) 24.6 sec.
(3) 24° 44′.

2. A target is to be placed on Cro' Nest. The distance from the sea-coast battery to target is 1990 yards; height of target above battery is 237 feet. Determine the angle of departure necessary to strike the target, using the 8-inch converted rifle.

$d =$ 7.95 inches;
Weight of projectile = 180 lbs.;
I. V = 1414. Answer: 5° 45′.

3. The 6-inch b. l. rifle requires according to the range table an elevation of 1° 51′ and a muzzle velocity of 1850 f. s. to strike an object at a distance of 2000 yards. On firing the range obtained was only 1800 yards, and investigation showed that the powder was damp.* What additional elevation would be necessary for a range of 2000 yards? $W =$ 100 lbs.

Answer: 0° 28′.

4. At the siege of Strasbourg in 1870, the Germans wished to breach the scarp wall of an outwork at 2000 yards distance; the ditch was known to be 50 feet wide, and the shell were to strike 12½ feet below top of counterscarp wall. An 8-inch howitzer firing a projectile weighing 180 lbs. with a muzzle velocity of 700 f. s. was employed.

Required the striking velocity and the angle of departure

Answer: $\begin{cases} 616 \text{ f. s.} \\ 11° 47′. \end{cases}$

5. At a range of 1200 yards a 64-lb. shell grazes the top of a traverse 8 feet high. How far beyond the traverse will the shot strike the ground?

$d =$ 6.171 inches;
Weight of projectile = 64 lbs.;
I. V. = 1260 f. s.

Answer: 153 feet or 51 yards.

6. A Martini-Henry rifle-bullet strikes a vertical target at 500 yards at a certain spot when the muzzle velocity is 1353 f. s. How much lower on the target will the same projectile

*See proportion, foot p. 7.

strike if the muzzle velocity is only 1300 f. s., the elevation and other conditions remaining the same?

$d = 0.45$ inch.

Weight of projectile = 480 grains = 0.06857 lb.

Answer: $21\frac{1}{4}$ inches.

7. Using the Hebler rifle, determine the maximum continuous dangerous space for a man kneeling.

$d = 0.296$ inch;

$w = 225$ grains $= 0.03214$ lb.;

I. V. = 1942 f. s.;

Height of a man kneeling = 42 inches.

Compare with Springfield rifle:

$d = 0.45$ inch;

$w = 500$ grains $= 0.07142$ lb.;

I. V. = 1316 f. s.

Answer: Hebler rifle, 458.0 yards.
Springfield rifle, 340.7 "

8. A 3-inch Eureka shell, weight 9 lbs., fired with 2 lbs. of powder, has an I. V. = 1495 f. s. With what charge should a 10-lb. shell be fired to have at 407 yards the same remaining velocity that the full charge gives at 2500 yards?

Answer: 11.5 ounces.

9. A 3.2-inch shell weighing 13 lbs. is fired with a muzzle velocity = 958 f. s. The target is at a distance of 407 yards, and the angle of sight is $4° 1'$. Determine the necessary breech-sight elevation and the quadrant elevation.

Answer: $e = 1° \ 19'$.
$q = 5° \ 20'$.

10. A 3.2-inch shell weighing 13 lbs. is fired with I. V. = 986 f. s. How high above the gun should be placed a horizontal bar at a distance of 80 feet, so that the shell shall strike the bar and hit a target on the same level as the gun, and at a distance of 1200 yards. Determine also the necessary breech-sight elevation.

Answer: Height = 4 ft. 6.5 ins.
$e = 4° \ 0' \ 22''$.

BALLISTIC TABLES.

Table I.

Value of K for the Cubic Law of Resistance, Ogival-headed Projectiles ($1\frac{1}{2}$ diameter heads).

Velocity.	*Value of* K.	*Velocity.*	*Value of* K.	*Velocity.*	*Value of* K.	*Velocity.*	*Value of* K.
f. s.		f. s.		f. s.		f. s.	
400	148·0	880	75·0	1360	106·7	1840	75·2
410	145·2	890	75·0	1370	106·3	1850	74·7
420	142·5	900	75·0	1380	105·8	1860	74·2
430	139·8	910	75·0	1390	105·3	1870	73·6
440	137·2	920	75·0	1400	104·7	1880	73·1
450	134·6	930	75·0	1410	104·1	1890	72·6
460	132·0	940	75·0	1420	103·5	1900	72·1
470	129·4	950	75·0	1430	102·9	1910	71·6
480	126·9	960	75·0	1440	102·3	1920	71·2
490	124·4	970	75·0	1450	101·6	1930	70·8
500	121·9	980	75·0	1460	100·9	1940	70·4
510	119·6	990	75·0	1470	100·1	1950	70·0
520	117·3	1000	75·0	1480	99·4	1960	69·7
530	115·0	1010	75·1	1490	98·6	1970	69·4
540	112·8	1020	75·3	1500	97·9	1980	69·2
550	110·7	1030	76·7	1510	97·1	1990	69·0
560	108·7	1040	80·8	1520	96·2	2000	68·8
570	106·7	1050	87·3	1530	95·3	2010	68·6
580	104·6	1060	94·0	1540	94·4	2020	68·4
590	102·5	1070	98·7	1550	93·6	2030	68·3
600	100·5	1080	102·2	1560	92·8	2040	68·2
610	98·6	1090	104·9	1570	92·0	2050	68·1
620	96·8	1100	106·9	1580	91·2	2060	68·0
630	95·1	1110	108·4	1590	90·4	2070	67·9
640	93·5	1120	109·2	1600	89·7	2080	67·9
650	91·9	1130	109·6	1610	89·0	2090	67·8
660	90·5	1140	109·6	1620	88·3	2100 ...	67·8
670	89·1	1150	109·6	1630	87·6	2110	67·7
680	87·7	1160 .. .	109·6	1640	86·9	2120	67·6
690	86·3	1170	109·6	1650	86·2	2130	67·6
700	84·9	1180	109·6	1660	85·5	2140	67·5
710	83·7	1190	109·6	1670	84·8	2150	67·4
720	82·6	1200	109·6	1680	84·2	2160	67·3
730	81·6	1210	109·6	1690	83·6	2170	67·2
740	80·6	1220	109·6	1700	83·0	2180	67·2
750	79·6	1230	109·5	1710	82·4	2190	67·1
760	78·7	1240	109·5	1720	81·8	2200	67·0
770	78·0	1250	109·4	1730	81·2	2210	66·9
780	77·4	1260	109·3	1740	80·6	2220	66·8
790	76·8	1270	109·2	1750	80·0	2230	66·8
800	76·2	1280	109·0	1760	79·5	2240	66·7
810	75·6	1290	108·8	1770	78·9	2250	66·6
820	75·2	1300	108·6	1780	78·4	2260	66·5
830	75·1	1310	108·4	1790	77·8	2270 ...	66·4
840	75·0	1320	108·1	1800	77·3	2280	66·2
850	75·0	1330	107·8	1810	76·8	2290 ...	65·9
860	75·0	1340	107·5	1820	76·2	2300	65·5
870	75·0	1350	107·1	1830	75·7		

Table II.

Time and Velocity Table, $Ct = \tau_{v'} - \tau_{v''}$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
40	20 5·0299	5·1349	5·2393	5·3432	5·4466	5·5494	5·6517	5·7534	5·8546	5·9553	·1028
41	6·0554	6·1550	6·2540	6·3525	6·4505	6·5480	6·6450	6·7414	6·8373	6·9327	·0975
42	7·0276	7·1220	7·2159	7·3093	7·4022	7·4947	7·5867	7·6782	7·7693	7·8599	·0925
43	20 7·9501	8·0398	8·1291	8·2179	8·3063	8·3942	8·4817	8·5687	8·6553	8·7415	·0879
44	8·8272	8·9125	8·9974	9·0819	9·1660	9·2497	9·3330	9·4159	9·4984	9·5805	·0837
45	9·6622	9·7435	9·8244	9·9050	9·9852	*0·0651	*0·1446	*0·2237	*0·3025	*0·3809	·0799
46	21 0·4590	0·5367	0·6140	0·6910	0·7677	0·8440	0·9200	0·9956	1·0709	1·1459	·0763
47	1·2205	1.2948	1·3687	1·4423	1·5156	1·5886	1·6613	1·7336	1·8056	1·8773	·0730
48	1·9487	2·0198	2·0906	2·1611	2·2313	2·3012	2·3708	2·4401	2·5091	2·5779	·0699
49	21 2·6464	2·7146	2·7825	2·8501	2·9174	2·9845	3·0513	3·1178	3·1841	3·2501	·0671
50	3·3159	3·3814	3·4466	3·5116	3·5763	3·6408	3·7050	3·7689	3 8326	3·8960	·0645
51	3·9592	4·0221	4·0848	4·1472	4·2094	4·2713	4·3330	4·3944	4·4556	4·5165	·0619
52	21 4·5772	4·6377	4·6979	4·7579	4·8177	4·8773	4·9367	4·9958	5·0547	5·1134	·0596
53	5·1719	5·2302	5·2882	5·3460	5·4036	5·4610	5·5182	5·5752	5·6320	5·6886	·0574
54	5·7450	5·8012	5·8572	5·9130	5·9686	6·0240	6·0792	6·1342	6·1890	6·2436	·0554
55	21 6·2980	6·3522	6·4062	6·4600	6·5136	6·5670	6·6202	6·6732	6·7260	6·7786	·0534
56	6·8311	6·8834	6·9355	6·9874	7·0391	7·0907	7·1421	7·1933	7·2444	7·2953	·0516
57	7·3460	7·3965	7·4469	7·4971	7·5471	7·5970	7·6467	7·6962	7·7456	7·7948	·0499
58	21 7·8438	7·8928	7·9417	7·9904	8·0389	8·0873	8·1356	8·1837	8·2316	8·2793	·0483
59	8·3271	8·3746	8·4220	8·4692	8·5163	8·5632	8 6100	8·6566	8·7031	8·7494	·0468
60	8 7957	8·8417	8·8877	8·9334	8·9791	9·0246	9·0700	9·1152	9·1603	9·2052	·0454
61	21 9·2501	9·2947	9·3393	9·3837	9·4280	9·4721	9·5161	9·5600	9·6037	9·6473	·0441
62	9·6908	9·7341	9·7773	9·8204	9·8633	9·9062	9·9489	9·9914	*0·0338	*0·0761	·0428
63	22 0·1183	0·1604	0·2023	0·2441	0·2858	0·3273	0·3687	0·4100	0·4512	0·4922	·0415
64	22 0·5332	0·5740	0·6147	0·6552	0·6957	0·7360	0·7762	0·8163	0·8563	0·8962	·0403
65	0·9359	0·9755	1·0151	1·0544	1·0937	1·1328	1·1718	1·2107	1·2495	1·2881	·0391
66	1·3267	1·3651	1·4034	1·4416	1·4797	1·5177	1·5555	1·5933	1·6309	1·6684	·0379
67	22 1·7059	1·7432	1·7804	1·8175	1·8545	1·8914	1·9281	1·9648	2·0014	2·0378	·0368
68	2·0742	2·1105	2·1466	2·1827	2·2186	2·2545	2·2902	2·3259	2·3614	2·3969	·0358
69	2·4322	2·4675	2·5027	2·5377	2·5727	2·6076	2·6424	2·6771	2·7117	2·7462	·0348
70	22 2·7806	2·8150	2·8492	2·8833	2·9174	2·9513	2·9852	3·0189	3·0526	3·0862	·0339
71	3·1196	3·1530	3·1863	3·2195	3·2526	3·2856	3·3185	3·3513	3·3840	3·4167	·0330
72	3·4492	3·4816	3·5140	3·5462	3·5784	3·6105	3·6424	3·6743	3·7061	3·7378	·0320
73	22 3 7694	3·8009	3·8323	3·8636	3·8949	3·9260	3·9571	3·9881	4·0189	4·0497	·0311
74	4·0804	4·1110	4·1416	4·1720	4·2024	4·2326	4·2628	4·2929	4·3230	4·3529	·0302
75	4·3828	4·4125	4·4422	4·4719	4·5014	4·5308	4·5602	4·5895	4 6187	4·6478	·0294
76	22 4·6769	4·7058	4·7347	4·7635	4·7922	4·8208	4·8493	4·8777	4·9060	4·9343	·0286
77	4·9624	4·9905	5·0185	5·0464	5·0742	5·1020	5·1296	5·1572	5·1847	5·2121	·0277
78	5·2394	5·2666	5·2937	5·3208	5·3478	5·3747	5·4015	5·4282	5·4549	5·4814	·0268
79	22 5·5079	5·5343	5·5606	5·5869	5·6130	5·6391	5·6652	5·6911	5·7170	5·7428	·0261
80	5·7685	5·7941	5·8197	5·8452	5·8706	5·8959	5·9212	5·9463	5·9714	5·9965	·0253
81	6·0214	6·0463	6·0711	6·0959	6·1205	6·1451	6·1696	6·1941	6·2184	6·2427	·0245
82	22 6·2669	6·2910	6·3151	6·3390	6·3629	6·3867	6·4104	6·4340	6·4576	6·4810	0237
83	6·5044	6·5277	6·5509	6·5740	6·5971	6·6201	6·6430	6·6658	6·6885	6·7111	·0229
84	6·7337	6·7562	6·7786	6·8009	6·8232	6·8454	6·8675	6·8895	6·9114	6·9333	·0221
85	22 6·9551	6·9768	6·9984	7·0200	7·0415	7·0629	7·0842	7·1055	7·1267	7·1478	·0214
86	7·1688	7·1898	7·2107	7·2315	7·2522	7·2729	7·2935	7·3140	7·3345	7·3549	·0206
87	7·3752	7·3954	7·4156	7·4357	7·4558	7·4757	7·4956	7·5155	7·5353	7·5550	·0199

TABLE II.—*Continued.*

Time and Velocity Table, $Ct = \tau_{v'} - \tau_{v''}$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
88	22 7·5746	7·5942	7·6137	7·6332	7·6526	7·6719	7·6912	7·7104	7·7295	7 7486	·0193
89	7·7677	7·7866	7·8055	7·8244	7·8431	7·8618	7·8805	7·8991	7·9176	7 9360	·0187
90	7·9544	7·9727	7·9909	8·0091	8·0272	8·0452	8·0632	8·0812	8·0990	8·1168	·0180
91	22 8·1346	8·1523	8·1699	8·1875	8·2050	8·2225	8·2399	8·2573	8·2746	8·2918	·0174
92	8·3090	8·3261	8·3432	8·3602	8·3772	8·3941	8·4109	8·4277	8·4445	8·4611	·0169
93	8·4778	8·4943	8·5109	8·5273	8·5437	8·5601	8·5764	8·5927	8·6089	8·6250	·0163
94	22 8·6411	8·6572	8·6732	8·6892	8·7051	8·7209	8·7367	8·7525	8·7682	8·7838	·0158
95	8·7994	8·8150	8·8305	8·8459	8·8613	8·8767	8·8920	8·9073	8·9225	8·9376	·0153
96	8·9528	8·9678	8·9828	8·9978	9·0128	9·0276	9·0425	9·0573	9·0720	9·0867	·0149
97	22 9·1014	9·1160	9·1306	9·1451	9·1595	9·1740	9·1884	9·2027	9·2170	9·2312	·0144
98	9·2454	9·2596	9·2737	9·2878	9·3018	9·3158	9·3298	9·3437	9·3575	9·3713	·0140
99	9·3851	9·3989	9·4126	9·4262	9·4398	9·4534	9·4670	9·4805	9·4939	9·5073	·0136
100	22 9·5207	9·5340	9·5473	9·5606	9·5738	9·5869	9·6001	9·6132	9·6262	9·6392	·0132
101	9·6522	9·6651	9·6780	9·6908	9·7036	9·7164	9·7291	9·7418	9·7544	9·7670	·0127
102	9·7796	9·7921	9·8046	9·8170	9·8294	9·8417	9·8540	9·8662	9·8783	9·8904	·0123
103	22 9·9024	9·9144	9·9262	9·9380	9·9496	9·9612	9·9727	9·9841	9·9954	*0·0066	·0115
104	23 0·0177	0·0287	0·0396	0·0504	0·0610	0·0716	0·0820	0·0923	0·1025	0·1126	·0105
105	0·1226	0·1325	0·1423	0·1520	0·1615	0·1710	0·1804	0·1897	0·1988	0·2079	·0094
106	23 0·2170	0·2259	0·2347	0·2435	0·2522	0·2609	0·2694	0·2780	0·2864	0·2948	·0086
107	0·3031	0·3114	0·3196	0·3278	0·3359	0·3439	0·3520	0·3599	0·3678	0·3757	·0080
108	0·3835	0·3913	0·3990	0·4067	0·4143	0·4219	0·4295	0·4370	0·4445	0·4519	·0076
109	23 0·4593	0·4667	0·4740	0·4813	0·4885	0·4958	0·5030	0·5101	0·5172	0·5243	·0072
110	0·5314	0·5384	0·5454	0·5524	0·5593	0·5662	0·5731	0·5800	0·5868	0·5936	·0069
111	0·6004	0 6071	0·6139	0·6206	0·6272	0·6339	0·6405	0·6471	0·6537	0·6603	·0066
112	23 0·6668	0·6733	0·6798	0·6863	0·6928	0·6992	0·7056	0·7120	0·7184	0·7248	·0064
113	0·7311	0·7374	0·7437	0·7500	0·7563	0·7625	0·7688	0·7750	0·7812	0·7874	·0062
114	0·7936	0·7997	0·8059	0·8120	0·8181	0·8242	0·8303	0·8364	0·8424	0·8484	·0061
115	23 0·8545	0·8605	0·8665	0·8726	0·8787	0·8847	0·8906	0·8965	0·9024	0·9083	·0059
116	0·9142	0·9200	0·9259	0·9317	0·9375	0·9433	0·9490	0·9548	0·9605	0·9663	·0058
117	0·9720	0·9777	0·9833	0·9890	0·9947	1·0003	1·0059	1·0115	1·0171	1·0227	·0056
118	23 1·0283	1·0338	1·0394	1·0449	1·0504	1·0559	1·0614	1·0669	1·0723	1·0778	·0055
119	1·0832	1·0886	1·0940	1·0994	1·1048	1·1101	1·1154	1·1208	1·1261	1·1314	·0054
120	1·1367	1·1420	1·1473	1·1525	1·1578	1·1630	1·1682	1·1734	1·1786	1·1838	·0052
121	23 1·1889	1·1941	1·1992	1·2043	1·2095	1·2146	1·2196	1·2247	1·2298	1·2348	·0051
122	1·2399	1·2449	1·2499	1·2549	1·2599	1·2649	1·2698	1·2748	1·2797	1·2847	·0050
123	1·2896	1·2945	1·2994	1·3043	1 3091	1·3140	1·3188	1·3237	1·3285	1·3333	·0049
124	23 1·3381	1·3429	1·3477	1·3524	1·3572	1·3619	1·3667	1·3714	1·3761	1·3808	·0047
125	1·3855	1·3902	1·3948	1·3995	1·4041	1·4088	1·4134	1·4180	1·4226	1·4272	·0046
126	1·4318	1·4364	1·4410	1·4455	1·4501	1·4546	1·4591	1·4636	1·4681	1·4726	·0045
127	23 1·4771	1·4816	1·4860	1·4905	1·4949	1·4993	1·5038	1·5082	1·5126	1·5170	·0044
128	1·5214	1·5257	1·5301	1·5345	1·5388	1·5431	1·5475	1·5518	1·5561	1·5604	·0043
129	1·5647	1·5690	1·5732	1·5775	1·5818	1·5860	1·5902	1·5945	1·5987	1·6029	·0042
130	23 1·6071	1·6113	1·6155	1·6196	1·6238	1·6280	1·6321	1·6362	1·6404	1·6445	·0042
131	1·6486	1·6527	1·6568	1·6609	1·6650	1·6690	1·6731	1·6772	1·6812	1·6852	·0041
132	1·6893	1·6933	1·6973	1·7013	1·7053	1·7093	1·7133	1·7173	1·7212	1·7252	·0040
133	23 1·7291	1·7331	1·7370	1·7410	1·7449	1·7488	1·7527	1·7566	1·7605	1·7644	·0039
134	1·7682	1·7721	1·7760	1·7798	1·7837	1·7875	1·7913	1·7952	1·7990	1·8028	·0038
135	1·8066	1·8104	1·8142	1·8179	1·8217	1·8255	1·8292	1·8330	1·8367	1·8405	·0038

TABLE II.—*Continued.*

Time and Velocity Table, $Ct = \tau_{v'} - \tau_{v''}$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
136	23 1·8442	1·8479	1·8517	1·8554	1·8591	1·8628	1·8665	1·8702	1·8738	1·8775	·0037
137	1 8812	1·8848	1·8885	1·8921	1·8958	1·8994	1·9030	1·9067	1·9103	1·9139	·0036
138	1·9175	1·9211	1·9247	1·9282	1·9318	1·9354	1·9390	1·9425	1·9461	1·9496	·0036
139	23 1·9532	1·9567	1·9602	1·9638	1·9673	1·9708	1·9743	1·9778	1·9813	1·9848	·0035
140	1·9883	1·9918	1·9952	1·9987	2·0022	2·0056	2·0091	2·0125	2·0160	2·0194	·0035
141	2·0228	2·0263	2·0297	2·0331	2·0365	2·0399	2·0433	2·0467	2·0501	2·0535	·0034
142	23 2·0569	2·0602	2·0636	2·0670	2·0703	2·0737	2·0770	2·0804	2·0837	2·0870	·0034
143	2·0904	2·0937	2·0970	2·1003	2·1036	2·1069	2·1102	2·1135	2·1168	2·1201	·0033
144	2·1234	2·1267	2·1299	2·1332	2·1364	2·1397	2·1430	2·1462	2·1494	2·1527	·0033
145	23 2·1559	2·1591	2·1624	2·1656	2·1688	2·1720	2·1752	2·1784	2·1816	2·1848	·0032
146	2·1880	2·1912	2·1944	2·1975	2·2007	2·2039	2·2071	2·2102	2·2134	2·2165	·0032
147	2·2197	2·2228	2·2260	2·2291	2·2322	2·2354	2·2385	2·2416	2·2447	2·2478	·0031
148	23 2·2509	2·2540	2·2571	2·2602	2·2633	2·2664	2·2695	2·2726	2·2757	2·2787	·0031
149	2·2818	2·2849	2·2879	2·2910	2·2940	2·2971	2·3001	2·3032	2·3062	2·3093	·0030
150	2·3123	2·3153	2·3183	2·3214	2·3244	2·3274	2·3304	2·3334	2·3364	2·3394	·0030
151	23 2·3424	2·3454	2·3484	2·3514	2·3543	2·3573	2·3603	2·3633	2·3662	2·3692	·0030
152	2·3722	2·3751	2·3781	2·3810	2·3840	2·3869	2·3899	2·3928	2·3958	2·3987	·0029
153	2·4016	2·4046	2·4075	2·4104	2·4133	2·4162	2·4192	2·4221	2·4250	2·4279	·0029
154	23 2·4308	2·4337	2·4366	2·4395	2·4424	2·4453	2·4481	2·4510	2·4539	2·4568	·0029
155	2·4597	2·4625	2·4654	2·4683	2·4711	2·4740	2·4768	2·4797	2·4825	2·4854	·0029
156	2·4882	2·4911	2·4939	2·4967	2·4996	2·5024	2·5052	2·5080	2·5108	2·5137	·0028
157	23 2·5165	2·5193	2·5221	2·5249	2·5277	2·5305	2·5333	2·5361	2·5389	2·5416	·0028
158	2·5444	2·5472	2·5500	2·5528	2·5555	2·5583	2·5611	2·5638	2·5666	2·5693	·0028
159	2·5721	2·5748	2·5776	2·5803	2·5831	2·5858	2·5885	2·5913	2·5940	2·5967	·0027
160	23 2·5994	2·6022	2·6049	2·6076	2·6103	2·6130	2·6157	2·6184	2·6211	2·6238	·0027
161	2·6265	2·6292	2·6319	2·6346	2·6373	2·6400	2·6426	2·6453	2·6480	2·6506	·0027
162	2·6533	2·6560	2·6586	2·6613	2·6640	2·6666	2·6693	2·6719	2·6745	2·6772	·0026
163	23 2·6798	2·6825	2·6851	2·6877	2·6903	2·6930	2·6956	2·6982	2·7008	2·7034	·0026
164	2·7061	2·7087	2·7113	2·7139	2 7165	2·7191	2·7217	2·7243	2·7268	2·7294	·0026
165	2·7320	2·7346	2·7372	2·7398	2·7423	2·7449	2·7475	2·7500	2·7526	2·7552	·0026
166	23 2·7577	2·7603	2·7628	2·7654	2·7679	2·7705	2·7730	2·7756	2·7781	2·7806	·0025
167	2·7832	2·7857	2·7882	2·7908	2·7933	2·7958	2·7983	2·8008	2·8034	2·8059	0025
168	2·8084	2·8109	2·8134	2·8159	2·8184	2·8209	2·8234	2·8258	2·8283	2·8308	·0025
169	23 2·8333	2·8358	2·8383	2·8407	2·8432	2·8457	2·8481	2·8506	2·8531	2·8555	·0025
170	2·8580	2·8604	2·8629	2·8653	2·8678	2·8702	2·8726	2·8751	2·8775	2·8799	·0024
171	2·8824	2·8848	2·8872	2·8896	2·8921	2·8945	2·8969	2·8993	2·9017	2·9041	·0024
172	23 2·9065	2·9089	2·9113	2·9137	2·9161	2·9185	2·9209	2·9233	2·9257	2·9281	·0024
173	2·9304	2·9328	2·9352	2·9376	2·9399	2·9423	2·9447	2·9470	2·9494	2·9518	·0024
174	2·9541	2·9565	2·9588	2 9612	2·9635	2·9659	2·9682	2·9705	2·9729	2·9752	·0023
175	23 2·9776	2·9799	2·9822	2·9845	2·9869	2·9892	2·9915	2·9938	2·9961	2·9985	·0023
176	3·0008	3·0031	3·0054	3·0077	3·0100	3·0123	3·0146	3·0169	3·0192	3·0215	·0023
177	3·0237	3·0260	3·0283	3·0306	3·0329	3·0351	3·0374	3·0397	3·0420	3·0442	·0023
178	23 3·0465	3·0488	3·0510	3·0533	3·0555	3·0578	3·0600	3 0623	3·0645	3·0668	·0023
179	3·0690	3·0713	3·0735	3·0757	3·0780	3·0802	3·0824	3·0847	3·0869	3·0891	·0022
180	3·0913	3·0935	3·0958	3·0980	3·1002	3·1024	3·1045	3·1068	3·1090	3·1112	·0022
181	23 3·1134	3·1156	3·1178	3·1200	3·1222	3·1244	3·1266	3·1287	3·1309	3·1331	·0022
182	3·1353	3·1375	3·1396	3·1418	3·1440	3·1461	3·1483	3·1505	3·1526	3·1548	·0022
183	3·1569	3·1591	3·1613	3·1634	3·1656	3·1677	3·1698	3·1720	3·1741	3·1763	·0021

TABLE II.—*Continued.*

Time and Velocity Table, $Ct = \tau_{v'} - \tau_{v''}$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
184	23 3·1784	3·1805	3·1827	3·1848	3·1869	3·1891	3·1912	3·1933	3·1954	3·1975	·0021
185	3·1997	3·2018	3·2039	3·2060	3·2081	3·2102	3·2123	3·2144	3·2165	3·2186	·0021
186	3·2207	3·2228	3·2249	3·2270	3·2291	3·2312	3·2333	3·2353	3·2374	3·2395	·0021
187	23 3·2416	3·2437	3·2457	3·2478	3·2499	3·2520	3·2540	3·2561	3·2582	3·2602	·0021
188	3·2623	3·2643	3·2664	3·2685	3·2705	3·2726	3·2746	3·2767	3·2787	3·2808	·0021
189	3·2828	3·2848	3·2869	3·2889	3·2909	3·2930	3·2950	3·2970	3·2991	3·3011	·0020
190	23 3·3031	3·3051	3·3072	3·3092	3·3112	3·3132	3·3152	3·3172	3·3192	3·3212	·0020
191	3·3233	3·3253	3·3273	3·3293	3·3313	3·3333	3·3353	3·3372	3·3392	3·3412	·0020
192	3·3432	3·3452	3·3472	3·3492	3·3511	3·3531	3·3551	3·3571	3·3590	3·3610	·0020
193	23 3·3630	3·3649	3·3669	3·3689	3·3708	3·3728	3·3747	3·3767	3·3786	3·3806	·0020
194	3·3825	3·3845	3·3864	3·3884	3·3903	3·3922	3·3942	3·3961	3·3980	3·4000	·0019
195	3·4019	3·4038	3·4057	3·4077	3·4096	3·4115	3·4134	3·4153	3·4172	3·4192	·0019
196	23 3·4211	3·4230	3·4249	3·4268	3·4287	3·4306	3·4325	3·4344	3·4362	3·4381	·0019
197	3·4400	3·4419	3·4438	3·4457	3·4476	3·4494	3·4513	3·4532	3·4550	3·4569	·0019
198	3·4588	3·4606	3·4625	3·4644	3·4662	3·4681	3·4699	3·4718	3·4736	3·4755	·0019
199	23 3·4773	3·4791	3·4810	3·4828	3·4846	3·4865	3·4883	3·4901	3·4920	3·4938	·0018
200	3·4956	3·4974	3·4992	3·5010	3·5028	3·5047	3·5065	3·5083	3·5101	3·5119	·0018
201	3·5137	3·5155	3·5172	3·5190	3·5208	3·5226	3·5244	3·5262	3·5280	3·5297	·0018
202	23 3·5315	3·5333	3·5351	3·5368	3·5386	3·5404	3·5421	3·5439	3·5456	3·5474	·0018
203	3·5492	3·5509	3·5527	3·5544	3·5561	3·5579	3·5596	3·5614	3·5631	3·5648	·0017
204	3·5666	3·5683	3·5700	3·5717	3·5735	3·5752	3·5769	3·5786	3·5803	3·5820	·0017
205	23 3·5837	3·5854	3·5871	3·5888	3·5905	3·5922	3·5939	3·5956	3·5973	3·5990	·0017
206	3·6007	3·6024	3·6040	3·6057	3·6074	3·6091	3·6107	3·6124	3·6141	3·6157	·0017
207	3·6174	3·6191	3·6207	3·6224	3·6240	3·6257	3·6273	3·6290	3·6306	3·6323	·0016
208	23 3·6339	3·6355	3·6372	3·6388	3·6404	3·6420	3·6437	3·6453	3·6469	3·6485	·0016
209	3·6502	3·6518	3·6534	3·6550	3·6566	3·6582	3·6598	3·6614	3·6630	3·6646	·0016
210	3·6662	3·6678	3·6694	3·6710	3·6726	3·6741	3·6757	3·6773	3·6789	3·6805	·0016
211	23 3·6820	3·6836	3·6852	3·6867	3·6883	3·6899	3·6914	3·6930	3·6946	3·6961	·0016
212	3·6977	3·6992	3·7008	3·7023	3·7039	3·7054	3·7070	3·7085	3·7100	3·7116	·0015
213	3·7131	3·7146	3·7162	3·7177	3·7192	3·7207	3·7223	3·7238	3·7253	3·7268	·0015
214	23 3·7283	3·7298	3·7313	3·7329	3·7344	3·7359	3·7374	3·7389	3·7404	3·7419	·0015
215	3·7434	3·7448	3·7463	3·7478	3·7493	3·7508	3·7523	3·7538	3·7552	3·7567	·0015
216	3·7582	3·7597	3·7612	3·7626	3·7641	3·7656	3·7670	3·7685	3·7700	3·7714	·0015
217	23 3·7729	3·7743	3·7758	3·7772	3·7787	3·7801	3·7816	3·7830	3·7845	3·7859	·0014
218	3·7874	3·7888	3·7902	3·7917	3·7931	3·7945	3·7960	3·7974	3·7988	3·8002	·0014
219	3·8016	3·8031	3·8045	3·8059	3·8073	3·8087	3·8101	3·8115	3·8129	3·8144	·0014
220	23 3·8158	3·8372	3·8186	3·8200	3·8214	3·8227	3·8241	3·8255	3·8269	3·8283	·0014
221	3·8297	3·8311	3·8325	3·8338	3·8352	3·8366	3·8380	3·8394	3·8407	3·8421	·0014
222	3·8435	3·8448	3·8462	3·8476	3·8489	3·8503	3·8517	3·8530	3·8544	3·8557	·0014
223	23 3·8571	3·8584	3·8598	3·8611	3·8625	3·8638	3·8651	3·8665	3·8678	3·8692	·0013
224	3·8705	3·8718	3·8732	3·8745	3·8758	3·8772	3·8785	3·8798	3·8811	3·8824	·0013
225	3·8838	3·8851	3·8864	3·8877	3·8890	3·8903	3·8916	3·8930	3·8943	3·8956	·0013
226	23 3·8969	3·8982	3·8995	3·9008	3·9021	3·9034	3·9047	3·9059	3·9072	3·9085	·0013
227	3·9098	3·9111	3·9124	3·9137	3·9150	3·9162	3·9175	3·9188	3·9201	3·9214	·0013
228	3·9226	3·9239	3·9252	3·9264	3·9277	3·9290	3·9303	3·9315	3·9323	3·9341	·0013
229	23 3·9353	3·9366	3·9379	3·9391	3·9404	3·9416	3·9429	3·9441	3·9454	3·9467	·0013
230	3·9479	3·9492	3·9504	3·9517	3·9529	3·9542	3·9554	3·9567	3·9579	3·9592	·0013

Table III.

Distance and Velocity Table, $Cs = \sigma_{v'} - \sigma_{v''}$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
40	2 5008·0	5050·2	5092·3	5134·2	5176·0	5217·6	5259·2	5300·6	5341·9	5383·0	41·7
41	5424·0	5464·9	5505·7	5546·4	5586·9	5627·3	5667·6	5707·8	5747·8	5787·8	40·4
42	5827·6	5867·3	5906·9	5946·4	5985·8	6025·0	6064·2	6103·3	6142·2	6181·0	39·3
43	2 6219·8	6258·4	6296·9	6335·3	6373·6	6411·8	6449·9	6487·9	6525·8	6563·6	38·2
44	6601·3	6638·9	6676·4	6713·7	6751·0	6788·2	6825·3	6862·3	6899·3	6936·1	37·2
45	6972·8	7009·4	7046·0	7082·4	7118·8	7155·0	7191·2	7227·3	7263·3	7299·2	36·3
46	2 7335·1	7370·8	7406·5	7442·1	7477·6	7513·0	7548·3	7583·6	7618·8	7653·9	35·4
47	7688·9	7723·8	7758·7	7793·5	7828·2	7862·8	7897·3	7931·8	7966·2	8000·5	34·6
48	8034·7	8068·9	8103·0	8137·0	8170·9	8204·8	8238·6	8272·3	8305·9	8339·5	33·9
49	2 8373·0	8406·5	8439·8	8473·1	8506·4	8539·5	8572·6	8605·6	8638·6	8671·5	33·2
50	8704·3	8737·1	8769·8	8802·4	8835·0	8867·5	8900·0	8932·3	8964·7	8996·9	32·5
51	9029·1	9061·2	9093·2	9125.2	9157·1	9189·0	9220·8	9252·5	9284·2	9315·8	31·9
52	2 9347·3	9378·8	9410·3	9441·6	9472·9	9504·2	9535·4	9566·5	9597·6	9628·7	31·3
53	9659·6	9690·6	9721·4	9752·2	9783 0	9813·7	9844·3	9874·9	9905·4	9935·9	30·7
54	9966·3	9996·7	*0027·0	*0057·3	*0087·5	*0117·7	*0147·8	*0177·8	*0207·8	*0237·8	30·2
55	3 0267·6	0297·5	0327·3	0357·0	0386·7	0416·3	0445·9	0475·4	0504·9	0534·3	29·6
56	0563·6	0592·9	0622·2	0651·4	0680·6	0709·7	0738·7	0767·7	0796·7	0825·6	29·1
57	0854·5	0883·3	0912·1	0940·9	0969·6	0998·2	1026·8	1055·4	1083·9	1112·4	28·6
58	3 1140·8	1169·2	1197·6	1226.0	1254·3	1282·5	1310·8	1339·0	1367·1	1395·2	28 3
59	1423·3	1451·3	1479·3	1507·3	1535·2	1563·0	1590·9	1618·7	1646·4	1674·2	27·9
60	1701·8	1729·5	1757·1	1784·6	1812·2	1839·6	1867·1	1894·5	1921·9	1949·2	27·5
61	3 1976·5	2003·7	2031·0	2058·1	2085·3	2112·4	2139·4	2166·4	2193·4	2220·4	27·1
62	2247·3	2274·2	2301·0	2327·8	2354·5	2381·3	2407·9	2434·6	2461·2	2487·7	26·7
63	2514·3	2540·8	2567·2	2593·6	2620·0	2646·3	2672·6	2698·9	2725·1	2751·3	26·3
64	3 2777·5	2803·6	2829·7	2855·7	2881·7	2907·7	2933·7	2959·6	2985·4	3011·2	26·0
65	3037·0	3062·8	3088·5	3114·2	3139·8	3165·4	3191·0	3216·5	3242·0	3267·4	25·6
66	3292·8	3318·2	3343·5	3368·8	3394·1	3419·3	3444·5	3469·6	3494 7	3519·8	25·2
67	3 3544·8	3569·8	3594·8	3619·8	3644·7	3669·5	3694·3	3719·1	3743·9	3768·6	24·8
68	3793·3	3818·0	3842·6	3867·2	3891·7	3916·2	3940·7	3965·2	3989·6	4014·0	24·5
69	4038·4	4062·7	4087·0	4111·3	4135·6	4159·8	4184·0	4208·1	4232·2	4256·3	24·2
70	3 4280·4	4304·5	4328·5	4352·4	4376·4	4400·3	4424·1	4448·0	4471·8	4495·5	23·9
71	4519·3	4543·0	4566·6	4590·2	4613 8	4637·4	4660·9	4684·4	4707·8	4731·3	23·5
72	4754·7	4777·9	4801·3	4824·6	4847·9	4871·1	4894·2	4917·4	4940·5	4963·6	23·2
73	3 4986·6	5009·6	5032·6	5055·5	5078 4	5101·3	5124·1	5146 9	5169·6	5192·4	22·8
74	5215·1	5237·7	5260 3	5282·9	5305·5	5328·0	5350·5	5373·0	5395·4	5417·8	22·5
75	5440·2	5462·5	5484·8	5507·1	5529·3	5551·5	5573·7	5595·8	5617·9	5640·0	22·2
76	3 5662·1	5684·1	5706·0	5728·0	5749·9	5771·7	5793·5	5815·3	5837·0	5858·7	21·8
77	5880·4	5902·0	5923·6	5945·1	5966·6	5988·1	6009·5	6030·9	6052·2	6073·6	21·5
78	6094·8	6116·1	6137·3	6158·4	6179·6	6200·7	6221·7	6242·7	6263·7	6284 6	21·1
79	3 6305·5	6326·4	6347·2	6368·0	6388·8	6409·5	6430·2	6450·8	6471·4	6492·0	20·7
80	6512·6	6533·1	6553·6	6574·0	6594·4	6614·8	6635·1	6655·4	6675·7	6695·9	20·4
81	6716·1	6736·3	6756·4	6776·5	6796·5	6816·5	6836·5	6856·4	6876·3	6896·1	20·0
82	3 6916·0	6935·7	6955·5	6975·1	6994·8	7014·4	7033·9	7053·4	7072·9	7092·3	19·6
83	7111·7	7131·0	7150·3	7169·6	7188·8	7207·9	7227·1	7246·1	7265·2	7284·1	19·1
84	7303·1	7322·0	7340·8	7359·6	7378·4	7397·1	7415·8	7434·4	7453·0	7471·5	18·7
85	3 7490·0	7508·5	7526·9	7545·3	7563·6	7581·8	7600·0	7618·2	7636·3	7654·4	18·2
86	7672·4	7690·5	7708·4	7726 4	7744·2	7762·0	7779·9	7797·6	7815·4	7833·0	17·8
87	7850·6	7868·2	7885·8	7903·3	7920·8	7938·2	7955·6	7973·0	7990·3	8007·6	17·4

TABLE III.—*Continued.*

Distance and Velocity Table, $Cs = \sigma_{v'} - \sigma_{v''}$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
88	3 8024·8	8042·0	8059·2	8076.3	8093·4	8110·4	8127·4	8144·4	8161·3	8178·2	17·0
89	8195·0	8211·9	8228·6	8245·4	8262·1	8278·7	8295·4	8312·0	8328·5	8345·0	16·6
90	8361·5	8377·9	8394·3	8410·7	8427·0	8443·3	8459·6	8475·8	8492·0	8508·2	16·3
91	3 8524·3	8540·4	8556·4	8572·4	8588·4	8604·3	8620·3	8636·1	8652·0	8667·8	15·9
92	8683·5	8699·3	8715·0	8730·7	8746·3	8761·9	8777·5	8793·0	8808·5	8824·0	15·6
93	8839·4	8854·8	8870·2	8885·5	8900·8	8916·1	8931·3	8946·5	8961·7	8976·8	15·3
94	3 8991·9	9007·0	9022·0	9037·0	9052·0	9066·9	9081·9	9096·7	9111·6	9126·4	15·0
95	9141·2	9156·0	9170·7	9185·4	9200·1	9214·7	9229·3	9243·9	9258·4	9272·9	14·6
96	9287·4	9301·9	9316·3	9330·7	9345·0	9359·4	9373·7	9387·9	9402·2	9416·4	14·3
97	3 9430·6	9444·7	9458·9	9473·0	9487·0	9501·1	9515·1	9529·1	9543·0	9557·0	14·0
98	9570·8	9584·7	9598·6	9612·4	9626·1	9639·9	9653·6	9667·3	9681·0	9604·6	13·7
99	9708·3	9721·9	9735·4	9749·0	9762·5	9775·9	9789·4	9802·8	9816·2	9829·6	13·5
100	3 9842·9	9856·3	9869·6	9882·9	9896·1	9909·3	9922·5	9935·3	9948·8	9961·9	13·2
101	9975·0	9988·1	*0001·1	*0014·1	*0027·1	*0040·0	*0052·9	*0065·8	*0078·7	*0091·5	12·9
102	4 0104·3	0117·1	0129·8	0142·5	0155·2	0167·8	0180·4	0192·9	0205·4	0217·8	12·6
103	4 0230·1	0242·4	0254·6	0266·8	0278·8	0290·8	0302·7	0314·5	0326·2	0337·8	11·9
104	0349·4	0360·8	0372·2	0383·4	0394·5	0405·6	0416·5	0427·3	0438·1	0448·7	11·0
105	0459·2	0469·6	0479·9	0490·0	0500·1	0510·1	0520·0	0529·8	0539·5	0549·2	9·9
106	4 0558·7	0568·2	0577·6	0586·9	0596·2	0605·4	0614·5	0623·6	0632·6	0641·6	9·2
107	0650·5	0659·3	0668·1	0676·9	0685·6	0694·2	0702·8	0711·4	0719·9	0728·4	8·6
108	0736·8	0745·2	0753·6	0761·9	0770·2	0778·4	0786·6	0794·8	0802·9	0811·0	8·2
109	4 0819·0	0827·1	0835·0	0843·0	0850·9	0858·9	0866·7	0874·6	0882·4	0890·2	7·9
110	0897·9	0905·7	0913·4	0921·1	0928·7	0936·4	0944·0	0951·5	0959·1	0966·6	7·6
111	0974·2	0981·6	0989·1	0996·6	1004·0	1011·4	1018·8	1026·2	1033·5	1040·9	7·4
112	4 1048·2	1055·5	1062·8	1070·0	1077·3	1084·5	1091·7	1099·0	1106·1	1113·3	7·2
113	1120·5	1127·6	1134·8	1141·9	1149·0	1156·1	1163·2	1170·2	1177·3	1184·4	7·1
114	1191·4	1198·4	1205·4	1212·4	1219·4	1226·4	1233·3	1240·3	1247·2	1254·1	6·9
115	4 1261·0	1267·9	1274·8	1281·7	1288·6	1295·4	1302·3	1309·1	1315·9	1322·7	6·8
116	1329·5	1336·3	1343·1	1349·8	1356·6	1363·3	1370·0	1376·7	1383·4	1390·1	6·7
117	1396·8	1403·5	1410·1	1416·8	1423·4	1430·0	1436·6	1443·2	1449·8	1456·4	6·6
118	4 1462·9	1469·5	1476·0	1482·6	1489·1	1495·6	1502·1	1508·6	1515·1	1521·5	6·5
119	1528·0	1534·4	1540·9	1547·3	1553·7	1560·1	1566·5	1572·9	1579·2	1585·6	6·4
120	1591·9	1598·3	1604·6	1610·9	1617·2	1623·5	1629·8	1636·1	1642·3	1648·6	6·3
121	4 1654·8	1661·1	1667·3	1673·5	1679·7	1685·9	1692·1	1698·2	1704·4	1710·5	6·2
122	1716·7	1722·8	1728·8	1735·0	1741·1	1747·2	1753·3	1759·4	1765·4	1771·5	6·1
123	1777·5	1783·6	1789·6	1795·6	1801·6	1807·6	1813·6	1819·6	1825·6	1831·5	6·0
124	4 1837·5	1843·4	1849·4	1855·3	1861·2	1867·1	1873·0	1878·9	1884·8	1890·6	5·9
125	1896·5	1902·3	1908·2	1914·0	1919·8	1925·6	1931·5	1937·3	1943·0	1948·8	5·8
126	1954·6	1960·4	1966·1	1971·9	1977·6	1983·3	1989·0	1994·8	2000·5	2006·2	5·7
127	4 2011·8	2017·5	2023·2	2028·9	2034·5	2040·2	2045·8	2051·4	2057·0	2062·7	5·6
128	2068·3	2073·9	2079·5	2085·0	2090·6	2096·2	2101·8	2107·3	2112·9	2118·4	5·6
129	2123·9	2129·4	2135·0	2140·5	2146·0	2151·5	2157·0	2162·4	2167·9	2173·4	5·5
130	4 2178·8	2184·3	2189·7	2195·1	2200·6	2206·0	2211·4	2216·8	2222·2	2227·6	5·4
131	2233·0	2238·4	2243·7	2249·1	2254·5	2259·8	2265·1	2270·5	2275·8	2281·1	5·3
132	2286·4	2291·8	2297·1	2302·4	2307·6	2312·9	2318·2	2323·5	2328·7	2334·0	5·3
133	4 2339·2	2344·5	2349·7	2355·0	2360·2	2365·4	2370·6	2375·8	2381·0	2386·2	5·2
134	2391·4	2396·6	2401·8	2406·9	2412·1	2417·3	2422·4	2427·6	2432·7	2437·8	5·2
135	2443·0	2448·1	2453·2	2458·3	2463·4	2468·5	2473·6	2478·7	2483·8	2488·9	5·1

TABLE III.—*Continued.*

Distance and Velocity Table, $Cs = \sigma_{v'} - \sigma_{v''}$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
136	4 2493·9	2499·0	2504·1	2509·1	2514·2	2519·2	2524·3	2529·3	2534·3	2539·4	5·0
137	2544·4	2549·4	2554·4	2559·4	2564·4	2569·4	2574·4	2579·4	2584·3	2589·3	5·0
138	2594·3	2599·2	2604·2	2609·1	2614·1	2619·0	2624·0	2628·9	2633·8	2638·8	4·9
139	4 2643·7	2648·6	2653·5	2658·4	2663·3	2668·2	2673·1	2678·0	2682·9	2687·8	4·9
140	2692·6	2697·5	2702·4	2707·2	2712·1	2717·0	2721·8	2726·7	2731·5	2736·3	4·9
141	2741·2	2746·0	2750·8	2755·7	2760·5	2765·3	2770·1	2774·9	2779·7	2784·5	4·8
142	4 2789·3	2794·1	2798·9	2803·7	2808·5	2813·2	2818·0	2822·8	2827·5	2832·3	4·8
143	2837·1	2841·8	2846·6	2851·3	2856·0	2860·8	2865·5	2870·2	2875·0	2879·7	4·7
144	2884·4	2889·1	2893·8	2898·6	2903·3	2908·0	2912·7	2917·4	2922·1	2926·7	4·7
145	4 2931·4	2936·1	2940·8	2945·5	2950·1	2954·8	2959·5	2964·1	2968·8	2973·5	4·7
146	2978·1	2982·8	2987·4	2992·1	2996·7	3001·3	3006·0	3010·6	3015·2	3019·9	4·6
147	3024·5	3029·1	3033·7	3038·4	3043·0	3047·6	3052·2	3056·8	3061·4	3066·0	4·6
148	4 3070·6	3075·2	3079·8	3084·4	3089·0	3093·5	3098·1	3102·7	3107·3	3111·8	4·6
149	3116·4	3121·0	3125·6	3130·1	3134·7	3139·2	3143·8	3148·3	3152·9	3157·4	4·6
150	3162·0	3166·5	3171·0	3175·6	3180·1	3184·6	3189·2	3193·7	3198·2	3202·7	4·5
151	4 3207·2	3211·8	3216·3	3220·8	3225·3	3229·8	3234·3	3238·8	3243·3	3247·8	4·5
152	3252·3	3256·8	3261·3	3265·8	3270·3	3274·8	3279·3	3283·8	3288·3	3292·8	4·5
153	3297·2	3301·7	3306·2	3310·6	3315·1	3319·6	3324·1	3328·5	3333·0	3337·5	4·5
154	4 3342·0	3346·4	3350·9	3355·3	3359·8	3364·3	3368·7	3373·2	3377·6	3382·1	4·5
155	3386·5	3391·0	3395·4	3399·9	3404·3	3408·7	3413·2	3417·6	3422·0	3426·5	4·4
156	3430·9	3435·3	3439·8	3444·2	3448·6	3453·0	3457·4	3461·9	3466·3	3470·7	4·4
157	4 3475·1	3479·5	3483·9	3488·3	3492·7	3497·1	3501·5	3505·9	3510·3	3514·7	4·4
158	3519·1	3523·5	3527·9	3532·3	3536·7	3541·1	3545·4	3549·8	3554·2	3558·6	4·4
159	3563·0	3567·3	3571·7	3576·1	3580·4	3584·8	3589·1	3593·5	3597·9	3602·2	4·4
160	4 3606·6	3610·9	3615·3	3619·6	3624·0	3628·3	3632·6	3637·0	3641·3	3645·7	4·3
161	3650·0	3654·3	3658·7	3663·0	3667·3	3671·6	3676·0	3680·3	3684·6	3688·9	4·3
162	3693·3	3697·6	3701·9	3706·1	3710·5	3714·8	3719·1	3723·4	3727·7	3732·0	4·3
163	4 3736·3	3740·6	3744·9	3749·2	3753·5	3757·8	3762·1	3766·4	3770·6	3774·9	4·3
164	3779·2	3783·5	3787·8	3792·0	3796·3	3800·6	3804·9	3809·1	3813·4	3817·6	4·3
165	3821·9	3826·2	3830·4	3834·7	3838·9	3843·2	3847·4	3851·7	3855·9	3860·2	4·3
166	4 3864·4	3868·7	3872·9	3877·2	3881·4	3885·6	3889·9	3894·1	3898·3	3902·5	4·2
167	3906·8	3911·0	3915·2	3919·5	3923·7	3927·9	3932·1	3936·3	3940·5	3944·7	4·2
168	3949·0	3953·2	3957·4	3961·6	3965·8	3970·0	3974·2	3978·4	3982·6	3986·7	4·2
169	4 3990·9	3995·1	3999·3	4003·5	4007·7	4011·9	4016·0	4020·2	4024·4	4028·6	4·2
170	4032·7	4036·9	4041·1	4045·2	4049·4	4053·6	4057·7	4061·9	4066·0	4070·2	4·2
171	4074·3	4078·5	4082·6	4086·8	4090·9	4095·1	4099·2	4103·3	4107·5	4111·6	4·1
172	4 4115·7	4119·9	4124·0	4128·1	4132·3	4136·4	4140·5	4144·6	4148·7	4152·9	4·1
173	4157·0	4161·1	4165·2	4169·3	4173·4	4177·5	4181·6	4185·7	4189·8	4193·9	4·1
174	4198·0	4202·1	4206·2	4210·3	4214·4	4218·5	4222·6	4226·7	4230·8	4234·8	4·1
175	4 4238·9	4243·0	4247·1	4251·2	4255·3	4259·3	4263·4	4267·5	4271·5	4275·6	4·1
176	4279·6	4283·7	4287·8	4291·8	4295·9	4300·0	4304·0	4308·8	4312·1	4316 1	4·1
177	4320·2	4324·2	4328·3	4332·3	4336·4	4340·4	4344·4	4348·5	4352·5	4356·5	4·0
178	4 4360·5	4364·6	4368·6	4372·6	4376·6	4380·7	4334·7	4388·7	4392·7	4396·7	4·0
179	4400·7	4404·7	4408·8	4412·8	4416 8	4420·8	4424·8	4128·8	4432·8	4436·8	4·0
180	4440·8	4444·7	4448·7	4452·7	4456·7	4460·7	4464·7	4468·7	4472·6	4476·6	4·0
181	4 4480·6	4484·6	4488·5	4492·5	4496·5	4500·5	4504·4	4508·4	4512·4	4516·3	4·0
182	4520·3	4524·2	4528·2	4532·2	4536·1	4540·1	4544·0	4548·0	4551·9	4555 9	4·0
183	4559·8	4563·7	4567·7	4571·6	4575·6	4579·5	4583·4	4587·4	4591·3	4595·2	3·9

TABLE III.—*Continued.*

Distance and Velocity Table, $Cs = \sigma_{v'} - \sigma_{v''}$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
184	4 4599·2	4603·1	4607·0	4610·9	4614·9	4618·8	4622·7	4626·6	4630·5	4634·4	3·9
185	4638·4	4642·3	4646·2	4650·1	4654·0	4657·9	4661·8	4665·7	4669·6	4673·5	3·9
186	4677·4	4681·3	4685·2	4689·1	4693·0	4696·9	4700·8	4704·6	4708·5	4712·4	3·9
187	4 4716·3	4720·2	4724·1	4727·9	4731·8	4735·7	4739·6	4743·4	4747·3	4751·2	3·9
188	4755·0	4758·9	4762·8	4766·7	4770·5	4774·4	4778·2	4782·1	4786·0	4789·8	3·9
189	4793·7	4797·5	4801·4	4805·2	4809·1	4812·9	4816·8	4820·6	4824·5	4828·3	3·8
190	4 4832·2	4836·0	4839·8	4843·7	4847·5	4851·4	4855·2	4859·0	4862·8	4866·7	3·8
191	4870·5	4874·3	4878·1	4882·0	4885·8	4889·6	4893·4	4897·3	4901·1	4904·9	3·8
192	4908·7	4912·5	4916·3	4920·1	4923·9	4927·7	4931·5	4935·3	4939·1	4942·9	3·8
193	4 4946·7	4950·5	4954·3	4958·1	4961·9	4965·7	4969·4	4973·2	4977·0	4980·7	3·8
194	4984·5	4988·3	4992·1	4995·8	4999·6	5003·4	5007·1	5010·9	5014·7	5018·4	3·8
195	5022·2	5025·9	5029·7	5033·4	5037·2	5040·9	5044·7	5048·4	5052·1	5055·9	3·7
196	4 5059·6	5063·4	5067·1	5070·8	5074·6	5078·3	5082·0	5085·7	5089·4	5093·1	3·7
197	5096·9	5100·6	5104·3	5108·0	5111·7	5115·4	5119·1	5122·8	5126·5	5130·2	3·7
198	5133·9	5137·5	5141·2	5144·9	5148·6	5152·3	5156·0	5159·6	5163·3	5166·9	3·7
199	4 5170·6	5174·3	5177·9	5181·6	5185·2	5188·9	5192·5	5196·2	5199·8	5203·4	3·6
200	5207·1	5210·7	5214·3	5218·0	5221·6	5225·2	5228·8	5232·5	5236·1	5239·7	3·6
201	5243·3	5246·9	5250·5	5254·1	5257·7	5261·3	5264·9	5268·5	5272·1	5275·7	3·6
202	4 5279·2	5282·8	5286·4	5290·0	5293·6	5297·2	5300·7	5304·3	5307·8	5311·4	3·6
203	5311·9	5318·5	5322·0	5325·6	5329·1	5332·7	5336·2	5339·7	5343·3	5346·8	3·5
204	5350·3	5353·8	5357·3	5360·9	5364·4	5367·9	5371·4	5374·9	5378·4	5391·9	3·5
205	4 5385·4	5388·9	5392·4	5395·9	5399·4	5402·9	5406·3	5409·8	5413·3	5416·7	3·5
206	5420·2	5423·7	5427·1	5430·6	5434·1	5437·5	5441·0	5444·4	5447·8	5451·3	3·5
207	5454·7	5458·1	5461·6	5465·0	5468·4	5471·9	5475·3	5478·7	5482·1	5485·5	3·4
208	4 5488·9	5492·3	5495·7	5499·1	5502·5	5505·9	5509·3	5512·7	5516·1	5519·4	3·4
209	5522·8	5526·2	5529·6	5532·9	5536·3	5539·7	5543·0	5546·4	5549·7	5553·1	3·4
210	5556·4	5559·8	5563·1	5566·4	5569·8	5573·1	5576·5	5579·8	5583·1	5586·4	3·3
211	4 5589·7	5593·0	5596·4	5599·7	5603·0	5606·3	5609·6	5612·9	5616·2	5619·5	3·3
212	5622·8	5626·1	5629·3	5632·6	5635·9	5639·2	5642·5	5645·7	5649·0	5652·3	3·3
213	5655·5	5658·8	5662·0	5665·3	5668·6	5671·8	5675·1	5678·3	5681·5	5684·8	3·2
214	4 5688·0	5691·2	5694·5	5697·7	5700·9	5704·2	5707·4	5710·6	5713·8	5717·0	3·2
215	5720·2	5723·4	5726·6	5729·9	5733·1	5736·3	5739·5	5742·6	5745·8	5749·0	3·2
216	5752·2	5755·4	5758·6	5761·8	5764·9	5768·1	5771·3	5774·4	5777·6	5780·8	3·2
217	4 5783·9	5787·1	5790·2	5793·4	5796·6	5799·7	5802·9	5806·0	5809·1	5812·2	3·1
218	5815·4	5818·5	5821·6	5824·8	5827·9	5831·0	5834·1	5837·3	5840·4	5843·5	3·1
219	5846·6	5849·7	5852·8	5855·9	5859·0	5862·1	5865·2	5868·3	5871·4	5874·4	3·1
220	4 5877·5	5880·6	5883·7	5886·8	5889·9	5893·0	5896·0	5899·1	5902·1	5905·2	3·1
221	5908·3	5911·3	5914·4	5917·4	5920·5	5923·6	5926·6	5929·6	5932·7	5935·7	3·0
222	5938·7	5941·8	5944·8	5947·8	5950·9	5953·9	5956·9	5959·9	5963·0	5966·0	3·0
223	4 5969·0	5972·0	5975·0	5978·0	5981·0	5984·0	5987·0	5990·0	5993·0	5996·0	3·0
224	5999·0	6002·0	6004·9	6007·9	6010·9	6013·9	6016·9	6019·8	6022·8	6025·8	3·0
225	6028·7	6031·7	6034·6	6037·6	6040·5	6043·5	6046·5	6049·4	6052·4	6055·3	3·0
226	4 6058·3	6061·2	6064·1	6067·1	6070·0	6072·9	6075·9	6078·8	6081·7	6084·7	2·9
227	6087·6	6090·5	6093·4	6096·3	6099·3	6102·2	6105·1	6108·0	6110·9	6113·8	2·9
228	6116·7	6119·6	6122·5	6125·4	6128·3	6131·2	6134·1	6137·0	6139·9	6142·8	2·9
229	4 6145·7	6148·6	6151·5	6154·4	6157·3	6160·1	6163·1	6166·0	6168·8	6171·7	2·9
230	6174·6	6177·5	6180·4	6183·3	6186·2	6189·4	6191·9	6194·8	6197·7	6200·6	2·9

TABLE IV.*

Inclination and Velocity Table, $Cd = \delta_{v'} - \delta_{v''}$.

v.	0	1	2	3	4	5	6	7	8	9
f. s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
40	0	·4838	·9640	1·4407	1·9137	2·3830	2·8488	3·3110	3·7689	4·2240
41	4·6757	5·1240	5·5688	6·0101	6·4482	6·8828	7·3141	7.7421	8·1660	8·5874
42	9·0056	9·4207	9·8327	10·2410	10·6467	11·0496	11·4494	11·8462	12·2397	12·6306
43	13·0187	13·4039	13·7862	14·1652	14·5419	14·9159	15·2872	15·6557	16·0211	16·3843
44	16·7450	17·1030	17·4585	17·8110	18·1614	18·5094	18·8549	19·1980	19·5383	19.8766
45	20·2125	20·5460	20·8772	21·2054	21·5320	21·8565	22·1788	22·4989	22·8169	23·1327
46	23·4463	23·7578	24·0671	24·3736	24·6788	24·9821	25·2834	25·5927	25·8801	26·1756
47	26·4691	26·7607	27·0503	27·3376	27·6234	27·9075	28·1897	28·4702	28·7486	29·0254
48	29·3006	29·5739	29·8455	30·1151	30·3833	30·6498	30·9147	31·1779	31·4393	31·6993
49	31·9576	32·2143	32·4695	32·7227	32·9747	33·2253	33·4743	33·7219	33·9679	34·2125
50	34·4557	34·6973	34·9375	35·1761	35·4134	35·6493	35·8837	36·1167	36·3480	36·5783
51	36·8073	37·0349	37·2613	37·4862	37·7099	37·9323	38·1534	38·3731	38·5914	38·8086
52	39·0246	39·2394	39·4529	39·6651	39·8762	40·0860	40·2947	40·5022	40·7083	40·9135
53	41·1175	41·3204	41·5221	41·7225	41·9221	42·1205	42·3179	42·5142	42·7095	42·9037
54	43·0967	43·2887	43·4795	43·6690	43·8578	44·0456	44·2324	44·4182	44·6031	44·7870
55	44·9698	45·1516	45·3325	45·5122	45·6910	45·8689	46·0457	46·2217	46·3964	46·5705
56	46·7437	46·9160	47·0874	47·2581	47·4277	47·5965	47·7644	47·9314	48·0973	48·2625
57	48·4270	48·5906	48·7534	48·9153	49·0764	49·2368	49·3963	49·5551	49·7130	49·8701
58	50·0265	50·1822	50·3370	50·4909	50·6442	50·7968	50·9487	51·0999	51·2505	51·4002
59	51·5492	51·6975	51·8451	51·9917	52·1378	52·2832	52·4280	52·5721	52·7155	52·8583
60	53·0003	53·1417	53·2825	53·4224	53·5618	53·7005	53·8386	53·9761	54·1130	54·2492
61	54·3847	54·5196	54·6539	54·7875	54·9205	55·0529	55·1846	55·3158	55·4462	55·5761
62	55·7054	55·8342	55·9623	56·0899	56·2169	56·3433	56·4690	56·5942	56·7188	56·8428
63	56·9663	57·0891	57·2114	57·3330	57·4542	57·5749	57·6950	57·8146	57·9338	58·0523
64	58·1703	58·2878	58·4046	58·5209	58·6367	58·7521	58·8669	58·9832	59·0949	59·2081
65	59·3209	59·4332	59·5449	59·6562	59·7669	59·8772	59·9869	60·0961	60·2047	60·3130
66	60·4207	60·5280	60·6348	60·7411	60·8470	60·9523	61·0572	61·1616	61·2654	61·3688
67	61·4719	61·5744	61·6766	61·7783	61·8796	61·9804	62·0808	62·1807	62·2802	62·3793
68	62·4779	62·5761	62·6739	62·7711	62·8680	62·9646	63·0607	63·1565	63·2519	63·3468
69	63·4414	63·5356	63·6294	63·7227	63·8157	63·9084	64·0006	64·0924	64·1838	64·2749
70	64·3656	64·4559	64·5459	64·6356	64·7249	64·8137	64·9022	64·9903	65·0779	65·1652
71	65·2522	65·3388	65·4250	65·5107	65·5962	65·6813	65·7660	65·8504	65·9345	66·0182
72	66·1015	66·1845	66·2671	66·3494	66·4313	66·5128	66·5940	66·6749	66·7553	66·8355
73	66·9153	66·9949	67·0740	67·1529	67·2314	67·3096	67·3875	67·4649	67·5422	67·6190
74	67·6955	67·7717	67·8476	67·9231	67·9983	68·0733	68·1479	68·2223	68·2964	68·3702
75	68·4436	68·5168	68·5896	68·6620	68·7342	68·8062	68·8778	68·9492	69·0204	69·0912
76	69·1617	69·2318	69·3017	69·3712	69·4404	69·5094	69·5780	69·6464	69·7145	69·7823
77	69·8497	69·9169	69·9838	70·0503	70·1166	70·1826	70·2483	70·3137	70·3787	70·4436
78	70·5082	70·5725	70·6365	70·7004	70·7639	70·8271	70·8901	70·9527	71·0149	71·0770
79	71·1388	71·2004	71·2617	71·3228	71·3837	71·4442	71·5045	71·5646	71·6244	71·6839
80	71·7432	71·8023	71·8611	71·9196	71·9779	72·0359	72·0937	72·1513	72·2086	72·2656
81	72·3225	72·3791	72·4354	72·4915	72·5473	72·6030	72·6584	72·7135	72·7685	72·8232
82	72·8776	72·9317	72·9856	73·0393	73·0927	73·1458	73·1988	73·2514	73·3038	73·3560
83	73·4079	73·4596	73·5111	73·5622	73·6132	73·6639	73·7145	73·7648	73·8149	73·8647
84	73·9143	73·9636	74·0127	74·0615	74·1101	74·1585	74·2067	74·2546	74·3023	74·3498
85	74·3971	74·4441	74·4910	74·5376	74·5839	74·6301	74·6760	74·7217	74·7670	74·8123
86	74·8573	74·9022	74·9468	74·9912	75·0355	75·0795	75·1233	75·1669	75·2104	75·2536
87	75·2966	75·3395	75·3821	75·4246	75·4668	75·5089	75·5507	75·5924	75·6339	75·6752

*By W. D. Niven, Esq., M. A., F. R. S.

TABLE IV.—*Continued.*

Inclination and Velocity Table, $Cd = \delta_{v'} - \delta_{v''}$.

v.	0	1	2	3	4	5	6	7	8	9
f.s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
88	75·7163	75·7572	75·7980	75·8385	75·8788	75·9190	75·9590	75·9988	76·0384	76·0778
89	76·1171	76·1562	76·1952	76·2339	76·2725	76·3109	76·3492	76·3873	76·4252	76·4629
90	76·5005	76·5379	76·5751	76 6121	76·6490	76·6857	76·7223	76·7588	76·7951	76·8312
91	76·8671	76·9029	76·9385	76·9739	77·0092	77·0444	77·0794	77·1142	77·1489	77·1835
92	77·2179	77·2522	77·2863	77·3203	77·3541	77·3878	77·4213	77·4547	77·4879	77·5210
93	77·5540	77·5868	77·6195	77·6520	77·6844	77·7167	77·7488	77·7807	77·8125	77·8442
94	77·8757	77·9071	77·9384	77·9695	78·0005	78·0314	78·0622	78·0929	78·1234	78·1538
95	78·1841	78·2142	78·2442	78·2741	78·3039	78·3335	78·3630	78·3924	78·4216	78·4508
96	78·4798	78·5087	78·5375	78·5622	78·5947	78·6231	78·6514	78·6796	78·7076	78·7356
97	78·7634	78·7911	78·8188	78·8463	78·8736	78·9009	78·9280	78·9551	78·9819	79·0087
98	79·0354	79·0621	79·0886	79·1150	79·1413	79·1675	79·1936	79·2195	79·2454	79·2712
99	79·2968	79·3224	79·3478	79·3731	79·3983	79·4234	79·4484	79·4734	79·4982	79·5230
100	79·5476	79·5722	79·5966	79·6210	79·6453	79·6695	79·6935	79·7175	79·7414	79·7652
101	79·7889	79·8124	79·8359	79·8593	79·8826	79·9058	79·9289	79·9519	79·9748	79·9976
102	80·0203	80·0430	80·0655	80·0879	80·1102	80·1324	80·1544	80·1763	80·1981	80·2197
103	80·2412	80·2625	80·2837	80·3408	80·3256	80·3462	80·3667	80·3869	80·4071	80·4270
104	80·4466	80·4661	80·4854	80·5045	80·5234	80·5420	80·5605	80·5787	80·5967	80·6145
105	80·6321	80·6495	80·6667	80·6835	80·7003	80·7169	80·7333	80·7495	80·7654	80·7813
106	80·7970	80·8126	80·8280	80·8432	80·8583	80·8733	80·8882	80·9029	80·9175	80·9319
107	80·9463	80·9606	80·9747	80·9886	81·0026	81·0164	81·0301	81·0437	81·0573	81·0707
108	81·0841	81·0973	81·1105	81·1236	81·1366	81·1495	81·1624	81·1751	81·1877	81·2003
109	81·2129	81·2253	81·2377	81·2501	81·2623	81·2745	81·2866	81·2986	81·3105	81·3224
110	81·3342	81·3460	81·3578	81·3695	81·3811	81·3927	81·4042	81·4156	81·4269	81·4382
111	81·4495	81·4607	81·4719	81·4829	81·4939	81·5049	81·5159	81·5268	81·5377	81·5485
112	81·5593	81·5700	81·5807	81·5913	81·6019	81·6124	81·6230	81·6334	81·6439	81·6543
113	81·6647	81·6750	81·6853	81·6955	81·7057	81·7159	81·7260	81·7361	81·7462	81·7562
114	81·7662	81·7761	81·7861	81·7960	81·8058	81·8156	81·8254	81·8351	81·8448	81·8545
115	81·8641	81·8737	81·8833	81·8929	81·9024	81·9119	81·9213	81·9307	81·9401	81·9495
116	81·9588	81·9681	81·9774	81·9866	81·9958	82·0049	82·0141	82·0232	82·0322	82·0413
117	82·0503	82·0592	82·0682	82·0771	82·0860	82·0948	82·1036	82·1124	82·1212	82·1299
118	82·1386	82·1473	82·1559	82·1645	82·1731	82·1817	82·1902	82·1988	82·2073	82·2157
119	82·2241	82·2325	82·2408	82·2492	82·2575	82·2657	82·2740	82·2822	82·2903	82·2985
120	82·3066	82·3147	82·3228	82·3309	82·3389	82·3469	82·3549	82·3629	82·3708	82·3787
121	82·3865	82·3944	82·4022	82·4100	82·4178	82·4255	82·4333	82·4410	82·4486	82·4563
122	82·4639	82·4715	82·4790	82·4865	82·4940	82·5015	82·5090	82·5164	82·5238	82·5312
123	82·5386	82·5459	82·5533	82·5606	82·5679	82·5751	82·5824	82·5896	82·5968	82·6040
124	82·6112	82·6183	82·6254	82·6324	82·6395	82·6465	82·6535	82·6605	82·6675	82·6744
125	82·6814	82·6883	82·6951	82·7019	82·7088	82·7156	82·7224	82·7291	82·7359	82·7427
126	82·7494	82·7561	82·7627	82·7694	82·7760	82·7826	82·7892	82·7957	82·8023	82·8088
127	82·8153	82·8218	82·8283	82·8348	82·8412	82·8477	82·8541	82·8604	82·8668	82·8731
128	82·8794	82·8857	82·8920	82·8983	82·9045	82·9107	82·9169	82·9231	82·9292	82·9354
129	82·9415	82·9477	82·9538	82·9599	82·9660	82·9720	82·9780	82·9840	82·9900	82·9960
130	83·0019	83·0079	83·0138	83·0197	83·0256	83·0315	83·0373	83·0432	83·0490	83·0548
131	83·0606	83·0664	83·0721	83·0779	83·0836	83·0893	83·0950	83·1007	83·1063	83·1119
132	83·1176	83·1232	83·1288	83·1344	83·1400	83·1455	83·1511	83·1566	83·1621	83·1676
133	83·1730	83·1785	83·1840	83·1894	83·1949	83·2003	83·2057	83·2110	83·2164	83·2217
134	83·2271	83·2324	83·2377	83·2430	83·2483	83·2536	83·2588	83·2641	83·2693	83·2745
135	83·2797	83·2849	83·2900	83·2951	83·3003	83·3054	83·3105	83·3156	83·3207	83·3257

Table IV.—*Continued.*

Inclination and Velocity Table, $Cd = \delta_{v'} - \quad_{v''}$.

v.	0	1	2	3	4	5	6	7	8	9
f.s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
136	83·3308	83·3359	83·3409	83·3459	83·3509	83·3560	83·3609	83·3659	83·3709	83·3759
137	83·3808	83·3857	83·3906	83·3955	83·4004	83·4053	83·4101	83·4150	83·4198	83·4247
138	83·4295	83·4343	83·4391	83·4438	83·4486	83·4533	83·4581	83·4628	83·4676	83·4723
139	83·4770	83·4817	83·4863	83·4910	83·4956	83·5003	83·5049	83·5095	83·5141	83·5187
140	83·5233	83·5279	83·5325	83·5371	83·5417	83·5462	83·5507	83·5553	83·5598	83·5642
141	83·5687	83·5732	83·5777	83·5821	83·5866	83·5910	83·5954	83·5999	83·6043	83·6087
142	83·6130	83·6174	83·6218	83·6261	83·6305	83·6348	83·6392	83·6435	83·6478	83·6522
143	83·6565	83·6607	83·6650	83·6693	83·6735	83·6778	83·6820	83·6862	83·6904	83·6946
144	83·6988	83·7030	83·7072	83·7114	83·7156	83·7197	83·7239	83·7280	83·7321	83·7362
145	83·7403	83·7444	83·7485	83·7526	83·7567	83·7608	83·7649	83·7689	83·7730	83·7770
146	83·7810	83·7850	83·7891	83·7930	83·7970	83·8010	83·8050	83·8090	83·8130	83·8170
147	83·8209	83·8249	83·8288	83·8327	83·8366	83·8406	83·8445	83·8484	83·8522	83·8561
148	83·8600	83·8639	83·8677	83·8715	83·8754	83·8792	83·8830	83·8869	83·8907	83·8945
149	83·8983	83·9021	83·9059	83·9096	83·9134	83·9172	83·9209	83·9247	83·9285	83·9322
150	83·9359	83·9396	83·9433	83·9470	83·9507	83·9544	83·9581	83·9617	83·9654	83·9691
151	83·9727	83·9764	83·9800	83·9837	83·9873	83·9909	83·9946	83·9982	84·0018	84·0054
152	84·0090	84·0126	84·0161	84·0197	84·0233	84·0269	84·0304	84·0340	84·0375	84·0410
153	84·0446	84·0481	84·0516	84·0551	84·0587	84·0622	84·0657	84·0692	84·0727	84·0762
154	84·0796	84·0831	84·0866	84·0900	84·0935	84·0969	84·1004	84·1038	84·1072	84·1106
155	84·1140	84·1174	84·1208	84·1242	84·1276	84·1310	84·1344	84·1378	84·1412	84·1445
156	84·1479	84·1513	84·1546	84·1579	84·1613	84·1646	84·1679	84·1713	84·1746	84·1779
157	84·1812	84·1845	84·1878	84·1911	84·1943	84·1976	84·2009	84·2041	84·2074	84·2107
158	84·2139	84·2172	84·2204	84·2237	84·2269	84·2301	84·2333	84·2366	84·2398	84·2430
159	84·2461	84·2493	84·2525	84·2557	84·2588	84·2620	84·2652	84·2683	84·2715	84·2746
160	84·2778	84·2809	84·2840	84·2871	84·2902	84·2933	84·2965	84·2996	84·3027	84·3058
161	84·3088	84·3119	84·3150	84·3180	84·3210	84·3242	84·3272	84·3302	84·3333	84·3363
162	84·3394	84·3424	84·3454	84·3484	84·3514	84·3544	84·3574	84·3604	84·3634	84·3664
163	84·3694	84·3724	84·3753	84·3783	84·3813	84·3843	84·3872	84·3902	84·3931	84·3960
164	84·3990	84·4019	84·4048	84·4078	84·4107	84·4136	84·4165	84·4194	84·4223	84·4252
165	84·4281	84·4310	84·4339	84·4367	84·4396	84·4425	84·4453	84·4482	84·4510	84·4539
166	84·4567	84·4595	84·4624	84·4652	84·4680	84·4709	84·4737	84·4765	84·4793	84·4821
167	84·4849	84·4877	84·4905	84·4933	84·4961	84·4988	84·5016	84·5044	84·5070	84·5099
168	84·5127	84·5154	84·5181	84·5209	84·5236	84·5263	84·5291	84·5318	84·5345	84·5372
169	84·5399	84·5426	84·5453	84·5480	84·5508	84·5534	84·5561	84·5588	84·5615	84·5641
170	84·5668	84·5695	84·5721	84·5748	84·5775	84·5801	84·5828	84·5854	84·5880	84·5907
171	84·5933	84·5959	84·5985	84·6012	84·6038	84·6064	84·6090	84·6116	84·6142	84·6168
172	84·6193	84·6219	84·6245	84·6271	84·6297	84·6322	84·6348	84·6373	84·6399	84·6424
173	84·6449	84·6475	84·6500	84·6525	84·6550	84·6575	84·6601	84·6626	84·6651	84·6676
174	84·6701	84·6726	84·6750	84·6776	84·6800	84·6825	84·6850	84·6875	84·6899	84·6924
175	84·6948	84·6973	84·6997	84·7022	84·7046	84·7071	84·7095	84·7119	84·7144	84·7168
176	84·7192	84·7216	84·7240	84·7264	84·7288	84·7312	84·7336	84·7360	84·7384	84·7408
177	84·7432	84·7455	84·7479	84·7503	84·7526	84·7550	84·7574	84·7597	84·7621	84·7645
178	84·7668	84·7692	84·7715	84·7739	84·7762	84·7785	84·7809	84·7832	84·7855	84·7878
179	84·7902	84·7925	84·7948	84·7972	84·7994	84·8017	84·8040	84·8063	84·8086	84·8109
180	84·8131	84·8154	84·8177	84·8199	84·8222	84·8244	84·8267	84·8289	84·8312	84·8334
181	84·8357	84·8379	84·8401	84·8424	84·8446	84·8468	84·8490	84·8513	84·8535	84·8557
182	84·8579	84·8601	84·8623	84·8645	84·8667	84·8689	84·8711	84·8732	84·8754	84·8776
183	84·8798	84·8819	84·8841	84·8863	84·8884	84·8906	84·8927	84·8949	84·8970	84·8992

Table IV.—*Continued.*

Inclination and Velocity Table, $Cd = \delta_{v'} - \delta_{v''}$.

v.	0	1	2	3	4	5	6	7	8	9
f. s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
184	84·9013	84·9035	84·9056	84·9077	84·9099	84·9120	84·9141	84·9162	84·9184	84·9205
185	84·9226	84·9247	84·9268	84·9289	84·9310	34·9331	84·9351	84·9372	84·9393	84·9414
186	84·9435	84·9456	84·9476	84·9497	84·9518	84·9538	84·9559	84·9580	84·9600	84·9621
187	84·9641	84·9662	84·9682	84·9702	84·9723	84·9743	84·9763	84·9784	84·9804	84·9820
188	84·9845	84·9865	84·9885	84·9905	84·9925	84·9946	84·9966	84·9986	85·0006	85·0026
189	85·0045	85·0065	85·0085	85·0105	85·0125	85·0145	85·0165	85·0185	85·0204	85·0224
190	85·0244	85·0263	85·0283	85·0303	85·0322	85·0342	85·0361	85·0380	85·0400	85·0419
191	85·0438	85·0458	85·0477	85·0496	85·0515	85·0535	85·0554	85·0573	85·0592	85·0611
192	85·0630	85·0650	85·0669	85·0687	85·0706	85·0725	85·0744	85·0763	85·0782	85·0801
193	85·0820	85·0838	85·0857	85·0876	85·0895	85·0913	85·0932	85·0951	85·0969	85·0988
194	85·1006	85·1025	85·1043	85·1062	85·1080	85·1099	85·1117	85·1136	85·1154	85·1172
195	85·1190	85·1208	85·1227	85·1245	85·1263	85·1281	85·1299	85·1317	85·1335	85·1353
196	85·1371	85·1389	85·1407	85·1425	85·1443	85·1460	85·1478	85·1496	85·1514	85·1531
197	85·1549	85·1567	85·1584	85·1602	85·1619	85·1637	85·1654	85·1672	85·1689	85·1707
198	85·1724	85·1741	85·1759	85·1776	85·1793	85·1810	85·1827	85·1844	85·1862	85·1879
199	85·1896	85·1913	85·1930	85·1947	85·1964	84·1981	85·1998	85·2014	85·2031	85·2048
200	85·2065	85·2081	85·2098	85·2115	85·2131	85·2148	85·2165	85·2181	85·2198	85·2214
201	85·2231	85·2247	85·2264	85·2280	85·2296	85·2313	85·2329	85·2346	85·2362	85·2378
202	85·2394	85·2411	85·2427	85·2443	85·2459	85·2476	85·2492	85·2507	85·2524	85·2540
203	85·2556	85·2572	85·2588	85·2604	85·2620	85·2635	85·2651	85·2667	85·2682	85·2698
204	85·2714	85·2729	85·2745	85·2760	85·2776	85·2791	85·2807	85·2822	85·2838	85·2853
205	85·2868	85·2884	85·2899	85·2915	85·2930	85·2945	85·2960	85·2975	85·2990	85·3005
206	85·3020	85·3035	85·3051	85·3066	85·3081	85·3095	85·3110	85·3125	85·3140	85·3155
207	85·3170	85·3184	85·3199	85·3214	85·3229	85·3244	85·3258	85·3273	85·3287	85·3302
208	85·3316	85·3331	85·3345	85·3360	85.3373	85·3388	85·3403	85·3417	85·3431	85·3446
209	85·3460	85·3474	85·3488	85·3503	85·3517	85·3531	85·3545	85·3559	85·3573	85·3587
210	85·3601	85·3615	85·3629	85·3643	85·3657	85·3671	85·3685	85·3698	85·3712	85·3726
211	85·3740	85·3754	85·3767	85·3781	85·3795	85·3808	85·3822	85·3836	85·3849	85·3863
212	85·3876	85·3890	85·3903	85·3917	85·3930	85·3943	85·3957	85·3970	85·3983	85·3996
213	85·4010	86·4023	85·4036	85·4049	85·4063	85·4076	85·4089	85·4102	85·4115	85·4128
214	85·4141	85·4154	85·4167	85·4180	85·4193	85·4206	85·4219	85·4232	85·4245	85·4258
215	85·4271	85·4284	85·4297	85·4309	85·4322	85·4335	85·4348	85·4360	85·4373	85·4385
216	85·4398	85·4411	85·4423	85·4436	85·4448	85·4461	85·4473	85·4485	85·4498	85·4510
217	85·4523	85·4535	85·4547	85·4560	85·4572	85·4584	85·4597	85·4609	85·4621	85·4633
218	85·4645	85·4658	85·4670	85·4682	85·4694	85·4706	85·4718	85·4730	85·4742	85·4754
219	85·4766	85·4778	85·4790	85·4802	85·4814	85·4825	85·4837	85·4849	85·4861	85·4873
220	85·4885	85·4896	85·4908	85·4920	85·4932	85·4943	85·4955	85·4967	85·4978	85·4990
221	85·5001	85·5013	85·5024	85·5036	85·5047	85·5059	85·5070	85·5082	85·5093	85·5105
222	85·5116	85·5128	85·5139	85·5150	85·5162	85·5173	85·5184	85·5195	85·5207	85·5218
223	85·5229	85·5240	85·5251	85·5262	85·5273	85·5285	85·5296	85·5307	85·5318	85·5329
224	85·5340	85·5351	85·5362	85·5373	85·5384	85·5394	85·5405	85·5416	85·5427	85·5438
225	85·5449	85·5460	85·5470	85·5481	85·5492	85·5502	85·5513	85·5524	85·5534	85·5545
226	85·5556	85·5566	85·5577	85·5588	85·5598	85·5609	85·5619	85·5630	85·5640	85·5651
227	85·5661	85·5672	85·5682	85·5693	85·5703	85·5713	85·5724	85·5734	85·5744	85·5755
228	85·5765	85·5775	85·5785	85·5796	85·5806	85·5816	85·5826	85·5836	85·5846	85·5856
229	85·5866	85·5876	85·5886	85·5896	85·5906	85·5916	85·5926	85·5936	85·5946	85·5956
230	85·5966	85·5976	85·5986	85·5996	85·6006	85.6015	85·6025	85·6035	85·6045	85·6055

CHAPTER XXI.

VARIETIES OF CANNON.

CLASSIFICATION.

The numerous ways in which cannon may be classified have been simplified by the almost universal adoption of those which are *breech-loading rifles*, *built up of steel.*

For convenience of treatment we may consider them according to their *proportions*, *construction* and *service.*

1. Proportions.

The facility with which breech-loading cannon of all lengths may be loaded has practically abolished the distinction between mortars and howitzers, although both terms are still used for pieces which do not differ materially in their proportions.

It has become customary to distinguish guns (Chapter I) from howitzers by calling the first named *rifles*, although all new howitzers are also rifled.

2. Construction.

As to construction, cannon are divided into muzzle-loaders and breech-loaders; some of the former class being still retained in service pending the preparation of those of the better type and also for subordinate purposes.

Breech-loaders may be divided into those having but one barrel, or *single fire* pieces, which are loaded by hand, and into *machine* guns, in which the loading is automatically performed by machinery. The former may be either the comparatively *slow fire* cannon, in which the cartridge and projectile are loaded separately, or the *rapid fire* in which

the ammunition makes but one package, as in small arms, and the recoil of which does not derange the aim.

Machine guns generally consist of a number of barrels so disposed, that while one is firing, the remainder may be loaded and prepared for loading. Like the rapid fire cannon these require metallic ammunition, and unlike them their size is imited by the weight of the required number of cartridges which can be conveniently kept in motion by the machinery; the latter is generally operated by hand.

3. Service.

According to their employment, cannon are divided into those for the *mountain*, *field*, *siege* and *sea coast* services.

The principal distinction here refers to the difficulties of transportation, for the rule is general that the most powerful cannon that can be efficiently transported should always be employed.

For field artillery especially, the principle of independence of function requires a very exact adaptation of the weight of the arm to the service required of it. Thus, we have, 1st, *Horse Artillery*, which, the cannoneers being mounted on horses, may accompany the Cavalry; 2nd, *Light Field Artillery*, which manœuvres with Infantry; and 3rd, *Heavy Field Artillery*, which forms batteries of position at important tactical points, and is intended to engage at long ranges.

This affords the following table:

CLASSIFICATION OF ARTILLERY ACCORDING TO

1. Proportions	Guns, for direct fire.		
	Howitzers, or Mortars for curved fire.		
2. Construction	Muzzle loading (obselete, retained)	Smooth bore.	
		Rifled.	
	Breech loading rifles	Single fire	slow.
			rapid.
		Machine guns.	

3. Service.	Mountain.	
	Field	Horse Artillery, very light.
		Light Field Artillery, medium.
		Heavy Field Artillery.
	Siege.	
	Sea Coast.	

SYSTEM OF ARTILLERY.

This term refers to the character and arrangement of the *matériel*,* as adopted by a nation at any particular epoch.

The principal requisites of a system of artillery are *simplicity*, *mobility* and *power*. To these the enormous armaments of the present day may add *economy*.

The improvements of the last four hundred years have had these qualities in view, the compromises between simplicity and mechanical efficiency, noted Chapter XVI, causing sometimes one, and sometimes another of these qualities to preponderate.

As in other nations the system of artillery in the United States service is still in an experimental state.

For lack of funds, withheld largely because of uncertainty regarding the direction of improvement, many obsolete weapons have been retained by us either unchanged, or *converted* so as to increase their power at a moderate expense.

The following description is therefore partly historical, and contains incidental reference to methods adopted in other countries whose political situation has made their immediate armament urgent. It is confined to slow fire guns, since other types of breech-loaders depend for their efficiency almost wholly upon the control of their recoil and upon the use of metallic ammunition; subjects not yet discussed. See Chapter XXIX.

* See Webster.

CONSTRUCTION.

I. MUZZLE LOADING CANNON.

United States.

The *field guns* used during the Civil War were of two kinds.

1. *The* 3 *inch wrought iron* (10 *pdr.*) *rifle.*

This was made by wrapping boiler plate around a wrought iron bar to form a rough cylinder, which was welded together under the rolls and finished in the usual manner.

It made a very strong, light gun well adapted to the Horse Artillery.

2. *The* 12 *pdr. Napoleon Gun, smooth bore.*

This was of bronze, cast solid. Its value depended upon the topography of the seat of war.

The broken surface of the Appalachian system and the heavy woods with which much of the country was covered restricted the fighting to ranges which, compared to those obtainable on the broad plains of Europe, are very moderate. For such ranges its heavy shell and well filled shrapnel were more effective than those of the rifle, and the initial velocity was so great that for ranges of about 1000 yards the trajectory of the smooth bore was flatter than that of the rifle.

The *siege guns*, in which mobility was less important, were of cast iron. Owing to the length of the bore and its relatively small diameter these guns were cast solid. The projectile weighed about 30 pounds.

One of these pieces, the Parrott, was strengthened by a wrought iron cylinder shrunk over the breech and reinforce.

In order to prepare so massive a forging a hot iron bar was coiled helically around a mandrel, brought to a welding heat and forged by axial blows of the hammer. To prevent

distortion during welding, the coil was held in a hollow cylinder. Several coils would be similarly welded end to end. The direction of the fibers gave great tangential tenacity, but for reasons given in Chapters XV, page 60, and XIX, page 12, the construction was faulty.

Sea coast guns were generally of cast iron, cast hollow on the Rodman principle. To some the Parrott construction was applied.

Since 1875 many Rodman guns have been converted on the Palliser (English) plan by reaming out the bores to receive a thick, wrought-iron tube, which was then rifled. Chapter XIX.

These tubes, first made by coiling as above described, were ultimately replaced by those of solid steel, the intrinsic strength of which was almost sufficient.

The wrought-iron tube was at first inserted from the muzzle; but, as it was liable to be carried out with the projectile, a stronger but much more costly breech insertion was employed.

With steel, which presented no false welds for the action of the powder gases, the muzzle insertion was resumed.

In this way many 10 inch smooth-bore Rodman guns were altered to 8 inch rifles. The 15 inch Rodman guns are retained unchanged for subordinate purposes.

Foreign Services.

Abroad a similar course was followed. In *France*, the old cast-iron guns were hooped with puddled steel, originally to retain the fragments on explosion. The bores were lined with a short steel tube. This method is now followed for subordinate pieces of large caliber.

England tried the Palliser plan of conversion for her old guns. For new guns wrought iron was at first exclusively employed; then wrought iron coils on a steel tube were used,

and finally with breech loaders steel throughout. The fear of the brittleness of steel, the consequent preference for the weaker though more ductile wrought iron, and the indifference to the molecular treatment of steel as practiced by their more exact neighbors, the French, have cost the English Government much loss in time and money.

To Krupp, in *Germany*, belongs the credit of first using steel in large masses. The weight of his ingots has increased since 1851 from two tons to seventy.

The construction of his cannon now requires relatively large units of construction. The tendency elsewhere is to reduce the weight of the maximum unit so as to avoid the large outlay for plant required only for its manufacture and handling. For it must be remembered that although cannon comprise the heaviest masses now made, yet their commercial importance is relatively small. Chapter XIV, page 2.

II. BREECH LOADING CANNON.

These may be classified according to the means by which the breech is closed; but, as this depends largely upon the form of gas check employed, this will be first discussed.

1. Gas Checks.

Many early efforts were made to prevent the escape of gas by some rigid fastening after the manner of a plug; but, owing to the erosion through the slightest crevice caused by dust, rust or fouling, the efficiency of these devices was short-lived.

The self-sealing gas check alone made breech loading practicable. Gas checks may be classified according as they are attached to or detached from the breech block.

Detached Gas Checks.

The ordinary metallic cartridge case is the best example of this class. The flexibility of its walls and its renewal at every fire peculiarly adapt it for this purpose.

But, since it would be impracticable to use cartridges of the size required for heavy cannon, the cartridge case may be replaced by a short permanent ring as shown in figure **1**.

This represents one form of an American invention, the Broadwell ring, *r*, with its obturator plate, *p*.

The gaseous pressure expands the thin edge laterally against the seat in the tube and also presses the ring bodily backward against the plate. The annular grooves, *g*, in the base of the ring serve as air packing; they also increase the intensity of the pressure on a vital surface, and, with the hollow, *h*, collect any fouling, which might otherwise occur on this surface.

The surface, *s*, is spherical so as to adjust itself easily to he spherical *seat* of the ring around the mouth of the chamber, past which the obturator plate is caused to slide by the motion of the breech block to which it is attached.

This form of gas check is difficult to maintain, as it is difficult to prevent entirely the escape of gas between the ring and the plate.

Attached Gas Checks.

These necessarily require some motion of the block in the direction of the axis of the piece and across the joint to be sealed.

Figure 2 represents the Freyre gas check of Spanish origin. It consists of a steel ring, *r*, of triangular cross-section surrounding a conical wedge, *w*. This last is formed with a spindle, *s*, passing axially through the breech block, *B*. The stem is surrounded by a spiral spring against which it acts by a shoulder. The thickness of the wedge is slightly less than that of the ring.

The gases press the wedge backward and thus expand the ring; when they cease to act the spring moves the wedge forward and thus prevents the ring from sticking in its seat.

Figure 3 represents the De Bange (French) gas check, de-

rived from that used in the Chassepot b l. rifle, a small arm firing a non-metallic cartridge The steel ring of figure 2 is replaced by a plastic ring, *r*, composed of a mixture of asbestos and tallow enclosed in canvass and having the joints through which the composition might extrude protected by metallic rings. When the *mushroom head*, *h*, is compressed axially the ring, *r*, expands laterally, giving a pressure per unit of area against the surface of its seat nearly equal to $\frac{p_0 R}{2\,l}$; in which R is the common external radius of the head and ring, and l is the length of its bearing.

A nut on the rear end of the spindle regulates the initial compression required for efficiency. A spring beneath the nut relieves it from shock as the head is thrown forward after firing by the elasticity of the tallow.

Comparison.

The Broadwell ring has to seal four surfaces not protected from dirt instead of but two, and the joint, most difficult to seal, is that which is most exposed to dirt.

Of the attached gas checks, the Freyre, being inorganic, is less subject to extreme variations of temperature; it also takes up less room in the thickest part of the gun. It is open to objection that a slight nick on the edge of the ring might render the entire apparatus worthless.

To the last consideration is due the almost universal employment of the De Bange gas check, since this has been found almost indestructible by the accidents of service and to resume its shape when deformed in firing.

2. Fermeture.

The fermeture (French, *fermer* to close) is the device by which the breech is opened and closed. Its principal requisites are safety and convenience. The form of fermeture depends largely upon the kind of gas check employed.

Two principal varieties exist, the Krupp and the French systems.

1. The KRUPP or wedge system, figure 4.

Description.

The breech block, *B*, moves transversely through a horizontal slot in rear of the chamber. The front face of the block is flat, and the rear surface a convex semi-cylinder whose axis is slightly inclined to the plane of the face. This avoids the sharp reëntrant angles noted, Chapter XV, page 21. It has been found expedient also to round the angles in front of the slot.

The upper and lower surfaces of the slot contain guides, *g*, which are parallel to the elements of the cylindrical surface and enter corresponding grooves in the block. The block thus receives a component longitudinal motion in the direction of the axis of the bore which prevents friction between the ring and the obturator plate, and also assists somewhat in pressing the cartridge home.

A hole, *h*, through the block permits the gun to be loaded when the block is withdrawn to the proper position. It is prevented from passing this point by a *stop bolt*, screwed through the body of the gun and having a blank end projecting into a groove on the upper surface of the block.

Locking.

To secure the fermeture a revolving *latch*, *l*, is employed. For small cannon using metallic ammunition this may be a simple turn-button operated by an exterior handle, *G*, and entering a recess in one of the faces of the slot.

With a less perfect gas check, means must be provided for pressing the obturator plate, *p*, against the ring, *r*, so that for larger guns the latch consists of a screw. In order to facilitate the operation of the fermeture, the fillets on one

side of the newel of the screw are removed so that a half-turn of the screw may engage or disengage the remaining fillets.

Translation.

For field pieces the block is withdrawn directly by hand, but heavy pieces are provided with a long screw, *S*, contained in a groove in the upper part of the block, and turning in two cylindrical collars, one at each end. The rotation of this screw in a half nut which is attached to the gun, causes relative motion to occur between the block and the gun.

Since for this motion speed is required, the screw is cut with a considerable pitch. As this causes a loss of the power required to start the block from its seat and to close it firmly, there is supplied an auxiliary locking screw, *d*, which passes through the latch, *l*. By a peculiar arrangement illustrated in a model in the Ordnance Museum, in closing the breech this screw first turns the latch and then by its slow pitch supplies the power required, and conversely in opening.

Both screws are operated by a T wrench, *G*, which is detached.

2. THE INTERRUPTED SCREW FERMETURE is commonly known as the French system, although its origin is probably American.

Description.

A cylindrical block fills the breech in the prolongation of the bore and in rear of the tube.

The block is held by a screw thread which engages with the base ring; this in turn is screwed to the jacket by a ratchet screw thread, Chapter XV, figure 47, and figures 2, 3 and 10, Chapter XXI.

To facilitate its operation, alternate sections, ordinarily of

60°, are removed from the adjacent surfaces of the block and base ring, so that after sliding the block nearly into place it may be easily locked.

Some device is required to support the block when withdrawn. For small pieces this is supplied by the *carrier ring*.

This ring is provided with two lugs forming, with corresponding cavities in the jacket and a vertical pin, a hinge on which it swings to the left and rear in opening.

A *stop*, *a b*, Chapter XV, figure 47, screwed to the carrier ring, enters a groove formed in one of the smooth sectors of the block. This groove terminates in front at a short distance from the face of the block, and in rear makes a return of 60° parallel to the screw thread.

The carrier ring also contains a shallow groove, *c d*, for the head of the lever, and the *latch*, *p*, the action of which is important. See figure 10.

The latch is pressed by a spiral spring radially inward against the block, so that its inner extremity describes on the smooth sector on which it rests a path parallel to the groove in which travels the stop. We will designate the rearmost end of this path by *r*, and the front end by *f*. At *r* and *f* are formed cavities into which the inner end of the latch may enter sufficiently to sink its outer end to the level of the outer edge of the carrier ring. Each cavity is connected with the intervening path by an inclined plane; the cavity at *f* is practically a cylinder.

On the rear face of the base ring is a conical dowel, the point of which, when the carrier ring is closed, enters a corresponding cavity in the adjacent face of the carrier ring. After passing this cavity, the point of the dowel enters a conical hole in the front surface of the latch, and thus serves to press it radially outward, so that when the carrier ring has been completely closed, the inner end of the latch will have been raised so far out of the cavity *f* that the block may slide

freely through the carrier ring. As it slides it forces the outer end of the latch into its seat in the jacket.

There are three concentric pieces, the block, the carrier ring and the jacket. The latch unites these alternately in pairs.

Operation.

Suppose the block to be closed and locked. Raise the lever and turn the block to the left until the stop prevents further rotation.

In so doing, the inner end of the latch rides up the inclined plane leading from *r*, and the outer end enters the jacket as shown in the end view of figure 47, Chapter XV. This prevents the obstruction to the withdrawal of the block caused by the simultaneous swinging of the ring which would otherwise occur.

The block can now ordinarily be freely withdrawn; but if, from the expansion of the gas check, it should not move freely, an eccentric projection on the head of the lever acts as a *cam** and starts the block from its seat.

It is well to observe that the rotation of the block being independent of that of the gas check, the binding of the latter does not resist the initial rotation above described.

On withdrawing the block to the extent allowed by the stop grove, the inner end of the latch drops into the cavity *f*; the carrier ring is then free to swing in continuation of the motion of withdrawal.

After loading, these motions are reversed. In closing the breech the latch locks the block and carrier ring together, since any slipping of the block through the ring would cause the edge of the gas check to strike against the base ring. This would be particularly objectionable in the Freyre check.

When the carrier ring comes against the rear face of the

* See Webster.

base ring the conical pin described lifts the pin from the hole, *f*, and permits the block to slide forward until ready to engage with the threads in the base ring.

After closing the breech the eccentric head of the lever enters the groove, *c d*; this prevents the unscrewing of the block by the tangential component of the pressure on the screw threads. This pressure is so great that it has been found necessary to protect the bearing surface of the groove, *c d*, by a plate of hardened steel.*

Variations.

For large pieces a more stable support than that offered by the thin carrier ring is required during the withdrawal of the block.

This is furnished by a tray which supports it for its whole length.

This tray is supported by a hinge bracket, called the *console*, which, being fastened to the face of the breech, allows the block and tray to be swung aside.

For such pieces the simple lever used in the field piece affords insufficient power.

It is accordingly replaced by more or less complicated machinery which, for the largest calibers, may be operated by steam, hydraulic or electrical power.

One of the most ingenious devices is that of the French engineer, Canet, who has an apparatus in which the continuous rotation of a crank performs all the varied operations of unlocking, withdrawing and swinging the block.

Vent.

The system adapts itself to the use of an axial vent which facilitates ignition. To permit renewal, the vent piece is

* The latest model (1890) exhibits slight changes in the details of the construction shown in figure 47, Chapter XV.

made removable ; and to avoid erosion, its front portion is of copper.

To avoid the danger of a premature discharge, the vent is preferably protected by a sliding shutter, a projection from which travels in a concentric groove in the rear face of the piece which is so formed that the primer cannot be inserted until the block is securely locked in place.

The complication attending the operation of an axial vent, the likelihood of accident to the gunners from the projection of the fragments of the ordinary primer and the necessary delicacy of the safety shutter when made on the small scale required for the field gun have so far caused these guns to be provided with a radial vent piece of copper leading to the top of the charge at about half its length.

Base Ring.

The seat of the block is of somewhat greater diameter than that required for loading in order to give a large bearing surface to the threads of the screw.

Under Barlow's law, this surface is less dilated by the gas pressure than one nearer to the axis ; and, since from a similar reason the greatest stress is borne by the foremost fillets, these do not approach as closely to the end of the tube as the construction might otherwise permit.

All exposed screw threads have their angles rounded, to avoid fracture and to resist deformation by the projectile in loading. For heavy pieces a *loading tray* is slipped into the opening so as to cover the thread in the base screw while the projectile is being pressed home.

The operation of the gun is very much facilitated by devices which avoid the translation of the breech block. One of these consists in giving the breech block a general conical shape so that it will swing directly into the position for locking.

The same end is accomplished in the Gerdon fermeture, figure 13, now on trial in the United States. After revolving the block through 90° so as to clear the two threaded sectors, it is swung to one side through a slot cut in the jacket. A radial slide on the rear face of the block acts both as a latch and as a shutter to the axial vent.

The parts are remarkably few and simple.

Comparison of the Two Systems.

1. Except where metallic ammunition is employed the French system permits the use of the best gas check.

2. It diminishes the weight of the gun for a given value of u and d. Chapter XII.

3. It serves to press the cartridge into place instead of *guillotining* it as in the Krupp.

4. The fermeture, when open, is less exposed to injury from a front fire.

5. It may be worked by power.

The Krupp system in its conception is of almost rustic simplicity.

This advantage is counterbalanced by the inferior gas check which is required when non-metallic ammunition is employed; also by the thickness and mass of the forging containing the slot, the presence of which must cause injurious internal strain in oil hardening.

The jar in opening it suddenly may deform and even bend the stop bolt. The parts are less securely protected in travelling. It has also the comparative disadvantages named in the discussion of the French system. The danger of premature discharge, though not so great as in the French system, is still said to exist.

U. S. SYSTEM OF ARTILLERY.

MOUNTAIN SERVICE.

The Hotchkiss 1.65-inch Rifle.

This gun weight but a little over 100 pounds and its carriage about twice as much, so that either makes but a fair load for a mule. Metallic ammunition is employed. The gun is a single piece of steel provided with the simplest form of Krupp fermeture as shown in figure 5 The operation of the fermeture can be readily seen from the figure and from previous discussions.

Its special feature is the *extractor*, *x*. This is a prismatic bolt, a hook on the front end of which engages with the flange of the cartridge (Chapter XVI, figures 8 and 9) as this is loaded

The extractor slides in a longitudinal groove, *g*, on the upper surface of the *slot* On its lower face is a tenon which enters a transverse groove, *g'*, in the upper face of the *block*.

The groove, *g'*, near the handle is straight and slightly inclined to the rear face, so as to give power in wedging the cartridge case from its seat. The screw thread on the latch also assists. At the other end it is so curved that when, in opening the breech, the loading hole comes opposite to the chamber the extractor will be suddenly drawn backwards, throwing the free cartridge case clear of the gun. The first of these operations is called the *extraction*, and the second the *ejection*

For simplicity this piece is fired with the ordinary friction primer. The blast from this raises the central portion of a thin, cup-shaped gas check within the cartridge, and the flame passes through the three holes shown in figure 8, Chapter XVI. As soon as the charge is ignited the back pressure of the gases closes the vent by reversing the action of the gas check.

Hotchkiss 3-Inch Mountain Rifle.

In order to permit the use of shrapnel a heavier mountain gun of 3-inch caliber has been recently produced. Figure 11.

Foreign Variations.

In order to increase both the power and portability of mountain cannon they are frequently made abroad in sections which are screwed together before firing. "Screw-guns" of 8-inch caliber have been successfully fired.

FIELD SERVICE.

The 3.2 B. L. Rifle shown in Chapter XV, figure 47, is the only new field piece now issued. (1891.) It is eventually intended for use as a Horse Artillery gun, and to be replaced for Field Service proper by a similar gun of 3.6 in caliber, firing a 20-pound shell. A 3.6 B. L. Mortar, figure 12, firing the same projectile, is also contemplated for delivering vertical fire against troops sheltered by temporary defences. It has a range of nearly two miles.

Foreign Variations.

It is proposed in France to have but one caliber, about 3 in. for all mountain and field service, viz., short, light, long and heavy pieces.

SIEGE SERVICE.

Siege cannon are intended for attacking and defending inland fortifications and the land fronts of sea-coast fortifications.

The term is usually applied to pieces which, although too heavy for field operations, are yet light enough to be transported over common roads upon the carriages from which they are fired.

This limits the weight of the gun and carriage together to that which may safely be transported across a pontoon bridge.

Siege Gun.

The 5-inch siege rifle, figure 6, resembles in its construction the field gun described.

Siege Howitzer.

Principles of Design.

Defences of masonry have been largely replaced by those which are armored, or, particularly for the besieging party, of earth.

While armor requires for its penetration the concentration of kinetic energy found in a projectile of relatively small diameter fired from a gun, the demolition of earth works demands rather the transfer of energy in the potential form. Such defences should therefore be attacked by cannon of the largest caliber consistent with portability.

If the maximum weight of the piece is fixed by the considerations previously named, then by the definition of Chapter I, a howitzer results.

The proportions of this piece are also demanded by the advantages pertaining to vertical fire againt the large and well defined area occupied by the besieged, against communications of the besieger which are screened from view, and against the roofs of turrets. The shorter the piece is in rear of the trunnions the more easily may high angles of fire from a given carriage provided with the ordinary elevating screw be attained.* The avoidance of preponderance and the requisite strength of the chase demand that the length in front of the trunnions be also reduced.

Such considerations have fixed the value of u at about 12 times the caliber, which is 7 inches.

It is intended to throw a projectile weighing about 100 pounds to a distance of about 3 miles.

*A new German howitzer has the trunnions placed almost at the breech. In this carriage the elevating screw is under the chase, as the arrangement adopted gives a considerable muzzle preponderance. See also Carriage for 7-inch Howitzer, Chapter XXIII, figure 6.

Owing to the strength of the construction a larger caliber might have been employed for the given weight, but in such a case the energy of recoil, (Equation 7, Chapter XIX,) would have been excessive.

Since with the value of $\overline{p}_0$ usual in built up steel guns, the short length of bore reduces the value of e, it is proposed to utilize the value of E permitted by the strength of the carriage by increasing the value of m.

This will permit the use of very long torpedo shells (Chapter XVI, page 20). The limit of E for the wheeled carriage having thus been reached, for high angles of fire which increase the stress upon the axle, E may be further increased by dismounting the wheels and laying the stock of the carriage on a platform It is now (1891) proposed to use in the *field* service a 6 inch B. L mortar throwing a 70 pound shell, to be mounted as above described

It is probable that in the future the obstruction to efficiency which is due to the requirement that the piece be transported on the carriage from which it is to be fired, will disappear before the adoption of special carriages designed with the view of efficiently satisfying their independent requirements.

Charges.

In order to vary the angle of fall* to suit the range and the kind of fire employed, the howitzer is fired with varying charges of powder as well as with varying angles of fire. In this it differs from the gun in which the charge is usually a constant and a maximum. See Chapter XXX, page 9.

7-Inch B. L Howitzer

The construction, figure 8, resembles that of the field piece, the principal difference being in the construction of the key ring. This consists of two semi-circular segments of rect-

* The angle with the horizontal made by the tangent to the trajectory on impact.

angular cross-section which are laid in a shallow groove in the tube so as to project above its exterior and to bear against the front face of the trunnion hoop. They are kept in place by the lap of the sleeve.

The friction developed by shrinkage between the jacket and the tube throws part of the longitudinal stress upon the tube from which the key ring transfers this stress to the trunnions.

A shoulder formed on the tube in rear prevents the forward motion of the tube from the friction between it and the projectile. See page 5. This feature is general.

The cavities in the ends of the trunnions are for the points of the *bail* in which the piece is slung in mounting.

SEA COAST CANNON.

These comprise rifles of and above 8-inch caliber, and 12-inch rifled mortars.

Rifles.

Figure 7 shows the 8-inch b l. steel gun with which most of the recent experiments have been made.

The construction resembles that heretofore discussed, except that the jacket is strengthened by two rows of hoops, which since the original design, have been extended to the muzzle. (Chapter XI, page 18, paragraph 4.)

Other S. C. Rifles.

The guns so far designed are of 8, 10, 12 and 16-inch caliber. Being intended for use with the largest charges of slow burning powder, they are made very long, the values of u ranging from about 24 to 27 calibers.

For the largest calibers it is proposed to dispense with trunnions which are to be replaced by several rows of circumferential ribs, by which, *as in cannon of the very earliest times*, the pieces are to be secured to their support. The necessary alterations in elevation will be given by varying the

inclination of the chassis, to which by this arrangement the recoil is always parallel.

Mortars.

The importance of nearly vertical impact against the decks of vessels at short ranges requires the mortars to fire with angles of elevation as great as 75°.

It is proposed to group them in sunken batteries of 12 or 16 mortars, united under the control of one officer. He will occupy a detached position free from smoke, and will be provided with an accurate range finder, and with means of communicating to the chiefs of pieces the direction and elevation required. A simultaneous volley from the battery will probably drive from its anchorage any vessel within range. This will be an important aid to the defense, since, as the bombardment of Alexandria in 1882 clearly showed, the accuracy of fire from a vessel is much diminished when the vessel is under way.

12-*inch B. L. Mortar.* (*Figure* 9.)

The immediate supply of these cannon demanded by our present necessities (1891) and the relatively low energies required to penetrate armored decks by vertical fire have so far permitted the body of these pieces to be made of iron cast on the Rodman principle and strengthened by two rows of steel hoops as shown in figure 9.

The recent failure at a pressure of less than 20,000 pounds of an unhooped 12-inch cast iron mortar would indicate the future use of steel throughout the piece as soon as the steel works and the gun factory shall have become able to supply a sufficient number of heavy steel guns.

The growing importance of vertical fire has caused the employment of mortars, even upon ship board, to be seriously considered.

The value of u in this piece is about 6 calibers.

UNITED STATES ARMY BREECH-LOADING RIFLED ORDNANCE, 1890.

CALIBERS.	Weight	CHARGE.		Powder pressure.	Initial velocity.	Muzzle energy.	NUMBER OF GUNS, ETC.	
		Powder.	Projectile.				Ordered.	Completed
MOUNTAIN AND FIELD ARTILLERY.	Pounds	Pounds.	Pounds.	Tons.	Feet.	Ft-tons.		
3-inch Mountain gun, steel.	218	0.88	12.	6.5	870.	63.	1	1
3.2-inch Light field gun, steel.	829	3.75	13.5	15.	1675.	263.	100	75
3.6-inch Field gun, steel.	1181	4.50	20.	16.	1554.	335.	1*	1
3.6-inch Field mortar, steel.	244	1.00	20.	8.	650.	58.	1§	1
SIEGE ARTILLERY.								
5-inch guns, steel.	3660	12.50	45.	16.	1830.	1000.	11	1
7-inch howitzer, steel.	3710	9.75	105.	12.5	1085.	857.	11	1
SEA-COAST ARTILLERY.	Tons.							
8-inch gun, steel.	14.5	130.	300.	16.5	1935.	7787.	25	2
10-inch gun, "	30.0	256.	575.	16.5	1940.	15000.	24	1
12-inch gun, "	52.0	440.	1000.	16.5	1940.	26000.	16	—
12-inch mortar, cast-iron, steel hooped.	15.25	80.	630.	12.5	1152.	5796.	74	1
12-inch mortar, steel.	13.0	100.	800.	16.	1150.	7334.	1	—

* Orders for 24 guns await completion of tests of type gun. § Orders for 16 mortars await completion of tests of type mortar.

CHAPTER XXII.

ARTILLERY CARRIAGES.

PRINCIPLES OF CONSTRUCTION.

Classification.

Artillery carriages may be classified according as they are intended to *support* the piece when fired, to *transport* it, and to *supply* it with ammunition and accessories. These functions are sometimes combined.

They may also be classified according to the *service* in which they are employed.

Requisites.

1. *Strength* to resist permanent deformation from the shock of recoil.

2. *Stability* in firing and on the march.

3. *Mobility* as regards the ease of service in battery, and of transportation when required.

4. Only a *moderate recoil* in firing, so as to facilitate the service of the piece and to avoid the exposure of the gunners when sheltered by defences. A compromise between these properties is often necessary.

I. CARRIAGES WHICH SUPPORT THE PIECE.

GENERAL DESCRIPTION.

These are called *gun carriages.* They may be either *stationary* or *wheeled.*

Stationary Carriages.

The simplest form exists in the iron *mortar bed* used for the old S. B. Mortars.

This consists of trapezoidal plates forming the *cheeks*, which support the trunnions of the piece at such a height as to permit it to receive the elevation required. The cheeks are connected by transverse diaphragms called *transoms* and bolts in order to form a strong frame. For heavy pieces each cheek may consist of two plates united to a T shaped bar as shown in figure 1. The cheeks may be stiffened in the direction of compressive stresses by bars included between the plates.

The bearings of the trunnions may be widened by *trunnion bed plates*, so as to diminish the pressure per unit of area which they are called upon to support. The mortar bed is made low for ease of loading and for stability. The principles of construction above noted are of general application.

Sea coast gun carriages are intended to be used in firing over a parapet or through an embrasure. In the first case they are called *barbette* carriages, and in the second, *casemate* carriages. Owing to the height of the piece above the ground and the low angles of fire employed, the stability of the system generally requires the support of the piece to be divided into the gun carriage proper, constructed like the mortar bed, and the *chassis*, which is a moveable railway capable of directing the piece in its recoil and of being *traversed* in azimuth.

Non-recoil carriages are separately treated hereafter.

Wheeled Carriages.

In the mountain, field and siege services, the gun carriage must also be adapted to transportation. This involves the use of wheels, which complicate the problem of controlling the recoil.

Parts.

Their essential parts are: —

1. The *stock*. This is a prolongation of the cheeks which, with the wheels, forms the *three* points necessary for stability.

A greater number of supports might affect the stability on uneven ground.

The stock serves also to point the piece, since it sustains the elevating screw, and with the aid of the handspike gives the necessary changes in azimuth

It also connects the front and rear wheels in transportation.

For modern carriages the stock consists of two sheet steel *flasks* or *brackets* which rest at the *head* of the stock upon the axle, and are united at the further end by the *trail plate* or *shoe*.

2. The wheels and the axle replace the continuous support afforded in stationary carriages by the cheeks.

PERIODS OF THE RECOIL.

The recoil may be separated into two periods:

1. That during which the projectile is acquiring energy in the piece.

2. That comprising the subsequent recoil. Since the carriage is found not to move materially until the projectile has reached the muzzle, and since the system is not rigid, the corresponding phenomena may be taken to be:

1. A series of shocks between the trunnions and their beds transmitted through the axles to the wheels and through the stock to the trail. The system is finally set in motion by these shocks.

2. The resulting motion of the system accelerated by the remaining gaseous pressure and retarded by friction and various artificial resistances.

Energy of Recoil.

The nature of the recoil is preferably studied by velocimeters of Class III. But, as this is difficult and requires the previous construction of the carriage, it is customary for theoretical discussions of a general nature to ignore the first

period, and to assume that the system is rigid and that the acceleration to the system during the second period is compensated for by the acceleration of the projectile noted in Chapter XI, page 18.*

We may therefore change Equation (7), Chapter XIX, to read

$$E_s = \frac{m\,e}{M_s} \quad \text{or} \quad V_s = \frac{m\,v}{M_s} \tag{1}$$

in which the subscript s refers to the entire system recoiling.

For greater accuracy, when the mass of the powder, or m', bears a considerable ratio to that of the projectile, we may use the following formula in which v' represents the mean velocity of the products of combustion, found by experiment to be about 3,000 *f. s.*

$$V_s = \frac{m\,v + m'\,v'}{M_s} \tag{2}$$

Distribution of Work of Recoil.

Since this work is distributed between the two periods, and since it is necessary to restrict the exent of the recoil, it becomes necessary, as in Chapter V, to determine the maximum stress which the system can safely endure and to maintain this stress as nearly constant as possible over the path of the recoil.

This principle, which underlies all recent improvements, in gun carriages, owes its importance to the recent increase of m and e and the decrease in M (Equation 7, Chapter XIX), due to the general use of *built up rifled cannon* firing *large* charges of *progressive* powder In fact, it may be said that the limit of the power of cannon, or h, page 21, Chapter XI, is fixed by the difficulty of controlling their recoil,

If we assume that the mobility of the system fixes the sum of the masses, M and M', composing the gun and carriage,

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* This assumption will be corrected as occasion arises hereafter.

the following discussion explains the prevailing practice of making M' approximately equal to M.

For, if we assume the carriage to be properly proportioned, general experience shows that its permanent deformation, Q, may be considered inversely proportional to its mass and directly proportional to the energy which it receives. So that

$$Q \propto \left(\frac{E}{M'} = \frac{m\ e}{M\ M'} \right). \tag{3}$$

If $M + M' = C$, Q will be least when $M = M'$.

The assumption on which this deduction is based, although confirmed by experience in the construction of carriages, anvils and armor, is not conclusive; since mechanical ingenuity may compensate for the loss of strength resulting from a diminution of M'.

REMARKS.

It is found that with quick powders the velocity of recoil during the first period is greater than with slow powders, the maximum momentum of the projectile being the same.

With slow powders the velocity during the second period is increased to such an extent by the high pressure as the projectile leaves the gun (Chapter XI, page 18), that special devices have become necessary to diminish the increased extent of the recoil.

The problem is so complicated that computations, principally by graphical methods, are mainly resorted to in order to determine the direction of the stresses, the corresponding dimensions being found empirically. It is highly probable that the gun carriages of the future, like many other constructions, will be the outgrowth of practical experience.

FORCES ACTING ON A GUN CARRIAGE.

Velocity of Translation.

If the axis of the bore intersects the axis of the trunnions at the centre of gravity of the piece, the force producing re-

coil is communicated to the carriage at the trunnion beds. The carriage being constructed symmetrically with regard to the axis of the piece, we may suppose that the wheels, trunnion beds and trail are all situated in the same plane and that the force producing recoil is applied at the point where the axis of the trunnions pierces this plane.

The direction in which this force acts will be that given by the *angle of fire* or the inclination to the horizontal of the axis of the piece.

Let v figure 2, be the position of the axis of the trunnions, and $m\,v = I_l$, represent the intensity and direction of the force, and θ the angle of fire. Let T be the point of contact of the trail and ground, l the distance of this point from the trunnions, and α the angle made by the line $T\,v$, with the horizontal. Let W_s be the weight of the system acting through the center of gravity G at the horizontal distance b from the point T. Let f be the coefficient of friction between the carriage and the horizontal platform on which it rests.

The vertical component of I_l, and W_s will cause friction between the carriage and the platform. The force of friction or $f\,(W_s + m\,v \sin\theta)$, will oppose motion. So that, representing by V the horizontal velocity of recoil, we have

$$V = \frac{m\,v \cos\theta - f\,(W_s + m\,v \sin\theta)}{M_s},$$

$$= \frac{m\,v\,(\cos\theta - f \sin\theta)}{M_s} - g\,f, \qquad (4)$$

in which $g\,f$ may be neglected.

The vertical component will be distributed along the supports in a manner determined by the construction of the carriage and the values of θ. For wheeled carriages f will have separate values for the wheels and the trail.

If, in Equation 4, we neglect the weight of the system in comparison with the vertical component of I_l, (or $g\,f$), we

find that V will reduce to 0, or that recoil will cease for a value of θ, such that, calling this angle θ_1

$$\tan \theta_1 = \frac{1}{f} \tag{5}$$

This is called the angle of no recoil.

Extent of Recoil.

The extent of the recoil will be

$$s = \frac{V^2}{2\, g f}. \tag{6}$$

If, as is usual, the platform be inclined at an angle, β, with the horizon so as to check the recoil, then for θ in the above equations should be written

$$\theta + \beta; \text{ and for } V,\ V \cos \beta.$$

In this case, since the energy of the recoil is absorbed not only by friction but by the work done in lifting W_s through a height $= s \sin \beta$, we have

$$s = \frac{V \cos \beta^2}{2\, g\, (\sin \beta + f \cos \beta)} \tag{7}$$

These equations are said to give correct values for stationary carriages, but do not apply very exactly to those which are wheeled.

Angular Velocity.

The force, I_1, also acts to rotate the carriage around the point T with a moment proportional to its lever arm, $l \sin (\alpha - \theta)$, so that the moment of this force will be $m\, v\, l \sin (\alpha - \theta)$.

This is opposed by the moment of the weight, or $W_s\, b$.

Then, since the angular velocity of the system is equal to the resultant moment of the impressed forces divided by the moment of inertia of the system, we have, representing the

angular velocity, about T by ω and the corresponding radius of gyration by k.

$$\omega = g \frac{mvl \sin(\alpha - \theta) - W_s b}{W_s k^2} \tag{8}$$

With this relation we may discuss in an elementary manner the stability of the system. For example, in the old S. B. Mortars, since W_s is small, α is made less than θ, so as to make ω negative.

Phenomena of Recoil.

Practically the phenomena are much more complex, since the rotation of the system is not immediate.

Wheeled Carriages.

In these the wheels tend to slide or rise to an extent determined by the resistance to sliding at T. Since T is not necessarily on a reciprocal axis of spontaneous rotation, the stock is subjected to a transverse stress. When, after rotation, the wheels fall, the axle receives a shock, and the trail being thrown up, the system recoils by rolling, and so on until the system comes to rest.

Rotation during the first period tends to derange the aim by what is called the angle of *jump*. See Chapter XX.

During the first period, since the trunnions fit rather loosely in their beds, a frictional moment is developed on the under side of the trunnions. This causes abnormal pressure on the head of the elevating screw, which, owing to the elasticity of the system, subsequently receives one or more severe blows from the breech. The effect is destructive, since the bearing of this screw upon its nut is restricted, and the necessary play between the screw and nut increases the striking velocity of the parts.

The objections may be mitigated by making the piece without preponderance, and by arranging the elevating screw so that its axis will be always normal to the surface which it

supports, since this will avoid the tendency to bend under pressure.

Other phenomena also occur.

The inertia of the wheels develops in the axle a considerable transverse stress.

In rifled pieces a rotary moment is developed which tends to raise from its bed the trunnion toward which the top of the projectile is revolving, and thus to raise one wheel higher than the other as the system jumps. The effect will be to concentrate most of the shock of the fall upon the lower wheel.

Stationary Carriages.

The chassis of stationary carriages revolves around a massive vertical pintle which may be placed in front of the chassis or at its middle.

While the former position is necessary for pieces firing through embrasures, in other cases the *center pintle* chassis is preferred, since a given change in azimuth covers less ground.

The tendency of the top carriage to jump is restrained by projections which engage under the chassis, but tend to lift the pintle from its socket. The pintle is also exposed to a horizontal stress nearly equal to the normal pressure between the carriage and the chassis multiplied by the sine of the inclination of the chassis to the horizon.

The strength of the pintle and its fastenings is therefore an important subject of consideration.

Angle of Greatest Recoil.

Since the energy of recoil is distributed between the motions of rotation and translation, the maximum velocity of recoil will follow from an angle of fire such that $\omega = 0$. Equation (8) does not contain all the data necessary for a

full discussion; but it may be shown that, if the weight of the system be neglected, and the notation of figure 2 be adopted, if we call the angle of greatest recoil, θ_2, then

$$\tan \theta_2 = \frac{h' - h}{a - f h} \tag{9}$$

Since $h' > h$, this value of θ_2 is always positive, or the maximum velocity of recoil will follow the use of an angle of fire >0. This value of θ should therefore be employed in all calculations relating to limiting the extent of recoil.

The angle θ_2, may also be called the *angle of no rotation*, *i. e.* the angle for which all the energy of recoil is expended in translation only. It may be taken to measure the exposure of the system to the injurious shocks resulting from rotation, since, in ordinary firing, θ is less than θ_2, and therefore rotation is ordinarily produced.

Equation 9 shows that θ_2 may be diminished by the following means:

1. By making $h' - h$ as small as possible. Owing to the length of a in stationary carriages this correction is principally confined to those that are wheeled. In these h' is made as small as facility of loading and the protection of the gunners by the parapet will permit, and h is increased by bringing the axle as near to the trunnions as the size, strength and weight of the wheel will allow.

2. By making a as long as conditions relating to mobility in transportation will permit.

3. By making f small.

In stationary carriages f is normally great, and, as hereafter seen, the resistance to sliding is generally artificially increased. For these carriages it is especially necessary that a be made large.

The stresses developed in field carriages by a large value of f, as when the site is sandy or the trail is rested against a

rock, are evidently prejudicial. They may sometimes be inevitable, as when firing across a valley at a high mark, since in such a case the trail may require sinking into a hole.

DEVICES TO CONTROL RECOIL.

These may be considered according to the end in view, as they seek, I, merely to limit the path; or, II, to regulate the resistance.

It is generally advisable to store up enough of the work of recoil to assist in bringing the piece back into battery. The return may be facilitated by the use of eccentric rollers.

These devices are often combined in the same carriage.

Class I. To the first class of devices belong those in which the energy is absorbed by friction or in which a weight is raised.

Stationary Carriages.

Friction Checks.

This variety is least valuable since it stores up no useful work. In the best types the friction, due to the normal component of the weight, is increased by the artificial pressure of a screw clamp.

The effect of a given pressure on the screw may be increased by increasing the number of surfaces upon which it acts. Thus, in Ericcson's *compressor* we have n parallel plates attached to the chassis and alternating between $n+1$ pieces so attached to the carriage that while they recoil with it, a slight initial play is allowed. Suppose this play to be destroyed by a normal pressure P. We shall then have for the friction of the compressor $F = Pf(2n)$ and from Equation (7)

$$S = \frac{(v \cos \beta)^2}{2g\left(\sin \beta + f \cos \beta + \frac{F}{W_s}\right)} \qquad (10)$$

The main objection to this system is the variable value of F, since this depends upon the judgment of the operator, and upon the state of the surfaces, and is greater for static friction, when the acceleration is greatest, than afterwards.

This arrangement is modified in the *Sinclair* check used with some converted U. S. Sea Coast guns.

This consists essentially of a clamp embracing a plate increasing slightly in thickness from front to rear. To prevent the plate from *buckling* in consequence of the *counter recoil* produced by the elasticity of the parts, the front end of the plate is free to move forward through its attachment to the chassis.

Wheeled Carriages.

These may be *braked* by various means. Among these is the *Hotchkiss* brake which consists of nuts threaded upon the axles between the wheels and serving, by friction produced against the hubs of the wheels, to keep the wheels from turning.

This brake is an example of the *friction clutch* often employed in the transfer of energy. When the tangential component of the force producing rotation exceeds that of friction, sliding takes place and the destruction of the resisting parts is averted.

This principle is sometimes applied to the elevating screw, since the clutch, which is required to vary θ only, will yield under the shocks of recoil and save the deformation of the parts.

A simple brake may be extemporized by lashing the wheels to the trail by a rope; but, as this strains the wheel, a better way, often used, is to rest the wheels on *shoes* attached by tension to the trail, as in wagons of commerce.

The latest patterns of brake admit of a partial distribution of the pressure, as explained later and in Chapter XXIII.

They may also be used in transportation without necessarily stopping the carriage, as is required when the shoe is used.

Raising a Weight.

If the piece raise its own weight, its exposure is increased; while, if it raise a counterpoise, it may itself descend. Such carriages are called *disappearing carriages.*

Moncrieff Carriage.—Figure 3

In this the flasks rock on the chassis so that the counterpoise, *W*, which was at first beneath the gun, has finally a considerable moment of restitution. By varying the curvature of the flasks this moment may be made to vary inversely with the acceleration of the recoil so that the stresses between the piece and the counterpoise may be nearly constant. Conversely, the return to battery will be gentle. The rack and pinion serve to retain the piece for loading, and to control its return to battery.

King's Carriage.

The chassis is steeply inclined to the rear, and the counterpoise, which is in a well, is lifted by a rope passing through the pintle.

This carriage, invented by Major King of the Engineers, is cheaper than the Moncrieff, and has been successfully tried in the United States.

In both carriages exposure may be minimized by aiming with mirrors, and by firing by an electrical contact automatically made when the piece comes into battery.

Regulation of Stress.

1. BY FLUID PRESSURE.

The method now generally adopted is the use of hydraulic or pneumatic buffers.

These consist essentially of a cylinder and a piston, relative motion between which results from the recoil. The effect is to force the fluid contained in the cylinder through orifices or *ports* which may be either constant or variable in size

In the pneumatic buffer the ports are in the cylinder heads; in the hydraulic buffer, as the liquid is to be used again, they are in the piston.

Pneumatic Buffer.

This, although simpler and requiring less attention than the hydraulic buffer, is more bulky, can be less easily regulated, and gives an injurious counter recoil.

Hydraulic Buffer.

Description. Let the arrangement be in principle such as shown in figure 4. C is the cylinder filled with a non-freezing mixture of glycerine and water; it is attached to the carriage P is the piston fixed on the rod R, which is secured to the chassis.

Many alternative arrangements are made.

By placing it under tensile stress during the recoil, the bending of the rod may be avoided.

The size of the ports in the piston may be varied by the profile of the ribs, r, which are fixed to the interior of the cylinder, or by a notched disc revolving on the piston and provided with projections which enter rifle grooves in the cylinder.

We will consider only the second period of recoil, and will neglect the friction of the liquid and that of the rod in its *stuffing box*, so that the pressure considered will be that required to give a constant acceleration to the fluid.

The value of this method appears from the fact that it may safely restrict the most powerful cannon to a recoil of about 3 calibers.

Notation.

Let:

A be the area of cross-section of the bore of the cylinder diminished by that of the piston rod and ribs.

a_0 the total initial area of the ports.

a this area at the end of the time, t, or after a displacement, x.

v' the corresponding velocity of the liquid current.

u the corresponding velocity of recoil.

V_0 the initial velocity of recoil, obtained either by measurement or by means of the formula

$$V_0 = \frac{\left(m + \frac{m'}{2}\right)}{M_s} v \left[\cos\left(\theta_2 + \beta\right) - f \sin\left(\theta_2 + \beta\right)\right] \tag{11}$$

derived from Equation (9), Chapter VII, and the remarks noted on pages 7 and 10 herein.

δ the density of liquid, or the *mass* of one unit of its volume.

P the pressure on the piston at any moment.

α the corresponding acceleration.

Extent of recoil. If the cylinder is full, the volume $a\,v'$, of liquid which in a unit of time passes from in front of the piston to the rear must be equal to the volume, $A\,u$ caused by the translation of the cylinder, whence

$$v' = \frac{A\,u}{a} \tag{12}$$

The mass of the liquid escaping in the time $\Delta\,t$ is then

$$m = \delta\,a\,v'\,\Delta\,t = \delta\,A\,u\,\Delta\,t \tag{13}$$

and its energy

$$\frac{m\,v'^2}{2} = \frac{\delta\,A^3\,u^3 \Delta\,t}{2\,a^2} \tag{14}$$

This is equal to the work done by P over the path $u \triangle t$, and therefore

$$P = \frac{\delta A^3 u^2}{2 a^2} \qquad (15)$$

Since P is constant, Equation (15) must be true, for the initial values of u and a, and therefore

$$P = \frac{\delta A^3 V_0^2}{2 a_0^2} \cdot \qquad (16)$$

From this, by equating the initial energy of the system, with the work of the resistances, including the lifting of the weight of the system, and the work of friction over the path S, we have

$$\frac{M_s V_0^2}{2} = S \left[\frac{\delta A^3 V_0^2}{2 a_0^2} + g M_s (\sin \beta + f \cos \beta) \right], \qquad (17)$$

from which S can be determined when A and a_0 are known, or from which a_0 can be determined when S and A are known.

Profile of the ribs. From the theory of energy we have

$$\frac{M_s V_0^2}{2} - M_s \alpha x = \frac{M_s u^2}{2}; \qquad (18)$$

or

$$u = V_0 \sqrt{1 - \frac{2 \alpha x}{V_0^2}} \qquad (19)$$

Also, since the recoil is uniformly retarded if we consider the resistance of the liquid only, we have

$$V_0^2 = 2 \alpha S$$

which value of V_0^2 in Equation (19) gives

$$u = V_0 \sqrt{1 - \frac{x}{S}} \qquad (20)$$

which from Equations (15.16) may be written

$$a = a_0 \sqrt{1 - \frac{x}{S}} \qquad (21)$$

If there are n similar ports, the area of each one is $\frac{a}{n}$. If each notch in the piston has a breadth, b, and a depth, d, and the rib has the same breadth and, as shown by figure 4, a variable depth, y, then

$$a = n\, b\, (d - y) \tag{22}$$

Which value of a substituted in Equation (21) and solved with respect to y gives

$$y = d - \frac{a_0}{nb}\sqrt{1 - \frac{x}{S}} \tag{23}$$

This is the equation of a parabola.

At the end of the recoil, when $x = S$, $y = d$, or the ports are completely closed

This formula applies only to the path of the recoil after the system has acquired its maximum velocity or during the second period.

While the projectile is in the gun the piece recoils from one to two inches and continues to gain velocity for four to six inches more so that the maximum velocity is not attained until after a recoil of five to eight inches

Figure 11 shows these phenomena and the effects of a suitable control, based largely upon the analysis of velocity curves obtained during the practically free recoil of the piece. The data were as follows: Weight of piece, about 100000 lbs.; of projectile, 754 lbs.; of powder, 244 lbs.; initial velocity, 1857 f. s.

Cylinder. The thickness of the walls of the cylinder may be determined from Equation (4), Chapter XIX, by placing

$$\overline{p}_0 = \frac{P}{A} = \frac{\delta}{2}\left(\frac{A}{a_0}\, V_0\right)^2.$$

The area A will be determined practically by the construction of the chassis. As the depth of the chassis limits the

diameter, it is customary to have two cylinders connected by a tube so as to equalize the resistances and prevent slueing.

Counter Recoil. Owing to the incompressibility of most liquids the tendency to counter recoil is slight; and, as the velocity of return is small, the weight of the system generally suffices to return it to battery.

Or the liquid may be forced into another vessel or a set of stationary vessels containing air or powerful springs, which store up energy to return the pieee to battery when a valve or latch is opened. Figure 5 illustrates the operation of such a carriage. When air is compressed by the liquid, the variety is known as the *hydro-pneumatic*

Regulation of Stress.

2 BY THE ELASTICITY OF SOLIDS.

The weight of the cylinder, and the difficulty of preventing leaks in the preceding apparatus, renders it objectionable in wheeled carriages, and more so for those used in the field service than for those in the siege service

A compromise has therefore been sought by the interposition of an elastic solid, the work done upon which in the first period will reduce the shock felt by the system. The restoration of this work is not essential, although it tends to distribute the stresses over the path.

Such an arrangement is shown in the Engelhardt (Russian) carriage, figure 6.

Engelhardt Buffer. The flasks are notched so as to allow the axle, *a*, a limited play. They are also similarly pierced for the cross-bar, *e*, each end of which is united to the outer end of *a*, by a brace, *k*. This keeps the axle from bending easily since the force of recoil is applied close to the wheels.

The transom, *c*, separates *e* from an elastic buffer, *b*. The buffer consists of layers of cork, rubber, or of *Belleville*

Springs (*post*), assembled on a bolt, *f*, the front end of which is secured to *e*, and the rear end of which is provided with a nut, *d*.

When discharged, the piece, with its flasks and *c* and *b*, slides back relatively to the wheels and *f* and *e*, so that *b* is compressed. A considerable proportion of the energy of recoil is thus absorbed before the wheels begin sensibly to move.

After recoil, the elasticity of *b* restores the parts.

Belleville Springs. These are saucer shaped discs of sheet steel, pierced by an axial hole by which they are united in pairs on a spindle, base to base. They are now much used under compression where space is limited.

Lemoine Brake. The French artillery have borrowed from the omnibus of Paris a more perfect but more complex brake, figure 12. On the march it may be set against the tire, by hand, as in the wagons of commerce. When the piece is fired, the relative motion of a mass, *m*, throws forward the elastic cross-bar, *b*, to each end of which is attached a tapering cord, *c*. Each cord after making several loose turns around the nave is secured to the brake beam, *B*. When *m* is thrown forward it is held in place by the serrated edge of an axial bar, *b'*, to which it is secured. The motion of *b* stretches the cords and tightens them around the naves so that they are further wound up by the revolution of the wheel in recoiling.

The greater the extent of the recoil at any instant, and therefore the less the velocity of recoil, the thicker will be the cord and therefore the greater will be the increment of the pressure of the brake upon the tire.

See also the U. S. *Buffington* brake in the next chapter.

PLATFORMS.

To insure continued accuracy of fire from the same site, it is absolutely necessary that the carriage should rest upon a solid and substantial platform.

The mobility of field pieces restricts this necessity to the sea coast and siege services.

In the sea coast service the platforms are constructed by the Engineer Department with the works which the cannon defend.

Wooden platforms are employed for siege pieces, in which long continued firing at one object as in breaching, would cut into the unprotected soil deep ruts, which would increase the difficulty of serving the piece and restrict both its horizontal and vertical field of fire.

The construction of the platform should be such that it may be taken up without injury for removal to another site.

Siege platforms consist of a certain number of pieces of wood; and in order that these pieces may be carried on the backs of soldiers from the depot to the battery, the weight of the heaviest piece should not exceed fifty pounds. Siege-platforms consist of *sleepers* (1), (fig. 7), and *deck-plank* (2). The general direction of the sleepers is parallel to the axis of the piece, and the deck-plank at right angles to it; this disposition of the parts offers the greatest resistance to the recoil of the carriage. The deck-planks are fastened together at their edges by dowels; the outer planks are secured by iron eye-pins, one at each end of a sleeper. The platform is secured in its place by driving stakes around the edges.

There are two principal platforms for the siege-service, viz., the *gun*-platform and the *mortar*-platform. The former is composed of twelve sleepers and thirty-six deck-planks; the mortar-platform of six sleepers and eighteen deck-planks.

A simple and strong mortar-platform, called the *rail-platform* may be used where trees or timber can be easily procured. This is composed of three sleepers and two rails, secured by driving stakes at the angles and at the rear ends of the rails. The rails are placed at the proper distance apart to support the cheeks of the bed.

II. TRANSPORTATION.

For certain light pieces, as machine guns, a two-wheeled vehicle is used. Where the weight of the load requires its distribution on several supports, the gun carriage is converted into a four-wheeled carriage by attaching it to another two-wheeled carriage, called the *limber*.

PRINCIPLES OF THE WHEEL.

In transportation the wheel is intended to transfer sliding friction from between the surfaces of the tire and the ground, where the coefficient of friction is large and variable, to the lubricated surfaces of an axle and its bearing, where the coefficient is small and nearly constant.

In this respect its value as a mechanical power varies directly with the radius of the wheel and inversely with that of the bearing.

The wheel, as shown by figure 8, increases also the lever arm, p, of the power, P, with respect to that, q, of the weight, W, to be raised over the obstacle, h.

On these accounts, since the diameter of the bearing, which is generally equal to that of the axle arm, is fixed by the maximum stress which the axle arm has to support, the mechanical advantage of the wheel increases with its diameter.

An increase in diameter as well as in the width of the tire, diminishes the pressure per unit of area between the tire and the ground, and therefore diminishes the *rolling friction*, or the work lost in permanently deforming the ground on which it travels. The elasticity of the wheel also favors this reduction; hence the use for railways of iron wheels on iron tracks.

The increase in size is limited by the weight of the wheel, the stability of the system on the march and in firing, and the convenience of loading. The mobility of transportation also limits the size, for all wheels in the same service being

interchangeable, the facility of turning depends upon their diameter, as will be shown.

As shown by figure 8, an advantage also follows from inclining the direction of the draught, particularly for the front wheels, which do most of the work of rolling friction and which therefore are designed to carry only about $\frac{1}{3}$ of the total load. Since the point at which the horse exerts his power is fixed by his conformation, it is evident that this advantage will be diminished with the increase of diameter of the wheel.

These considerations have generally fixed the diameter of all field artillery wheels at about 5 feet, and their weight at about 200 pounds.

Siege wheels are made heavier and larger.

CONSTRUCTION OF THE WHEEL.

The requisites of size, weight, elasticity and facility o. repair demand a more general use of wood in the wheel than in other parts of the carriage, and involve a marked application of the principle of independence of function. This will appear by comparing the rudimentary wheel, still used in remote districts, consisting of a disc cut from the trunk of a tree, with the complex elastic structure employed in the bicycle.

The *Archibald* wheel, figure 9, now much used in the U. S., resembles that now generally employed in other services, although it applies less fully the principle above named.

Starting from the center, which is the best way of considering any circular structure, we find:— N, N', the *nave* or *hub*. This receives the pressure of the axle arm on a lubricated surface, and distributes the pressure to the spokes. The nave is made in two parts, to facilitate repairing the spokes. The portion of the nave in contact with the axle, or the *axle box*, is so shaped as to receive the lubricant in the cavity, O.

In some foreign wheels and in the ordinary wooden wheel the *axle box* consists of a separate piece, so that it may be replaced when worn. Since friction is less between dissimilar metals than between surfaces of the same metal, and in order to cause the wear to take place most on the part which can be most easily replaced,* the axle box, when separate, is preferably made of phosphor-bronze, while the nave, as in the Archibald wheel, may be made of malleable cast iron. The metal nave marks a great improvement over the wooden nave formerly employed. The cross section of this piece made it difficult to season, its softness caused it to wear from the alternate compression of the vertical and extension of the horizontal spokes, and it was especially exposed to decay from moisture lodging in the angles between the spokes.

S, *S*, are the spokes, which transmit the weight to the rim. For elasticity and facility of repair they are made of oak or hickory. Their inner extremities are shaped like voussoirs, which abut closely upon the box to avoid destructive play. In the Archibald wheel the voussoirs are made a trifle large and simultaneously set together by a powerful radial press which subjects them to a stress many times greater than they are likely to receive in service.

R is the *rim* which distributes the weight over the ground. For the same reasons as the spokes, and because mud adheres less to wood than to iron, the rim is made of oak. In order to avoid cutting too much across the grain the rim consists of a number of segments called *felloes* or *fellies*.

T is the tire, shrunk on to bind the parts together and to protect the rim from wear. As it may require shortening in order to produce the necessary compression on parts which have become loose from wear, it is usually made of wrought iron or of low steel.

*This is an important principle in machine design.

The figure shows also various bolts and clips and the *linch pin* and *washer*, the functions of which are evident.

Dish.

The spokes are so arranged as to form a conical surface which is called the *dish*. The principal object of the dish is to give stiffness to the wheel, since (figure 13) on a transverse slope or on uneven ground, the lower wheel, which bears the greatest share of the weight, will resist the lateral thrust of the axle by a compressive stress upon the spoke. If the spokes lay in the plane of the rim, there would be an alternating transverse stress on the ends of the spokes; this stress would make them work loose in their sockets and accelerate the destruction of the entire machine.

Axle.

The *axle* or *axle tree* consists of the *body* and the *arms.* The arms are conical so as to give the greatest strength with the least mean diameter. In some vehicles, the wear between the arm and box is taken up by means of washers of varying thickness.

The axis of the arm is inclined slightly downward, forming the *hollow*, and to the front, forming the *lead.* Both together constitute the *set* of the arm.

In a dished wheel the hollow frees from transverse stress the "working spoke," which is that which bears the greatest load; it also relieves the linch pin from thrust. For a given width of carriage body it allows the axle body to be made shorter and therefore stronger; and, from the inclination of the plane of the rim, it tends to throw the mud clear of the carriage.

The effect of the lead is to diminish the transverse stress upon the front spoke in meeting obstacles.

Axle Body.

Although the interval between the cheeks transfers the transverse stress upon the axle to points near the wheels, it was found necessary in former carriages to reinforce the axle with a wooden body. In modern carriages this is sometimes replaced with two grooved plates which clamp the cylindrical axle between them and are extended to the front and rear so as to stiffen the axle in recoiling. They also serve to fasten the axle to the cheeks. The axle may, without sensible loss of strength, be made hollow, and three-fourths of its weight when solid.

THE STOCK.

The prolongation of the cheeks is called the *stock*. The use of metal instead of wood, has permitted a return to the construction of the great French designer, General *Gribeauval*, in whose gun carriages the flasks were parallel extensions of the cheeks.

The metallic flasks now used converge to the trail.

In the *stock trail system*, recently in use, the cheeks contained between them a single piece of wood called the stock.

Besides its functions under fire the stock of the gun carriage unites the two axles of the four-wheeled vehicle, as does the *reach* or *perch* of the ordinary vehicle. For artillery carriages used simply for transportation, such as the caisson and the forge, the stock is a single piece of wood joining the body to the limber.

Turning Angle.

The dimensions of the stock affect the mobility in turning. This is often measured by the *turning angle*, which is half the horizontal angle through which the pole can revolve when the carriage is at rest. Practically, the space required to turn the carriage will vary with:

1. The length and width of the line of horses and their gait.

2. The distance of the pintle from the vertical plane tangent to the rear face of the front wheels.
3. The thickness of the stock at the point rubbed b" the wheels in turning.
4. The length of the stock.

Owing to the first condition above named a turning angle of 60° is generally considered as sufficient. This may be increased by increasing the distance of the pintle from the front axle, but this is apt to cause the pole to "thrash."

Pintle.

The distance of the pintle in rear of the axle in connection with the moment of the trail, affects also the pressure on the necks of the wheel horses caused by the moment of the pole.

In siege carriages and in those used only for draught, the pintle is placed at some distance to the rear; or a similarly placed transverse *sweep bar* is used, which supports the weight of the stock.

But, in field carriages for which flexibility of attachment and mobility are essential, the pintle is placed more to the front and the evil corrected as far as possible by distributing the load or by various mechanical means.

In this arrangement the preponderance of the system composed of the gun and its carriage is an important factor. If the trunnion beds are moved towards the limber the pole is lifted, but the labor of limbering is increased, and the stability of the carriage in firing is diminished. (Eq. 9.)

To diminish the labor of limbering, the pintle is placed as low as permitted by the requirement that as much free space as possible should be left beneath the axles for mobility on ground covered with large stones, stumps, etc.

In the siege service, as the piece does not require to be brought into action rapidly, and as the limber carries no extra load, the piece may be shifted to the *traveling trunnion*

beds which, on the march, are in front of those from which the piece is fired.

THE LIMBER.

Nomenclature.

The wooden field limber, figure 10, is composed of an *axle tree* (1); a *fork* (2); two *hounds* (3, 3); a *splinter bar* (4); two *foot boards* (5, 5); a *pole* (6); the *pintle hook* and *key* (7); two *pole yokes* (8, 8); and *pole pad* (9).

Although destined to be soon replaced by one composed more largely of steel, it is here discussed as it illustrates some valuable principles.

The *hounds* serve to support the ends of the limber chest and the foot boards, and also to transmit the draught of the horses from the *splinter bar* to the axle.

The *pole* or *tongue* is employed to stop the carriage and to give it direction. As it is liable to be broken, it is practically made in two pieces, of which the *fork*, which is least exposed to accident, forms one. The fork then is a socket for the pole, and braces the entire frame by its attachment to the axle body and the parts in front.

The pole should be so attached to the fork that it may be readily replaced when broken.

The *pole yokes* transfer the weight of the free end of the pole to the necks of the wheel horses and the soft *pad* protects the leading horses from harm.

Attachments.

The metallic limber body consists of channel irons and T angle irons united in various ingenious ways. The rigid splinter bar may be replaced by the ordinary jointed *double tree* and *single trees*. These permit the horses to work more independently of each other than the splinter bar does, but are probably not so strong. A joint is always a cause of expense and generally a source of weakness.

In the British service the pole is replaced by shafts. Since the pace of the team is regulated by that of the slowest horse, this arrangement, while more manageable than the pole, and therefore better fitted for the showy evolutions of a drill, is objectionable for the march, since the work which devolves on the shaft horse diminishes his endurance.

The Limber Chest.

This serves to carry ammunition, and also furnishes a seat for some of the cannoneers. The gun carriage is often arranged to carry two cannoneers on side seats, in order to diminish the time required for coming into action, a matter which, owing to the precision, rapidity and range of infantry fire, is becoming of vital importance. The carriage also often carries two rounds of canister for use at close quarters.

The principal distinction between limber chests depends upon how the lids are placed.

If on top, the chest may easily be made waterproof in fording streams; but the contents are less accessible. If behind, the lid may form a convenient tray for preparing fuzes, &c. This arrangement is more liable to accidental opening than the former, and waterproof packages for the cartridges may be necessary.

The ammunition chests in the U. S. Service are still constructed of wood. In other countries sheet steel is generally used. For what reason is unknown; since, if not unduly heavy, they are not proof against infantry fire.

THE MORTAR WAGON.

This is used for transporting siege projectiles, mortars and their beds, and spare guns.

The body consists of a strong, rectangular frame provided with a stock by which it is attached to the siege limber. At the rear of the body is placed a windlass which aids in loading

heavy weights. Stakes may be placed around the sides to sustain boards used in retaining loose objects.

Since rifle projectiles are always issued boxed instead of loose, as was the former custom, the necessity of the mortar wagon for their transportation no longer exists; but its general utility is great. It will probably be used hereafter for transporting siege guns for considerable distances, since the height of the carriage from which they are now fired renders them unstable on rough roads.

A special wagon with a *crank axle,* so arranged as to carry the load close to the ground without diminishing the height of the wheels would appear to offer special advantages.

III. CARRIAGES FOR SUPPLY.

New U. S. System.

These include, 1st, the *caisson*, for carrying a larger quantity of ammunition than can be carried by the limber, and also a spare pole, wheel, handspikes, buckets and tools; 2nd, the *forge* and *battery wagon*, containing a larger assortment of tools and material for repairs; 3rd, the *artillery store wagon*, an ordinary four-horse wagon, containing extra small arms and ammunition and the men's knapsacks, etc., so as to confine the load of the fighting teams to the necessities of action.

REMARK.

The increased weight of each round of modern ammunition and the necessity for an even greater number of rounds than formerly sufficed, increases the difficulty of supply.

It is proposed abroad to increase the number of caissons per piece and to retain the supply in the limber for extreme emergencies.

CHAPTER XXIII.

VARIOUS ARTILLERY CARRIAGES.

The U. S. Field Carriage. Figures 1–4.

Construction.

This carriage, designed by Colonel Buffington of the Ordnance Department, is made of steel, since, owing to the large value of h of this gun (Chapter XI, page 21), wooden carriages, and even some differently constructed of steel, were found insufficiently strong.

The principal features relate to the construction of the *axle body*, of the *stock* and to the operation of the *brake.* The hollow cylindrical axle is strengthened by axle plates, figure 2, which stiffen it in the direction of the recoil. The stock consists of two brackets, each of which is made of two nearly symmetrical sheets of steel stamped hot between dies so as to give the corrugated cross section indicated in figure 3. When riveted through the webs, each bracket forms a strong, light truss, resisting stress both in its own plane and transversely.

The lower flanges of the outer plates project inwardly and serve to unite the brackets to the axle plates. The brackets are further united by transoms, three of which with a hinged lid form the trail box for the oil can and tools which have become a necessary portion of the equipment.

The carriage is provided with two axle seats for cannoneers.

The wooden handspike is permanently hinged to the trail.

Elevating Screw.

The space between the brackets allows the breech to descend sufficiently for the high angles of fire used with low

charges against troops sheltered from view, and the crank which operates the elevating screw is placed at the side, so that under these conditions it will be readily accessible.

The nut of the elevating screw oscillates slightly on trunnions, and the head of the screw is connected by a fork to an axis parallel to and beneath the trunnions, so that, as the angle of fire changes, the axis of the screw will be nearly normal to that of the gun.

Brake.

The great strength of this carriage has permitted the employment of Colonel Buffington's brake, figure 4.

This consists of an L shaped rod, the stem of which is surrounded by a spiral spring contained within a tube; the rod swings freely from a loose joint situated eccentrically above the axle.

The length of the brake is such that when held vertically the hook will pass over the wheel; and, being allowed to fall to the rear, it will engage with the tire at some point as *a*.

When the wheel revolves in the recoil, the friction at *a* tends to extend the rod. But this compresses the spring and increases friction, so that as the velocity of recoil decreases, the resistance to rolling increases, and the retardation approaches constancy, at least during the critical period preceding sliding.

The recoil has thus been reduced from 26 feet to 8 feet, without injury to the carriage.

In transportation the brake is secured vertically to one of the seat arms. It may also be used as a traveling brake.

Limber and Caisson.

These carriages are constructed substantially on the lines previously named. Steel angle irons are largely used for the frame.

The chests, which are of wood, open on top and are only high enough to receive the projectile standing; this brings the center of gravity very low.

The cartridges lie in a compartment between the two end compartments reserved for the projectiles, which thus serve to protect the powder from hostile fire.

For safety, no friction primers are carried with the powder, as was formerly done. Unbroken packages are placed in outside cases, and loose primers are carried with the tube pouch in the trail box.

The four chests per piece can carry 42 projectiles each, with a greater number of cartridges for curved fire.

The Siege Carriage. Figure 5.

The principal feature of this carriage is its height. For the protection of the gunners the axis of the trunnions is placed 6 feet above the ground.

In order to prevent the system from tipping forward in limbering, the trunnions are so placed that when limbered the center of gravity of the system will fall between the axles.

The wheels, axle plates and brakes are such as just described.

The carriage is intended to transport the piece only for short distances about the work which it defends.

REMARK.

A small hydraulic buffer connecting the stock with a pintle sunk into the platform between the wheels, and two movable chocks, assist in controling the recoil. The chocks rotate around the pintle with the gun and serve to return the piece into battery.

The Siege Howitzer Carriage. Figure 6.

The piece is mounted in two trunnion carriages, *a*, upon the inclined slides, *b*, upon which it is allowed a recoil of six

inches. The recoil upon the slides is checked by the hydraulic cylinders, *c*, and the courses of Belleville springs, *d*. The latter serve to return the piece to the firing position. They rest against the traveling trunnion beds, *e*, and the rods upon which they are strung pass through holes in these beds.

The flasks, *f*, are of rolled steel plate ½ inch thick, and are flanged inward except on their upper edges. From each flask is cut a large triangular piece in order to diminish its weight; the edges of the apertures being flanged inward as above. The flasks are united by three transoms, *g*, and the double transom, *h*, to which is fastened the piston rod of the hydraulic brake.

The flasks rest upon the axle through two iron forgings, *i*, and are strengthened by two supporting plates, *j*.

In order to facilitate the elevation of the piece a peculiar arrangement is employed. This consists of the elevating rack, *l*, which is attached to the piece, and the worm, *m*; the shaft, *n*, and the hand-wheel, *o*. The worm is attached to the right trunnion carriage, and in recoiling slides along the shaft, *n*. A *spline* (see Webster) on the shaft permits the worm to slide along the shaft, and yet constrains it to follow in any position the rotation given to the hand-wheel, *o*.

The advantage claimed from this design is that the recoil of the piece upon the carriage so diminishes the maximum stress upon the flasks and trail that their weight may be greatly reduced. A portion of the weight so saved is used to strengthen the axle and the wheels.

Weight of wheels, 375 pounds, each.

Weight of carriage, complete, 3200 pounds.

Pressure of trail on platform, 1300 pounds.

Height of trunnions, 6 feet.

Barbette Sea Coast Carriage.*

The principal feature of the *gun* carriage is borrowed from the old "flank-defense howitzer" carriage.

Its object is to return the piece to battery and by diminishing the variable work of sliding friction to increase that of the hydraulic buffer, which can be made constant.

Each cheek carries two rollers; that in rear is on an eccentric axle and that in front is on a concentric axle. When the piece is in battery the front rollers are nearly in contact with the chassis rail; while those in rear are usually raised from it, but may be thrown in contact by means of the eccentric. The lower front angles of the cheeks are truncated, so that, when the carriage is thus tilted to the front, all the rollers come into play and the piece may be moved from battery with comparative ease.

In firing, the rear rollers are out of gear so that the vertical thrust of recoil is borne by the lower face of the cheek and the axles are not endangered.

As the carriage recoils the rear rollers strike inclined planes bolted to the upper surface of the chassis rails and tilt the carriage sufficiently to cause it to move by rolling until it returns again to battery.

Muzzle-loading guns are retained from battery by means of an automatic latch.

MODERN TYPES OF SEA COAST CARRIAGES.

Owing to our deficiency in modern cannon the U. S. have not yet (1891) decided on any special pattern of sea coast carriage; but the following examples, derived from the French service, probably contain the essential features of the system to be adopted for the barbette carriages, as soon as the new cannon shall have been supplied.

* The Sea Coast Battery at West Point contains several specimens of this type.

The types of disappearing carriages and those designed for turrets are too numerous for description here. They generally apply the principles previously discussed with those treated in the course of Military Engineering.

Gun Carriage.

Figures 7, 8 represent the elements of a modern sea coast barbette gun carriage. It consists of three main parts: 1st, the top carriage, *T*, consisting essentially of the buffer; 2nd, the chassis, *C*, the lower part of which is circular; by means of a great number of loose conical rollers, it revolves upon the circular pintle platform, *P*. This platform, cast in a single piece, rests upon a proper foundation.

To avoid the complications due to sliding friction during recoil, the top carriage also moves on rollers recessed in the chassis rail.

Pointing in azimuth is performed by an endless chain engaging in a *sprocket** bed around the platform. The chain passes over a windlass, *W*, which is rotated by the crank, *K*.

The loading scoop, *s*, is on a lever, L, which is rotated by a geared crank so as to bring both the charge and the projectile into the position of loading.

In order to minimize the number of men required for loading, the act of lowering the scoop stores up energy in certain springs so that the maximum pressure which can be counted on shall be continuously applied, as in the hydraulic buffer.

The steel shield, *p*, protects the gunners from light projectiles.

Advantages. The carriage is low, stable, and as seen in figure 9, very compact. The use of rollers increases its mobility and their number distributes the thrust over a large area. All wheels are protected and the traversing chain is of

* See Webster.

rustic simplicity and easy of repair, even in action. The arrangement of the scoop facilitates loading since its load may be placed by simply tilting the hand truck on which it is brought from the magazine.

Sea Coast Mortar Carriages. Figures 10, 11.

Although of an entirely novel design, the carriage in figure 10 resembles essentially the gun carriage just described. The nomenclature is the same in both figures.

The chassis is divided into two portions, C_1, C_2; the surface of contact being cylindrical about the axis of the trunnions. By this arrangement for all angles of fire the axis of the gun is always in the plane of the axes of the hydraulic cylinders, so that the friction in starting is not increased by the pressure causing recoil.

Rotation from recoil is prevented by the clips, c, c, etc.

The diminished intensity of the maximum vertical pressure has caused this carriage to be adopted in the French Navy; for ships now, as well as forts, are beginning to utilize the advantages of vertical fire.

In another type of mortar carriage, shown in figure 11, also under trial in the U. S., the chassis is made in one piece, the direction of the recoil being downward at a constant angle of 60° This is a mean between the limiting angles of θ for mortar fire, viz.: 45° and 75°. The mortar is returned to battery by springs that are compressed during the recoil.

Another form of loading scoop is also shown.

This is known as the Easton-Anderson carriage, of English design.

CHAPTER XXIV.

HORSE AND HARNESS.

The horse transports his load in two ways. 1st, as a pack horse; 2nd, as a draught horse.

PACK HORSE.

The daily work of a pack horse is about equal to that of five men similarly employed; or, if he moves at a walk, he may carry a load of 200 pounds 25 miles a day, or 5000 mile-pounds.

If he trots, the increased expenditure of muscular energy reduces his daily work about one-third.*

In the above the weight of the horse is neglected, and it is assumed that, though this daily work may be temporarily exceeded, the excess cannot be long continued without injury.

The mule, owing to his build, carries more than the horse; he eats less and is surer of foot. He is therefore generally used in the mountain service.

DRAUGHT HORSE.

Load.

Although a horse can pull much less than he can carry, the advantages of the wheel enable him to draw over ordinary roads a load weighing about seven times as much as his pack. With a pull of 80 pounds the daily work of a draught horse

* It has been found that for any animal the maximum rate of work per unit of time (or $p\ v$, Chapter XI, page 4) is attained when the velocity is about $\frac{1}{3}$ of the maximum velocity unloaded, and the load about $\frac{1}{3}$ of the maximum load at the lowest positive velocity.

is generally given as 1600 pounds $\times$ 23 miles, or 36800 mile-pounds of load, or 1840 mile-pounds of actual work.

Owing to their interference with each other's motions, the maximum load drawn by teams of horses increases less rapidly than does the number of horses in draught. Thus, when the teams comprise respectively 2, 4, 6, 8 horses, the maximum loads which they can continuously draw are in the relation per team, of the numbers 9, 8, 7, 6.

These considerations, the mobility of the system (Chapter XXII, page 1), the increased weight of forage and length of column required, have generally fixed the limit of efficiency at the six-horse team.

It is estimated that when a draught horse carries a rider, his efficiency is diminished $\frac{1}{2}$ at a walk and $\frac{2}{3}$ at a trot. Consequently, supplying the data given, the maximum load for a team of 6 horses moving at a trot will be about

$$\tfrac{7}{9}\left[\overset{\text{near files.}}{\frac{3 \times 1600}{3}} + \overset{\text{off files.}}{\frac{3 \times 1600 \times 2}{3}}\right] = 3733 \text{ pounds;}$$

or 622 pounds per horse.

This may be considered a *physical constant*, the best method of distributing which between the objects transported and the means of transportation is still open to inquiry.

Various conditions must be allowed for: On one hand are bad roads, insufficient food, rapid movements for short times, and forced marches. On the other hand, the reduction in the load caused by the expenditure of ammunition, the dismounting of the cannoneers, and the infrequency of the trot.

Upon these considerations are based the following approximate loads per horse.

Horse artillery, 650 pounds.

Light field artillery, 700 pounds.

Heavy field artillery, 850 pounds.

Siege artillery, 1000 pounds.

REMARK.

The 12 pdr. Napoleon gun, which was the heaviest field gun used in our civil war, and which traveled over roads quite as bad as any used in foreign wars, gave a load of 645 pounds per horse, and was found amply mobile. The load per horse for the 3.2 inch B. L. R., field, is 632 pounds.

Angle of Draught.

The power of an animal in draught may be supposed to consist in his ability to maintain himself rigidly in a position such that the moment of his weight may be increased without increasing the lever arm of the resistance.

Thus in figure 1, let f be the position on the ground line, $f\ g$, of the hind feet of the horse in draught. Let s be the shoulder of the horse or the point at which he applies his power to the trace, $s\ c$, which is attached to the carriage at the point c. Let W be the weight of the horse, and l be the distance from f on the line $f\ s$, of the vertical passing through his center of gravity. Let τ be the tension on the trace, the length of which $s\ c = t$, and let R be the horizontal component of τ, producing uniform motion of the point c in a horizontal plane.

Let i be the variable angle with the ground line of the line $s\ f$, and φ be the variable angle between $s\ f$ and $s\ c$. Let h, drawn from f, perpendicular to $s\ c$, be the lever arm of the resistance. Let the same symbols "primed" represent a new position of the system caused by the horse bending his knees in pulling. For simplicity, we will suppose that his fore feet are off the ground and that his hind legs are not extended so as to increase l, as these suppositions tend to neutralize each other. Also that the center of gravity is on the line $f\ s$.

The construction of the figure shows that, as the point s moves to s', c will move to c', and that R will increase until

the compression along $s f$ causes the horse to bend so that l will shorten.

The stress R may under these circumstances be deduced as follows:

From the equality of moments we have $W l \cos i = \tau h$, and from the figure

$$R = \tau \cos (i - \varphi) = \frac{W l \cos i \cos (i - \varphi)}{h}$$

Graphical construction shows that, as i diminishes, h and φ will diminish slowly, and $i - \varphi$ will rapidly approach zero. R will have its maximum value when s falls on the line $c p$, either from raising the point of attachment to the load at c or from the descent of the point of application of the power at s. This value is not realized in practice, since, in addition to the effect noted above, as i decreases the force of friction at f decreases and the feet tend to slip.

A proper inclination of the trace is therefore valuable since it enables R to be increased according to the ability and willingness of the horse, and also that it enables him to draw by increasing the friction between his feet and the ground.

By experiment it was found that when the horse is free, the maximum practical value of R, or about 0.6 W, was attained for a value of $i - \varphi = 12°$. When the horse had a rider, $i - \varphi$ could profitably be reduced to 7°. From these data it is estimated that, since $\tan 12° = 0.2$, a draught horse should carry $\frac{1}{5}$ of his load on his back.

The preceding general considerations apply to the case of men pulling on ropes or pushing on capstan bars, etc. They partly explain also that, while for the horse the maximum value of $R = 0.6\ W$, for man, it is found practically that $R = W$.

Arrangement of the Horses.

Owing to the difficulty of coördinating the movement of the horses the single file is used only when the gait is slow

and the road smooth, so that the shaft horse will not be unduly fatigued by frequent changes of direction.

When the double file is used, the control of the direction is shared by the horses of the wheel team, provided the carriage have a pole.

This team is preferably attached to a movable *double tree*, Chapter XXII, page 28, since this shows by its inclination whether the horses are pulling evenly, and also transfers the draught to the axis of the carriage. For these reasons it is often called the *evener*.

By attaching the traces to the *single trees* hooked on to each end of the double tree, greater flexibility is attained; and, since the shoulders of the horse are naturally brought into bearing alternately, he is less apt to be chafed by the sliding of the collar.

He may also, when harnessed, be more readily hitched and unhitched.

In commerce the leading team is generally attached to an evener fastened to the front end of the pole. This is objectionable since it confuses the functions of the pole. A better method, sometimes followed, is to support the evener by the pole, and to connect it with the axle by an independent tensile member, as by a chain.

In the present arrangement, the objections to supporting the weight of the evener on the end of the pole, and herefore on the necks of the wheel team, are avoided, and the traces of each team are connected with those in rear by an arrangement which permits continuous draught without causing the effort of willing horses to be neutralized by the laggards.

The team between the *leaders* and the wheelers is called the *swing team*. The horse on the left of each team is called the *near* horse and that on the right the *off* horse.

Requirements.

The preceding considerations illustrate the application of the principle of independence of function to meet the requirements of artillery harness which, as stated by another writer, may be thus abridged.

"No horse should be restrained by the efforts of another, and the direction of the traces should be most favorable for draught. The drivers should be able to harness and unharness promptly, by night as well as by day, when benumbed by cold and when excited by danger. The fall or loss of a horse should not be a permanent obstacle to the advance, and disabled horses should be easily replaced."

U. S. Artillery Harness. Figure 2.

WHEEL HARNESS.

This is composed of four essential systems, three of which occur in all harnesses except for horses in the lead. The systems are:

1st. The *head gear* to guide and hold the horse.

2nd. The *saddle* to transport the driver, who, for the independent control of his team, is mounted.

3rd. The *draught harness* which enables the horse to move the carriage forward.

4th. The *breeching* for moving it backward.

1. The head gear consists of the bridle and halter. To the bit of the off horse is attached the lead rein, one end of which is held by the driver.

2. All horses are saddled, the off horse carrying the driver's valise, and, when necessary, an extra cannoneer.

3. The draught harness and the breeching constitute two independent systems symmetrically arranged.

The former is composed of the following parts.

The *hames*, *h*, figure 2, are two curved irons shaped like the signs ( ). They are connected together below by an

iron clasp, and adjusted at the top by a leather strap so as to embrace the neck and form a rigid frame against which the horse may thrust. To diminish the pressure per unit of area on the horse's shoulder* the hames rest on a similarly shaped cushion, the *collar.* To each hame is attached by a flexible hinge a stout leather *tug*, *t.* This terminates in an iron ring through which passes the trace chain, *c*, terminated by the *toggle*, *t'*. The latter connects the front trace chain of the wheel horse with the rear trace chain of his leader, and so on throughout the column. When in motion, the tug ring plays on the trace chain and thus makes the leading horses independent of those in rear.

The length of the rear trace chain may be varied by a toggle to suit the conformation of the horse.

The *safe*, *s*, protects the shoulder from chafing.

The *loin strap*, *l*, sustains the trace when relaxed, and the *belly band* beneath the saddle keeps it from rising over the back in turning.

4. The breeching is composed of the broad *breech strap*, *b*, figure 3, corresponding to the collar; it is supported by the *hip strap*, *h*. Corresponding to the traces is a Y-shaped system consisting:—1st. Of the continuous *breast strap*, *bs*, which, passing around the breast, is united at each end to the breech strap. It is supported in front by iron links hanging from the hames. 2nd. The stem of the Y is formed by the *pole strap*, *p*, connected at one end to the breast strap by an iron double-loop, shaped like a figure 8, and leading obliquely downward and inward to the end of the pole. The functions of the pole thus correspond to those of the splinter bar in rear.

* This end is served in modern practice by using hames of sheet steel formed to fit the shoulder. The same principle is applied in the cavalry saddle.

Pole Yoke.

The weight of the pole is supported by the *pole yoke*, which is connected by a short chain to the clasp of the hames. The branches of the yoke are so hinged to a collar revolving loosely around the pole that they can play only in a plane passing through the axis of the pole.

This allows the horses to travel freely at different levels and prevents the lateral *thrashing* of the pole.

LEAD HARNESS.

The leading horses have longer traces than the wheelers and have no breeching; otherwise their harness is identical.

Improved Harness. Figure 4.

The harness devised by Major Williston of the Artillery, which is now undergoing trial, resembles that above described except in the following principal points.

1st. For interchangeability, the saddles and bridles are the same as those used by the cavalry, and saddle bags replace the valise.

2nd. The wheel traces are attached to single trees which may be hooked to the saddle when not in use.

3rd. The breeching is that used in commerce. The stem of the Y passes under the horse to a transverse bar in front, which corresponds to the evener, and is called the *neck yoke*.

This is the most important change from the regulation harness. It prevents the breech strap from slipping upward in stopping suddenly, and also avoids the oblique thrust on the horse's neck which tends to make him fall.

The neck yoke also controls the pole better than the hinged pole yoke.

4th. The bridle rein of the off horse passes through a pulley on his saddle, so that, in holding him back, the oblique stress above mentioned is further avoided.

5th. The collar, instead of being continuous, is hinged above and is provided with a fastening below in easy reach.

6th. The horse of the chief of piece is provided with a light draught harness, consisting of a breast collar and traces, with which in an emergency the other horses may be assisted. When not in use the traces are folded across the horse's withers.

This harness is distinguished for the ease with which the horses may be detached from the carriage in all conditions of service.

LEATHER.

That used in harness is classified according to its thickness, into *harness*, *bridle* and *collar* leather.

The leather from the necks, shanks, flanks and bellies, or the *offal*, figure 5, is rejected as too spongy for use, so that only about one-half of the hide is employed. Of this, the *butt* is the best portion.

The lighter hides are slit axially into *sides*.

CHAPTER XXV.

ARTILLERY MACHINES.

Object.

Artillery machines are employed to mount and dismount cannon and to transport artillery material from one part of a work to another. They comprise the *gin*, the *gun lift* and *jacks* of various forms; and wheeled vehicles such as the *sling cart*, the *truck*, etc.

Machines Used in Mounting Cannon.

The *gin* consists of a tripod composed of two *legs* which form a shear or derrick, and a *pry pole* by which the legs are lifted and braced.

The hoisting apparatus consists of a block and fall suspended from the apex and operated by a windlass supported by the legs in a position convenient for the use of handspikes.

The use of the gin is confined to relatively light weights. Heavy weights are preferably lifted by the hydraulic jack and loose blocking.

The *hydraulic jack* is a compact form of the hydraulic press, which contains within itself the reservoir of liquid required. It is provided with valves by which the direction of the motion of the ram may be varied.

Other jacks apply the principles of the lever and the screw, and are correspondingly named.

The *gun lift* consists of two massive trestles so framed that they may be easily dismounted for transportation.

Each trestle carries on its beam a hydraulic jack; the latter by means of a lever raises a bar of iron which passes vertically through the beam and the lever, between the jack and the fulcrum of the lever. This bar is pierced at short intervals by holes, and its lower end is formed into a hook.

Both bars having been attached to the weight, and a pin having been passed through the hole in the bar next above the lever, the ram of the jack is raised to its full extent. A pin is then inserted through the hole next above the beam and the ram is lowered. The upper pin is then shifted downward and the operation continued.

For comparatively light weights a single trestle may be employed like a gin.

Machines Used in Transportation.

Heavy weights are usually transported by the aid of capstans and rollers.

When space permits, cannon may be rolled bodily by *parbuckling*. In such cases a *muzzle collar* of the maximum diameter of the piece corrects the circular path which the conical mass tends to describe.

Heavy weights may also be rolled through the narrow passages of forts on a low framework called the *cradle*.

The wheels of sling carts are large and have but little dish. Since, like the gin, they suspend the load, they are relatively weak, and hence are used for lighter weights than the cradle.

By mechanical appliances mounted on the axle, the weight may be lifted from the ground, and during transportation may be permanently secured to the axle by hooks which relieve the more delicate mechanism from shocks.

The means of lifting are the screw, and the hydraulic jack which works on the principle explained for the gun lift.

For light weights the eccentric position of the hooks may enable the weight to be raised by lifting the pole before the weight is attached and afterwards by depressing it. This means of lifting is applied to the iron sling cart. The field limber may be similarly used to carry a piece, the carriage of which is disabled.

In transportation the pole of the sling cart is supported by the limber.

CHAPTER XXVI.

HAND ARMS.

The weapons carried by the soldier, or *portable arms* may be divided into *hand arms* and *small arms.*

The former class is known in French as "armes blanches;" the latter requires, as in cannon, a preliminary study of the ammunition employed. See Chapter XXVII.

Classification.

Hand arms are divided into

1st. Thrusting arms which act by the point.

2nd. Cutting arms which act by the edge.

These functions may be combined in the same weapon, though at some sacrifice of efficiency.

Thrusting Arms.

The body of a thrusting weapon should be straight so as to avoid a rotary moment on impact, and the center of gravity should be placed near the handle. This may be attained by fluting the blade, or by suitably weighting the handle.

The principal thrusting weapons are the *straight sword,* the *lance* and the bayonet.

The sword is composed of the *blade,* the *hilt* by which it is held, and the *guard.* A knob sometimes acts to counterpoise the blade as in the foil.

The lance is composed of a short steel blade fixed to the end of a wooden handle about 10 to 16 feet in length. The handle is furnished with a leather arm-loop placed over the center of gravity.

After a long period of comparative disuse, in spite of the greatly increased efficiency of small arms, its use abroad is now becoming more general. In this country it has never been successfully employed.

The bayonet is useful principally for guard duty and for its moral effect. Like other hand arms, it has the merit of "never missing fire."

The objections attending its weight and that of its scabbard, and its eccentric position in firing may be partly overcome by combining its functions with those of the ramrod. Attempts have also been made to turn it into an intrenching trowel. The tendency is now to shorten it to the proportions of a dirk, which may form a useful knife.

Cutting Arms.

The efficiency of these arms is promoted by increasing the distance of the center of gravity from the handle, and by giving a curvature to the cutting edge so as to develop on impact a tangential or slicing component which will call into play the serrated edge possessed by even the sharpest knife. This enables the weapon to rupture in detail the muscular fibers on which it acts.

Description.

The principal cutting weapon is the *saber*. Sabers are classified according to their use. In the U. S. service there are two kinds, viz.: the cavalry saber and that for the light artillery.

The cavalry saber, being used on horseback for thrusting as well as for cutting, has but a slight curvature, a long blade, and a basket hilt (properly a *guard*) which carries the center of gravity toward the handle.

The light artillery saber being intended for hand to hand conflict by troops, who for the service of their batteries are

dismounted, is shorter than the cavalry saber, is more curved, and has a guard composed of a single scroll of brass.

Remarks.

The present tendency is to make the artilleryman depend for his personal defence upon the gun which his duty to the other troops compels him to serve to the very last extremity. He should therefore be free from any incumbrance which will distract him from his proper functions.

In order to avoid the exposure of the person in cutting, many cavalry officers are in favor of avoiding the objections to the combined functions of the cavalry saber by using it solely for thrusting.

On the other hand the swordsmen of East India, than whom there are few more expert, prefer blades which are greatly curved, the radius of curvature of some being about 18 inches.

The following discussion illustrates the effect of curvature, frequently utilized in the useful arts.

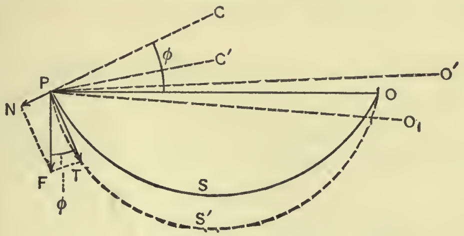

Fig. 1

Let $O\ S\ P$ be the edge of a curved blade rotated around O and striking at P with a blow, $P\ F$, at right angles to $O\ P$. Then $P\ T = P\ F \cos \varphi = P\ F \cos F\ P\ T$ is the tangential component, and this will be measured by $P\ F \cos C\ P\ O$, which gives an easy method of discussing the effect of curvature. If, as in the artillery saber, S', the radius of

curvature, be shortened by placing the center at C'; or, if as in some Eastern blades which have a tangential handle and also in the common scythe, the center of rotation be placed above the line $P\ O$, the value of $\cos \varphi$ will be increased and so will the proportionate value of the tangential component.

On the other hand if, as in the cavalry saber, the handle be lowered as to $O_{\prime}$ in order to increase the tangential component in thrusting, the slicing component will decrease.

CHAPTER XXVII.

SMALL ARM AMMUNITION.

The Relation between Arms and Ammunition.

As seen in Chapter V, the efficiency of all fire arms has been dependent principally upon the nature of their ammunition.

This may be called the *food* of the gun as the means of conveying it to the chamber is actually called the *feed*. As a rule the gun must be made to fit the ammunition as a shoe should be made to fit the foot.

MUZZLE LOADING AMMUNITION.

Powder and ball were originally carried loose; but for some time the greater rapidity of fire with arrows at the ranges common to both weapons, caused the latter to be preferred.

Gustavus Adolphus made important improvements in the ammunition.

He first provided separate receptacles for each powder charge; these were called *cartridges* from their paper envelopes. (Latin *charta*, paper.)

He subsequently combined the powder with the projectile in the paper wrapper, which, until about 1865, formed the principal ammunition for small arms. See Figure 1.

In addition to the comparative disadvantages of muzzle loading arms cited in Chapter XI, may be named the variable amount and condition of the powder in the chamber, since the powder was but imperfectly protected from moisture and was liable to be wasted in loading. There was also

the danger of inadvertently loading the piece with more than one cartridge at a time. Nearly one-half of the muskets abandoned at the battle of Gettysburg were found to contain more than one cartridge.

In spite of the theories of those who feared that increased rapidity of fire would lead to a disastrous expenditure of ammunition, there has always been the feeling expressed by Frederick the Great in saying, that other things being equal, "He who fires fastest hits most."

BREECH LOADING AMMUNITION.

Non-metallic Ammunition.

The state of the arts required the first breech loading ammunition to be formed after the manner of that just described; and, as it was impossible to permanently prevent the escape of gas by the close fitting of the parts of the breech, the joint required for rapid loading was generally placed in front of the chamber, from which position the soldier would suffer least from the discharge.

To facilitate loading the section of the barrel containing the chamber was caused to oscillate about an axis in rear; so that, the paper cartridge having been broken for loading, the bullet acted as a stopper to prevent the exposure of the loose powder before the piece was closed.

This structure distinguishes a large class of arms, now obsolete, which are known as having *movable chambers*. This includes the *Hall rifle*, used in this country in the early part of the century. It is believed to be the first breech loading small arm used by troops.

The operation of such guns was necessarily slow and defective.

METALLIC AMMUNITION.

Origin.

The primed metallic cartridge case, invented in France, was first used by troops during our Civil War. It contained

all the components of the ammunition, under invariable conditions, in an envelope which formed a gas check, and was therefore adapted to arms in which the chamber was fixed.

Being rigid and of exact dimensions it could be and was at first most extensively used in *magazine arms*, in which the operations of loading are automatically performed.

Rim Fire.

In order to support it against the blow which exploded the fulminating priming, and to extract the empty case, it was provided with a rim. For simplicity of manufacture, and because the arms in which it was principally employed contained the cartridges in tubular magazines and were carried by mounted troops, the fulminate, *f*, was placed within the rim, as shown in figure 2.

This construction, although confusing the functions of the rim and the primer, was intended to prevent accidental explosions in the magazine.

For the small charges of powder then used, the metal could be made thin enough for certainty of fire, since it was composed of soft copper.

Figure 2 shows that such a cartridge, having what is termed a *folded head*, is necessarily unsupported by the walls of the chamber for a length at least equal to the thickness of the metal forming the rim. Consequently, as charges and pressures were increased, the *rim fire* cartridges were found to shear across the edge of the chamber; and the copper was so deficient in elasticity that they would resist extraction.

The quantity of fulminate contained in the rim was much greater than was required for ignition at any one point, and further tended to destroy the fold. The distribution was imperfect and misfires were frequent.

The cartridge could not be reloaded.

Central Fire.

As metallic ammunition became more generally employed in all arms, these objections led to the use of the center fire cartridge, now universally employed; these objections led also for a time to the disuse of magazine arms.

The adoption of central fire permits the case to be strengthened indefinitely in the *shearing plane*, and to be made of an elastic material like brass, the special elasticity of which, developed by its manufacture, facilitates its extraction. It also permits the reloading required by the great expenditure of ammunition in target practice.

Folded Head.

The first center fire cartridges were made with folded heads, as the arts then furnished no other method of forming the rim. To avoid shearing, a thin, cup-shaped, gas check, as shown in figure 3, was, and is still employed. This contains a central hole to allow the flame from the fulminate, *f*, to pass through the vents, *vv*, in the anvil, *a*.

The Ordnance Department for several years made the copper *cup-anvil* cartridge shown in figure 4. In this it was attempted to combine in one piece the functions of the gas check and of the anvil. But these were inconsistent, and the cartridge, although avoiding objections urged against a perforated head which contained a loose primer, was abandoned.

The limit of resistance to shearing was soon reached, because, owing to the manufacture, the maximum thickness of metal is that of the head. So that if a thicker or more elastic metal was used misfires would result, unless the energy of the blow required for ignition was so much increased that the rapidity of fire was diminished.

The flat anvil, figure 5, demanded by the oblique firing pin of the Springfield rifle, requires a more powerful blow than does that shown in figure 3, and the thickness of metal

requires the firing pin to be sharp. On the other hand, the anvil of figure 3 is well adapted to the axial blow of a flat pointed pin. This requires less work in cocking and is less apt to pierce the cap.

Solid Heads.

The state of the arts now permits the U. S. cartridge to be made with a solid head, as in figure 5. The shearing plane lies in front of the edge of the chamber even when, owing to the yielding of its support, the case may be forced backward in firing.

Certainty of ignition now requires that the anvil shall be renewed at every fire. Consequently the primer is assembled before issue with its anvil and fulminate complete. The resulting variation in figure 3 is shown in figure 6.

An objection to the solid head cartridge arises from its unequal expansion when fired. The mouth, being thin, is more firmly held by friction against the walls of the chamber than is the thicker portion in rear, so that the latter may slide backward to the extent permitted by its support. Cases which have been often reloaded are found to tear across by longitudinal stress.

The Morse cartridge, figure 7, provides for this by making the head entirely separate from the body of the case.

Remark.

The influence of improvements in metallic ammunition has probably reached its limit in the cartridge employed in rapid firing cannon, Chapter XXIX, page 18. The size of the cartridges which these employ is limited by the weight which one man can conveniently handle.

METALS USED FOR CARTRIDGE CASES.

Copper was first employed on account of the ease with which it could be worked. When alloyed with a small pro-

portion of zinc it was until recently preferred by the U. S. to brass, which, when in contact with gun powder, undergoes in time a molecular change that renders it as brittle as baked clay. It is said that the discovery of this defect in the Russian ammunition postponed the war of 1877.

The deficient elasticity of copper accounts for the prevalence of the lever used for extraction in early breech loading arms, and for their comparative slowness of fire.

Brass is cheap and so elastic that guns in which it is used may be opened by the direct action of an axial bolt. For the reasons given, Chapter XXVIII, page 6, the rapidity of fire of such arms is increased. This is the metal now generally employed. In order to protect it from the powder the cavity may be varnished or tinned.

The elasticity of brass adapts it to reloading since *resizing* is less necessary than with copper.

The operation of resizing is required by unavoidable differences in the chambers of different guns. The brass cartridges used in the rifle, Cal. 0.45 may often be reloaded for use in the *same gun* without resizing them; but owing to the greater pressures found in the new Cal. 0.30 rifle, firing smokeless powder, resizing is always required for this arm. See Chapter XXVIII, page 3.

Low steel, when protected from oxidation, is proposed as a cartridge metal, on account of its strength elasticity and freedom from structural change.

MANUFACTURE OF METALLIC AMMUNITION.

The cartridge case may be made in two general ways, viz.: 1st, by coiling by hand a thin sheet of metal into a tube; 2nd, by drawing the tube from a thicker disc as described in Chapter XVIII, page 2.

1. Wrapped Metallic Cartridges. Chap. XVI, figure 8

The metallic sheet is trapezoidal so as to increase the thickness of the walls near the head. This gives the exterior the conical form required for extraction, while the interior being cylindrical retains its hold on the bullet. It also increases the thickness of the flange by which the case is riveted to the separate disc that forms the head.

This method, the origin of which is evident, avoids the use of the expensive machinery used in the second process, so that in an emergency the manufacture could be easily improvised.

The cartridge is serviceable, but neither waterproof, rigid, nor exact enough in its dimensions, for all the requirements of service.

2. Drawn Cartridges.

The operation of drawing necessarily leaves the exterior of the tube cylindrical, so that the required variation in thickness is obtained by varying the diameter of the punch.

The primary *draws* are facilitated by removing by annealing, (*i. e.*, heating followed by quenching), the special elasticity developed by the previous operations. Chapter XV, page 22.

After having been drawn to a length slightly in excess of that required, the tubes are trimmed to an exact length to prepare them for the operations of heading.

The mandrel, figure 8, supports the trimmed case in a closely fitting die. A *bunter* of the proper dimensions first forms the pocket for the primer, and a second operation with a bunter, such as shown, causes the metal to flow into the annular space provided for the rim. The pocket is then vented.

To facilitate extraction the case is tapered by forcing over it a conical die. The cylindrical seat for the bullet is simultaneously formed.

Components

The U. S. *anvil* is made from a copper wire of rectangular cross section containing on one side a continuous groove. From this are punched a series of circular discs which form the anvils. The edges of the discs are notched so as to form a passage way for the flame of the fulminate, *f*, through the notches, into the groove which bridges over the vent, *v*, in the head of the cartridge.

The *bullet* is composed of an alloy of lead and tin; the latter metal, although it increases the difficulty of manufacture, gives the hardness required to resist deformation in the gun. Chapter XXVIII, page 3.

The bullet is made by compression between dies which part on an axial plane. See figure 9. The cavity in the base of the bullet may be varied to bring the bullets to an exact weight.

The bullet is lubricated by being forced through a vegetable wax so as to fill the *cannelures*, or grooves. This is preferred to a fat, as it does not corrode the metals in store.

Common Operations

In the *loading machine* a measured charge of powder is first deposited in the case and slightly compressed so as to increase the density of loading. The bullet is next inserted and secured by crimping the case upon it.

The finished cartridges are all *inspected* for weight and dimensions.

The first is accomplished by a weighing machine which rejects all that weigh less than a prescribed minimum. The principal object of this operation is to detect charges in-

sufficient to expel a projectile which might cause a subsequent discharge to burst the gun.

The gauging machine makes sure that every cartridge will enter the gun. The gauging die, which is slightly smaller than the minimum chamber, verifies the length of the cartridge to the rear from the circle of contact between the bullet and the rifling, the profile between these planes, and the maximun radius of the rim.

For safety the primer is sunken below the plane of the head.

The automatic operation of the machinery has greatly reduced the cost of manufacture, and has thus removed one of the principal objections to metallic ammunition.

The inspection merely precedes the *proof*. Chapter XVII, page 18. This consists in firing a portion of the daily product to verify the certainty of fire, the strength of the case, to determine the volume of the charge, the compression required for the standard velocity and above all to test the accuracy of fire.

U. S. SMALL ARM AMMUNITION.

The following varieties are now made (1891):

1. The rifle ball cartridge, $\frac{70}{500}$, or about 70 grains of powder and a 500 grain bullet. $I\ V = 1280\ f\ s$.

2. The carbine ball cartridge, $\frac{50}{405}$. $I\ V = 1150\ f\ s$.

3. The revolver ball cartridge, $\frac{30}{230}$. $I\ V = 730\ f\ s$.

4. The rifle and carbine blank cartridge, filled with compressed powder that is protected by a varnished paper cup, and retained by crimping the case so as to facilitate loading.

5. The revolver blank cartridge as in 4.

Important changes in this ammunition are now pending. Their principles will be hereafter discussed in connection with the arm. It is significant to observe that now, as heretofore, the adoption of the new arm awaits the perfection of its ammunition. Chapter XXVIII, page 19.

CHAPTER XXVIII.

SMALL ARMS.

Classification.

Small arms may be classified according to the *service* in which they are employed, as this determines the maximum length of barrel, given to the *rifle*, the *carbine*, and the *pistol.*

Muzzle-loading arms, and breech-loaders having movable chambers being now obsolete, breech-loading small arms with fixed chambers may be classified into *single loading* and *magazine* arms.

The latter class is now supplanting the former, because of the moral and physical advantage of being able at will to increase the rapidity of musketry fire.

Historical Sketch.

Some of the objections formerly made against the breech-loader have been discussed in Chapter XXVII. To these may be added the former fear that the mechanism might not endure the accidents of service.

But the Prussian wars of 1864 and 1866, and the more extended campaigns of 1870, proved that after a victory there is generally time enough for repairs.

During the siege of Plevna in 1877, these conclusions were emphasized by the use by the Turks, for the first time in Europe, of the American Winchester repeater.

Although of a model now considered imperfect, its success was conclusive.

It is now realized that the change from muzzle-loading to breech-loading having established the advantages of rapidity, the choice of a magazine arm is a detail to be determined by

independent considerations. The selection is attended with many complications which, as in the past, relate principally to the ammunition. Some of these will be hereafter discussed in detail; but it may be premised that, while the power of the weapon depends principally upon the ability of its (human) carriage to resist recoil; its continued operation depends upon the number of cartridges which this carriage can conveniently transport.

The development is thus limited by a physical constant.

COMPONENT PARTS OF B. L. SMALL ARMS.

I. THE BARREL.

Weight.

Except for considerations relating to the recoil and the practical necessities of service, the general use of steel would permit the barrel to be considerably reduced in weight.

Caliber.

Although the best results follow from adapting to each arm its own ammunition, yet in order to meet emergencies the cartridges for the rifle and the carbine may be interchanged. These arms are therefore of the same caliber.

For the reasons stated in Chapter XVI, since the adoption of the rifle principle the tendency has been to reduce the caliber. The limit is fixed by questions of internal ballistics, and also by the nervous shock communicated to the animal struck. Upon the shock is thought to depend the "stopping power" of a bullet that does not kill.

Until lately the limit was generally taken at about 0.45 inch, but recent experiments have induced many countries to reduce it still further to about 0.30 inch.

The propriety of the change is still debated, and like many others requires the test of war. The advantage may consist in this: that a shock which might be insufficient to stop a

man in the heat of a close action may, at the long ranges which the reduced caliber provides, be severe enough to cause him to withdraw. But this would not apply to horses.

Rifling.

The cross-section of the rifling depends principally on the nature of the bullet. If this be of a soft material, like lead, the lands may be broad as in the Springfield rifle and conversely, figure 15, if the metal be hard. The grooves should be shallow and so formed as to be readily cleaned.

The increase of spherical density, which results from reducing the diameter of a projectile of which the length, and therefore the sectional density, is kept nearly constant, has required a considerable increase in the twist, so that special precautions have been required to prevent the projectile from shearing. Chapter XVI, page 10.

In the caliber 0.45 bullet this was done by alloying the lead with tin, Chapter XXVII, page 8; the new bullet is moreover coated with a thin jacket of a harder metal. Chapter XXVII; Plates.

II. THE STOCK.

This forms the handle by which the barrel is directed. It is made of wood on account of its lightness and strength and its deficient conductivity of heat.

The form of the stock depends on the conformation of the average man.

The butt is widened and curved so as to diminish the pressure per unit of area due to the recoil. It is bent for convenience in aiming. A rotary component of recoil is thereby developed, which, if the *crook* be excessive, may cause inconvenience to the firer.

The stock is necessarily weakened by being cut across the grain to form a grasp, and more so by the present develop-

ment in the volume of the parts about the breech It is consequently frequently made in two pieces, the *tip stock* being of a rigid material, such as black walnut, and the *butt stock* preferably tough, as of elm. Chapter XV, page 12.

The support in rear of the barrel should be of sufficient area to avoid permanent deformation; and that beneath the barrel should not be unduly rigid, since otherwise the barrel may be distorted by the effects of moisture upon the wood.

III THE SIGHTS.

The position of the rear sight is determined by the limit of distinct vision, and is so taken that the two sights and the object shall collectively be most plainly seen.

The sights are separated as far as convenience permits, so as to rectify their alignment with the object. See Chapter XXX, page 7. They admit of a permanent correction for jump and a variable correction for range, drift and the effects of wind.

The increasing flatness of the trajectory and the growing rapidity of fire will, except for sharpshooters, probably diminish the number of adjustments now given to the rear sight.

It is probable also, that instead of providing an extension to the slide for use at extreme ranges, a separate pair of sights will be placed on the side of the arm. The ordinary functions of the members of this pair will be reversed; that is, the rear sight will be fixed and the front sight movable downward, so that a considerable elevation may be attained without great variation in the relative positions of the eye of the marksman and the point of his body which receives the recoil.

It may be remarked that the requirements of sights for war service and for target practice at *known distances* are in many essentials incompatible.

IV. THE MOUNTINGS.

The bands, screws, pins, etc., are intended to connect the parts; and the butt plate, tip and the extension of the guard beneath the small of the stock are intended to protect from wear and to strengthen the relatively perishable wood.

V. THE BREECH MECHANISM.

Functions.

The functions of the breech mechanism are five, viz.: to open, load and lock the breech, to fire the charge, and to remove the empty shell.

The manner in which these functions are performed depends primarily upon the manner of opening and closing the breech, as is shown by the following scheme:

Classification of B. L. Small Arms.*

Breech-loading small arms have either a				*No.*	*Examples.*
fixed chamber closed by a movable	Barrel, which	slides		1.	(Rare).
		rotates about an axis which is	$\parallel$ to axis of gun	2.	Revolvers †
			$\perp$ to axis of gun	3.	Shot guns.
	Breech block which	slides	$\parallel$ to axis of gun	4.	Bolt guns.
			$\perp$ to axis of gun	5.	Sharps, (Krupp).
		rotates about an axis which is	$\parallel$ to axis of gun	6.	Joslyn, Warner.
			$\perp$ to axis of gun. in front of block	7.	Springfield, Remington.
			$\perp$ to axis of gun. not in front of block.	8.	Martini.
movable chambers (obsolete)				9.	Hall, Burnside.

Discussion of Table.

The mass of the barrel renders the classes, 1, 2, 3, unsuitable for the military service except when, as in revolvers, the mass is greatly reduced.

* For a fuller discussion, see Report Chief of Ordnance, 1873.

† The classification of these is difficult. For some reasons they may be considered as movable chambers, and in other respects they may be considered as an aggregation of barrels of reduced length.

Classes 5, 6, 8, are objectionable, as their operation does not assist in loading the cartridge, but rather, as the French say, to *guillotine* it.

They possess, however, the advantage of naturally resisting the pressure which tends to blow open the breech or to "*unlock*" it.

Class 7 naturally forms a lever, formerly useful in forcing into the chamber a deformed cartridge or in extracting one that stuck. Arms of classes 4, 5 and 8 were frequently provided with levers.

Bolt System.

But, as the quality of the ammunition has improved, the arms of class 4 *without levers*, have grown into general use.

The following are the principal objections which have hitherto prevented the more general adoption of the bolt gun, although its advantages were recognized by the Prussians as early as 1847.

1. The risk of premature discharge from striking an oversensitive cartridge in loading.

This was long considered an insuperable objection, but, as will be seen, has been overcome by very simple means.

2. The danger resulting from the necessity of loading the piece at a full cock.

This objection neglected the supreme advantage of the rapidity of fire which results from suppressing a discontinuous motion,* and which is further increased by the facility with which the reciprocating motion of the bolt adapts itself to the demands of magazine arms.

To illustrate the latest type of this arm, the American *Lee* system is described, as it contains in probably the best

*The word is used as in the drill book.

and simplest form the elements of the mechanism required for performing the functions above named.*

Lee System (as single loader) Figures 1 and 2.

Description.

The *receiver*, of approximately cylindrical form, is screwed to the breech and receives the mechanism. It is bored out and slotted to permit the axial motion of the bolt. The slot is widened to the front to form the *well* of the receiver, through which the operations of loading and ejection are performed.

The rectangular shoulder at *a* forms a support for the *locking mass*, *a'*, of the bolt in firing, and the oblique edge at *b* gives a short, spiral motion to the bolt as the locking mass is approaching or leaving its support.

The system is mortised vertically through the well to receive the magazine. As this is a special feature of the arm, its consideration is deferred until the features common to the best bolt guns have been discussed.

The reciprocating motion of the bolt sets the whole mechanism in motion.

The *handle* is placed in rear and is curved downward so that the hand need not leave it in firing.

A lug diametrically opposite to the locking mass engages with a corresponding recess in the bore of the receiver, so

*The Prussian Needle Gun used a combustible cartridge case, the fouling from which tended to obstruct the chamber; the joint was most imperfectly sealed, the flames escaping not only around the end of the bolt, but into the channel traversed by the firing needle. The tactical advantages of the arm, however, offset these very serious objections, so that it was retained unchanged until adapted to metallic ammunition after the war of 1870.

Its opponent in this war, the Chassepot, was of similar construction, but possessed for the end of the bolt a gas check, from which that of Colonel De Bange is derived.

that, by making the support symmetrical, certain objectionable vibrations of the barrel may be avoided.

The bolt contains an axial *firing pin* which is surrounded by a *spiral main spring* and secured to the *hammer*.

The bolt carries in front and to the right a hook shaped *extractor*, which, like the hammer, is so disposed as to share only in the motion of translation which the bolt receives. The extractor is retained by a flat spring which serves also to key the system together.

Operation.

To open the piece, raise the handle so that the locking mass may lie in the prolongation of the slot, and withdraw the bolt.

The incipient rotation of the bolt is ingeniously commuted into one of translation at each of its ends; as follows: —

In rear, a radial projection on the bolt strikes an oblique surface on the hammer and forces it back relatively to the bolt until the point of the firing pin is *retracted*, or withdrawn behind the plane surface in contact with the cartridge. To avoid premature explosion the point of the firing pin is kept back until the desired moment of discharge.

In front, the spiral motion due to the surface, *b*, forces the bolt slowly back from the barrel so that power is obtained to start the fired cartridge case from its seat. This slow and therefore powerful motion of *extraction* is commonly used. A rapid motion might cut through the cartridge rim and disable the rifle.

As the bolt is withdrawn, the extracted case follows until it passes from the chamber. The rim then strikes the *ejector stud*, a projection on the bore of the receiver opposite to the path of the extractor. The case is thereby rapidly revolved about the hook and *ejected*, or thrown clear of the gun.

A cartridge may then be dropped into the well, the bottom

of which is nearly continuous with the lower element of the chamber. A reversal of the motions forces the cartridge into place and locks the breech.

The surface, *b*, now serves to prevent the shock referred to on page 6, and also to make the motion of the hand continuous.

In the final motion of closing, the mainspring is fully compressed, or the piece is cocked, by the interposition of the *sear*, the nose of which arrests the forward motion of the hammer while the bolt moves on.

The U shaped *sear spring* acts against the trigger through the sear; so, that when the trigger is drawn, the sear spring is compressed, the sear is lowered and the hammer allowed to fall.

Remarks.

Opening, closing and loading. These operations are safely and rapidly performed.

Locking. The method is of great simplicity and affords a solid support. Jointed surfaces, however well made, permit an objectionable displacement under the stress of firing.

Firing. The coiled spring is admirably adapted to the purpose, since, owing to its developed length, the stress on any of its spires is slight; and, owing to its position on the pin, it will continue to work, even if broken.

Extraction and ejection. These are readily performed, even with inferior ammunition.

Assembling. The parts are few in number, strong and simple. They are arranged so as to avoid the effects of rust and dust, and are so connected as to be readily dismounted for cleaning without the use of special tools.

MAGAZINE ARMS.

If by any means a succession of cartridges can be automatically placed in front of the bolt as it is closing, a magazine gun will result.

This has been accomplished in many ways which may be classified. 1st. According as the magazines are *tubular*, or *box shaped*. 2nd. According as they are permanently *fixed* to the gun, or are *detachable*. The tubular magazines are always fixed.

TUBULAR MAGAZINES.

These may lie either, 1st, in *front*, as beneath the barrel, or 2nd, in the cylindrical volume forming the small of the stock and its prolongation in rear.

A spiral spring forces the contents of the tube toward the receiver, and a valve regulates their entrance.

In the first class a *carrier*, operated by the withdrawal of the bolt, raises the cartridges successively from the mouth of the tube to the mouth of the chamber. See figure 3 for one form of carrier.

The operation is that of the *bell crank*. Chapter XXIX, figure 7[a].

Advantages.

This form of magazine, used in the French Lebel Rifle, adapts itself to the profile of the gun. When in front, the capacity is large for cartridges which are short and thick, and a simple trap door on the side permits the magazine to be filled without opening the breech, *i. e.*, *without unloading the gun.*

Disadvantages.

The cartridges lie end to end, and in firing are exposed to shocks which may explode them or deform them sufficiently to interfere with the regularity of the feed.

The feed acts in the direction of the longest dimension of the cartridge.

For the front magazine the weight is not well distributed; and for that in the butt the capacity is small, and the filling

of the magazine is complicated with the unloading of the gun.

The operation of filling is slow, since the cartridges are passed in singly; and, since nothing external indicates the state of the supply, the control of the fire by the soldier, and of the soldier by the officer is impaired.

The "Cut-off."

By a device which may limit the withdrawal of the bolt, the magazine may be "*cut-off*" and its contents reserved for a suitable necessity. The piece meanwhile is used as a single loader.

Such attachments are fragile and in moments of excitement are confusing. When tried under such circumstances, they have been found unsuited to the conditions of service.

BOX MAGAZINES.

By placing the cartridges side by side in a box, many of the objections urged against the tube disappear. The principal point to be decided relates to whether the box shall be detachable or fix

1. Detachable Box.

An example of this type is seen in the Lee magazine, figure 2, which consists of a box of sheet steel, in which the cartridges lie over the feed spring, *N*.

The box is readily inserted through the mortise in the well of the receiver into the position shown.

The operation of the bolt passes the cartridges in succession into the chamber, and acts as a valve to regulate the ascent of those remaining to be fired.

A number of these magazines are carried by the soldier, who is expected to use his arm as a single loader until he receives the order to fix magazines.

This facilitates control by the officer, but the uncertainty of the soldier as to the state of the supply may lead him to go through the motions of firing with an empty arm.

The principal objection to the system applies to the excessive weight and cost of the box as a package, if many magazines are carried; and, if but few, to the probability of losing so important a component in the act of replacing it under fire.

2. Fixed Box.

1. A prominent arm of this type is the Austrian *Mannlicher* rifle, figure 4.

The cartridges are held by their bases in a sheet metal frame, the whole package being bodily inserted into the magazine through the well of the receiver, where it is retained by a spring latch, *r*. A *follower*, *f*, impelled by a strong spring, *s*, lifts the column so that the top cartridges are successively shoved into the chamber by the bolt. The fall of the empty case through the bottom of the magazine warns the soldier that the magazine is exhausted.

In a recent model the heads of the cartridges are so held by the frame that they lie in the same plane. With this model no special care is needed in inserting the frame into the magazine; while in that shown, the obliquity of the frame, caused by the step-like arrangement of the heads, may cause confusion.

The device for locking the arm consists of a brace, *b*, attached to the bolt. It is forced downward in front of a shoulder, *c*, in the receiver, by a wedge-shaped projection below an axial stem to which the knob, *k*, is attached. By simply pulling on the knob, the brace is lifted from its seat by the wedge, and the brace, knob and bolt slide out together.

This arrangement avoids the rotation of the bolt required in the Lee and in almost every other bolt gun.

This arm cannot be used as a single loader.

2. The *Schulhoff* magazine rifle, figure 5, may be used as such or as a single loader.

The cartridges are carried in an annular box, beneath the receiver.

The axial shaft, *s*, carries a radial plate, or *follower*, *f*, that is turned in one direction by the act of opening the lid, *l*, figure 6, and in the other direction by a spiral spring (not shown) surrounding the shaft, which is twisted in the act of opening.

The cartridges may be thrown in loosely, or may be loaded in mass from the *quick loader* shown in figure 7.

A circumferential slide, *c*, operated by the thumb piece *t*, forms a very simple cut-off.

The strength and solidity of the magazine enables it to be slit, so that the state of the supply may be seen at a glance.

The position of the box enables it to be filled without unloading the arm.

Quick Loader.

A cheap quick loader, figure 7, containing the supply of cartridges for one magazine enables them to be transferred to it in mass when rapid recharging is required. For this purpose the lower end of the quick loader being placed over the mouth of the magazine the pressure of the thumb of the operator on top of the column of cartridges forces them down into the magazine against the resistance of the magazine spring.

They are retained by a valve at the mouth of the magazine, and the quick loader is then thrown away. The valve serves to retain successive cartridges singly loaded.

Form Proposed.

The eventual preference of the fixed or the detachable box magazine will probably be largely determined by moral considerations.

The dispersed formations of future wars will probably require a more extended exercise of discretion in the lower grades than has hitherto been customary. The question arises, how far down will the discretionary control of fire extend?

It is now proposed to attach the box magazine permanently to the receiver, and ordinarily to load the arm continuously *through the magazine*, so that the cartridge last inserted shall be the first to be fired and that the number remaining shall be *automatically* held in reserve.

It is possible that the time gained by making the operation of the piece simple and *invariable*, as in the type proposed, may be so utilized in the general instruction of the troops that it will not be considered necessary to burden them with an inferior weapon in order to control their fire.

REQUISITES OF A MAGAZINE ARM.

The preceding considerations enable us to name the following necessities:

1. The best ballistic conditions attainable. These may modify the size and proportions of the cartridges, and so affect the capacity of the magazine.

2. Consecutive rapidity of fire as a single loader as great as that of any other arm, and the greatest possible intermittent rapidity when the magazine is employed.

3. The possibility of filling the magazine with single cartridges, or "in mass," without unloading the piece.

4. A maximum capacity which is yet to be determined by experience. It will probably be about 5 shots in the magazine.

5. A ready view of the state of the supply.

6. The most simple construction compatible with the maximum efficiency under the conditions of service.

Beyond a certain point objections to complexity become

pedantic, since experience shows that the instinct of self-preservation may be counted on for the care necessary to maintain an efficient arm.

THE SPRINGFIELD RIFLE.

History.

This arm, although originally intended as a means of utilizing the large supply of muzzle loading muskets left by the Civil War, has acquired a standing which, in 1886, caused its preference by 73 per cent of the officers to whom were submitted, for comparative trial in service, three of the best magazine arms.

Apart from the excellence of its manufacture and the ease with which it may be operated with but one hand, this preference may be attributed to the independent action of a form of lock, the outgrowth of centuries of experience; and the perfection of the apparatus for extraction and ejection.

The design of the cam latch and of the firing pin are exposed to criticism.

As a reserve supply of this arm is likely to be retained for many years after the adoption of a magazine gun, a few of its principles are described. The nomenclature is supposed to be known.

Operation. Figure 8.

Locking.

When the piece is fired the tendency of the block to swing upward out of the receiver, *A*, is corrected by the loose fit of the hinge pin, *E*, in its hole. The block, therefore, slides bodily to the rear until stopped by the interposition of the body of the cam latch *F*, between the block and the breech screw, *C*. The journals of the cam latch are loose in their bearings so that they may be free from strain.

The centre of pressure on the breech screw is brought

as nearly as possible in the prolongation of the axis of the bore so as to diminish the tangential component of the pressure, which tends to revolve the cam latch and throw open the block. This is imperfectly resisted by the friction developed by the normal pressure between the surfaces in contact, and also by the combined action of the thumb piece and the hammer, the functions of which are thereby perverted.

Extraction.

The power needed for extraction results from the compound lever formed by the breech block and the extractor, *J*.

Ejection.

In opening the block the revolution of the extractor compresses the coiled ejector spring, *K*, until the action line of this spring passes from above the axis of rotation to below it.

The expansion of the spring then rapidly revolves the extractor. This impels the cartridge case against the ejector stud *L*, which deflects it upward and throws it clear of the gun.

Firing Mechanism.

The lock, figures 9, 10, 11, consists of the lock plate, to which the parts are attached and by which the mechanism is secured to the stock by the side screws.

The hammer, *A*, outside the lock plate, and the tumbler, *B*, inside of it form mechanically but one piece, the arrangement adopted being required for the protection of mechanism from dirt.

The tumbler, *B*, is connected with the mainspring by a swivel, *J*, so disposed that the resistance to cocking the piece shall be nearly constant. This is accomplished by the variation in the lever arm of the mainspring; as the resistance of the mainspring due to its compression increases, the action line of the resistance passes nearer to the axis of the tumbler,

while the lever arm of the power, the thumb, is constant. The tumbler is thrice notched to receive the nose of the sear, *E*. This, under the action of the sear spring, *G*, maintains the hammer at the distances from the head of the firing pin required for convenience of transportation, safety of loading, and certainty of fire, respectively.

The bridle, *C*, holds the parts together.

The oblique blow of the firing pin is objectionable. Chapter XXVII, page 5.

RECENT DEVELOPMENT OF SMALL ARMS.

PHYSICAL CONSTANTS.

Owing to the mechanical improvements in the construction of arms and ammunition the ballistic development of the small arm is now limited by the soldier's endurance of its recoil. Similarly its tactical employment is limited by his ability to transport the burden of its ammunition ; for the maintenance of the rapid fire of the extended lines now rendered possible is a problem which increases in difficulty as the fire increases in rapidity and range.

1. Recoil.

These constants are influenced by racial peculiarities, and may be considerably modified by training ; but the proper training of large armies in the endurance of recoil implies so great a cost, that the present tendency is to render the recoil supportable by inexperienced troops, so that the accuracy of their fire may not be impaired by their apprehension of its effects.*

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*The effect of racial peculiarities, and incidentally of training, is shown by the following data which relate to arms of caliber about 0.45 in.

In the relatively small armies of Great Britain and the U. S. the energy of recoil is about 14 ft. pounds.

The low average stature of the French fixes a limit of about 11 ft.

2. Burden.

Training in weight-carrying is not expensive, and its importance is becoming recognized by the frequency with which practice marches are made. As in the artillery service Chapter XXIV, page 2, a judicious distribution of the burden between the arm and its ammunition depends greatly upon the former's recoil.

MODIFICATIONS OF THE RECOIL.

The recoil may be reduced by modifying the arm or the ammunition.

1. Modifications in the Arm.

If the ballistic conditions are kept constant, the weight of the arm may be reduced, and a greater number of cartridges be carried, by:—

1. The use of an elastic cushion attached either to the gun or to the clothing.

These plans are found impracticable.

2. Increasing the mass of the gun in firing by adding to it that of a portion of the ammunition, as in magazine guns.

The correction is variable and sometimes injurious to accuracy.

3. Storing up the energy of recoil as by the compression of a spring, which, by its resilience may operate the piece.

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pounds. But in Germany, although the ballistic conditions are nearly identical with the French, the desire for durability has developed the heaviest small arm known. Notwithstanding the strength of the Germans the recoil is only 10 ft. pounds.

In Italy, as in our service during the Civil War, about 7 ft. pounds is allowed.

This is the limit reached by the present reduction in caliber. The return to the former standard, page 23, is significant of its practical constancy.

This has been tried, but so far without success, owing to the complicated nature of the mechanism required.

4. The pressure due to the recoil may be distributed over an increased area of the person by the proper use of the gun sling. By lying down to fire, the path of the recoil is shortened and the pressure on the body increased.

General consent seems to have established the weight of the rifle at between 8.5 and 9.5 pounds.

2. Modifications in the Ammunition.

1. *Caliber and Recoil Constant.*

The advantages of any particular caliber being general, that of all military rifles at any epoch is approximately constant. It has recently been about 0.45 inch. See figure 12.

When the caliber and the weight of the arm are constant, the recoil can be reduced only at the expense of the ballistic properties of the arm. But these being maintained at the highest value consistent with the recoil endurable in any particular case, the Equation $M\ E = C = \frac{(m\ v)^2}{2}$ shows that modifications in the ammunition must be confined to factoring the momentum of the projectile. The following considerations illustrate the effect of variations in m and v, their product in any one case being constant.

a. If we increase m at the expense of v, we lose in dangerous space at short and decisive ranges but conversely at long distances. Chapter XX, page 40.

b. During the wars of 1870 and 1877 it was found advisable to deliver at extreme ranges an almost vertical fire against masses of troops.

It has since been found that the extreme range increases more rapidly with the sectional density of the bullet than with its initial velocity. The present U. S. bullet was accord-

ingly increased in weight from 405 to 500 grains, and an extreme range of two miles was attained.

c. It is found that the accuracy of fire at moderate *known* distances is incompatible with the high velocities required in actual service. This is probably due to the vibration of the barrel. Page 4.

Conclusion.

Owing to the impossibility of simultaneously satisfying the requirements of the different ranges, it is considered that efficiency at long ranges should be sought by the use of special means, such as machine guns firing heavy projectiles. For small arms it is considered that accuracy should become subordinate to flatness of trajectory for ranges exceeding 600 yards, at which individuals cease to be distinguished by the unaided eye; and that the trajectory should be so flat that but one height of the rear sight would be required within that distance, and the smallest number of changes beyond it.

Differences of elevation within the limits of graduation would be adjusted by varying the coarseness of the front sight. Chapter XXX, page 2.

2. *Caliber Variable.*

These conditions can be attained, and the number of cartridges in a given burden increased, by reducing the caliber. Under the conditions named on page 2, the limit of reduction has been fixed by the difficulties of manufacture and by those relating to the cleaning of the bore.

Small Caliber Rifle.

The following general principles govern the changes in ammunition resulting from the reduction in caliber. For simplicity of treatment we will first assume the muzzle velocity unchanged from the larger caliber.

Bullet. The sectional density, and therefore the length of

the bullet, has remained approximately constant; since, as shown in Chapter XVI, page 4, an increase in the sectional density would increase the value of p_0, unless the muzzle velocity were reduced.

The strength of the barrel is not materially greater than that of the caliber 0.45, and, owing to the reduction in the area corresponding to the bottom of the bore, the increase in the strength of the fermeture is only relative; therefore, the maximum value of p_0 formerly allowed cannot be greatly exceeded.

The sectional density being constant, the reduction in caliber reduces the mass of the bullet, and therefore, although the ballistic properties of the arm (being dependent only upon the sectional density or C, Chapter XX, and the muzzle velocity) would not be affected, the recoil would be reduced in the ratio $\left(\frac{m'}{m}\right)^2$. With the weights of bullet given in the following table, this would reduce the recoil from about 14 foot-pounds to about 3 foot-pounds.

Powder. This reduction being excessive, the normal endurance of the soldier against recoil is utilized by increasing the weight of the charge, and therefore the muzzle velocity.

The ballistic properties of the arm are therefore improved, figure 12, but the internal pressure* would be excessive unless

* If in Equation (D), Chapter XII, we place $K_0\, a^2 \triangle = C$, and represent by $\delta = \frac{W}{d^2}$ the sectional density of the projectile, and by $n = \frac{w}{W}$ the ratio between the weight of the powder and that of the projectile, we have, after reduction,

$$p_0 = C\,\delta\, n^{\frac{3}{4}}.$$

But if, as in the case considered, δ is constant, p_0 will vary with $n^{\frac{3}{4}}$.

From this it follows that if the same charge of the same kind of powder were used in the Hebler rifle as in the Springfield, the pressure would be nearly doubled.

the powder were made specially progressive. The value of $\frac{a}{l}$ for the cartridge shown in figure 14 is 11.43. See Table I, Chapter XII.

In spite of these precautions the main difficulty in the new small caliber high-powered guns is due to the excessive pressures developed.

Powder.

The principal difficulties found in realizing the advantages of a reduction in caliber exist in the powder.

It was thought by Professor Hebler, of Germany, to whom much of the credit of the proposed change is due, that these difficulties could be overcome by compressing the powder as in a rocket, in a cartridge case like the Morse. Figs. 13, 14.

The objections to this method noted, Chapters XII, page 21; XVI, page 45, and the large volume of smoke resulting from rapid fire, cause many to prefer a high explosive, such as described, Chapter XIV, page 15; in spite of its recognized objections. The complete solution of the problem is still deferred.

Projectile.

The projectile proposed is distinguished by its penetration, its cleanliness as regards the bore, and the nature of the wounds which it inflicts, page 2. When they are flesh wounds they are punctured rather than lacerated; but when they involve the bones these are shattered.

Cartridge Case.

In order to avoid the increase in the length and weight of the breech mechanism, resulting from the relative increase in the length of the cartridge case, this is made bottle-shaped, as in figure 14.

This unfits it for reloading with compressed powder, unless the Morse cartridge be used; the latter has been found too delicate to endure reloading by troops.

In some cases the volume of the magazine has been diminished by eliminating the rim and replacing it by a V shaped groove, in which the hook of the extractor may engage, figure 16. In order to facilitate reloading with perforated cylinders of powder, previously compressed, such as *c*, the cavity is cylindrical; the reduction in diameter being made by a brass ring, *r*. The blow of the hammer is supported by the *front* of the cartridge case.

Comparison.

The following table illustrates the advantages of the reduced caliber, since it compares the present Springfield rifle, which is one of the best of the arms recently used, with the Hebler, which is a fair type of the arms proposed:

	Springfield.	Hebler.
Caliber, inches	0.450	0.296
Bullet, wt. grains,	500	225
Powder, wt. grains,	70	83
Sectional density,	0.353	0.367
Spherical density,	3.6	5.6
Twist in inches,	22	4.58
Twist in calibers, ratio about,	3	1
Initial velocity, f. s.,	1280	1942
Cartridges, ratio of weights,	100	85.
Arm, weights pounds,	9.3	9.9
Maximum dangerous space, yds.,	380	440
Accuracy at 440 yds., ratio	1	3
Muzzle energy, foot-pounds,	1818	1882
Recoil energy, foot-pounds,	13.95	6.11

REVOLVERS.

As a military weapon the revolver is useful principally in enabling a horseman to use but one hand in delivering a rapid fire. In closed masses its employment is dangerous,

since it is difficult to fire to the front without striking the horse or the leading files, and the shortness of the piece leads to accidents to those alongside.

It is therefore considered generally an inferior arm, and one to be used only for personal defence and in maintaining discipline upon the field. Its ballistic properties need not be greater than necessary to stop a man at 50 or 60 yards.

The revolver is one of the oldest forms of magazine arms. Its present perfection is due to the invention of Col. Colt of Hartford, who combined the cocking of the hammer with the revolution of the cylinder.

Owing to the considerable moment of inertia of the loaded cylinder, this tends when rapidly revolved to pass the position in which the axis of the chamber next to be fired coincides with that of the barrel. This is the principal difficulty found in the construction of these arms.

To facilitate their operation, revolvers are sometimes made *self-cocking*, the action of the trigger causing all the motions to be performed. For greater *continuous* rapidity of fire, in which these arms, like many magazine rifles, are deficient, the cartridges may be simultaneously extracted by sliding or swinging the barrel and cylinder away from the breech. The chambers may then be simultaneously reloaded by using ammunition packed in clusters.

The complexity of these refinements and the limited scope of the revolver generally cause simpler patterns to be preferred.

MANUFACTURE OF SMALL ARMS.

Where Made.

The service rifle and carbine are made by the Ordnance Department at the National Armory. Pistols and such other special arms as may from time to time be needed are bought from private establishments.

How Made.

Efficiency in service and ultimate economy in manufacture require that the similar parts of arms of the same model shall be interchangeable. This is secured by the principle of gauging, noted in Chapters IV and XVII.

Gauging.

The general design of a gun having been perfected, an exact working model is carefully prepared. The component parts are so formed as to be as far as possible adapted to the operation of the varieties of the lathe. Chapter XVII, page 13.

Each of the components is then examined with reference to its *gauging points*. These are the surfaces between which the most exact relations are required.

For surfaces of revolution like the barrel, or parts intended to revolve like the Springfield breech block and the tumbler, the gauging points are established with reference to the axis of rotation.

For pieces subject to compression, like the bolt of the Lee rifle, the greatest pains would be taken with the distance from the rear face of the locking mass, a', to the front face of the bolt; and, in the receiver, with the distance from the shoulder, a, to the plane containing the mouth of the chamber, since the difference of these distance must be kept invariable in order to insure the proper working of the ammunition.

While the first of these is readily gauged, the second involves the relations between the barrel and the receiver; each of which must be similarly watched with reference to their abutting surfaces.

When the number of such surfaces is considerable, as in the Springfield mechanism, the sum of their possible errors requires the closest gauging of each link of the chain of parts.

Many forms of gauges are employed. They may be

classified like patterns as positive or negative gauges, the latter being sometimes simple notches, and sometimes matrices so formed as to contain exactly pieces of an irregular shape.

To retard their wear, the working surfaces of gauges are made of hardened steel; and, as steel tends in hardening to change its form, these surfaces are finished in the hardened state.

The number of gauges required not only for the finished parts but for the intermediate stages demand that, before the first arm of a series be produced, many thousand dollars shall be expended in preparation.

When the gauges and the corresponding tools and fixtures are made, the work goes on rapidly; for the functions of each workman are independent, and no time is wasted in fitting the product of different hands.

In illustration: The model may cost $600 — the first hundred guns made from the gauges $100 each, and the first ten thousand, all equal in quality to the model, $15 each.

The sense of feeling is so much more acute than that of sight, that by the use of guages differences far within the limits of ordinary measurement may be detected. The thousandth part of an inch is the customary unit, and this may be subdivided practically according to the requirements of the work.

The system more than anything else promotes the "division of labor," upon which industrial prosperity depends; and, by substituting an absolute for a discretionary standard, it educates in a remarkable manner the workman upon whose skill the value of the product is practically based.

OPERATIONS OF MANUFACTURE.

Barrels.

The principal operations are *rolling*, *boring*, *turning*, *straightening* and *rifling*. The *rolling* is done as previously

described, Chapter XV, page 44. While hot the rough ends of the tube are sawed off and it is straightened under a drop hammer, after which it is annealed by the residual heat. When cold the hard scale is removed by pickling in diluted acid.

In the preliminary *borings* the revolving auger is drawn through the barrel instead of being pushed, so as to keep the hole straight.* The bore is then enlarged by rapidly revolving reamers whose cross sections are square.

In *turning* the slide rest is guided by a template so as to produce a conical surface, and the barrel is kept from springing by the back rest.

Straightening is performed by light blows of the hand hammer applied at points which are indicated by the shadow of a straight edge reflected from the walls of the bore. This operation requires a peculiar knack which very few can acquire.

In order to secure uniformity in the *rifling*, a number of cutters equal to that of the grooves is provided and these are transferred automatically between adjoining grooves at the end of each stroke of the axial rifling rod.

This rod receives a combined motion of translation and rotation, by which, as in rifled cannon, the spiral motion is produced.

While in an intermediate stage, the barrel is *proved* by firing a very large charge of both powder and lead.

The final proof of the efficiency of the mechanism and of the accuracy of the arm is made with service ammunition.

MANUFACTURE OF THE MINOR PARTS.

The form is defined roughly by forging between dies.

* The adoption of the 0.30 caliber will increase the difficulty of boring since the barrel may require to be rolled solid and bored under compression.

The slow operation of a very powerful press permits many parts to be reduced to nearly their finished dimensions when cold.

The principles of milling, Chapter XVII, are used whenever practicable. The most complicated arms have thus been made without requiring the use of the file.

The most interesting operations are those required to produce irregular forms.

Profiling.

The profiler is a sort of milling machine in which relative motion in three coördinate directions can be produced between the revolving mill and the work. To limit the relative displacements the following arrangement is provided. See figure 17.

To a table moving in a horizontal plane the work, *W*, is clamped at a fixed distance from a hardened steel model, *M*, of the finished part.

At the same distance from the mill, *m*, and with its axis vertical, is a blank pin, *p*, of corresponding dimensions. We thus have two pairs of parts; one pair consisting of the model and the work and the other of the pin and the mill, with relative motion between the pairs. When the mill begins to cut it is necessary only to cause the pin to follow the profile of the model in order to reproduce it in the work.

The intricate *bed* or matrix of the lock is thus formed with the greatest accuracy in about one minute.

Eccentric Turning.

This operation was devised by Thomas Blanchard, an employee of the National Armory, for the purpose of forming the gun stock. It has been applied to many other useful purposes, as in the manufacture of shoe lasts, spokes, and even of statuary. Its principle is as follows: See figure 18.

In ordinary turning the cutter does not sensibly change its

distance from the axis during one revolution of the work and therefore leaves behind it a partically concentric surface.

But in eccentric turning the cutter, *C*, which revolves after the manner of a mill about an independent axis parallel to that of the work, is caused to oscillate slowly in a plane normal to the axis of rotation during each revolution of the work, *W*. This oscillation is produced by an iron model, *M*, revolving with and parallel to the work and resting against a blank wheel, *B*, attached to the oscillating frame which supports the cutter.

In concentric turning the cutting speed is due to the tangential velocity of the work, and in eccentric turning the high speed required in wood working is due to that of the cutter. The motion of translation is similarly performed in both cases.

The cutting edges are placed at progressively increasing radial distances, so as to cut to different depths during each revolution of the cutter. This principle is frequently applied in revolving tools.

The developed length of the cuts required to turn a gun-stock is 13 miles; the operation takes about 8 minutes.

Blacking and Browning.

To protect the parts from rust and to prevent them from flashing in the sun, small pieces are blackened by heating them until they will ignite the oil with which they are covered.

The outside of the barrel is oxidized by coating it with a dilute acid mixture and exposing it in a warm, damp place. The loose coating of red oxide having been brushed off, a permanent layer of black rust remains. Some parts are rapidly oxidized by immersing them in fused nitre.

CHAPTER XXIX.

CANNON WITHOUT RECOIL.

The advantages of rapid fire from cannon would be neutralized by the time required to readjust the aim, were it not that means have been found to so control the recoil that the piece may return, between shots, to its original position and direction. Such systems may be said to be practically without recoil.

They may be divided into two general classes. 1st. Those in which the mass of the system is so great in proportion to the momentum of the projectile that the velocity of recoil may be neglected. 2nd. Those in which so much of the energy of recoil of the piece is absorbed by an elastic resistance that the piece will be automatically returned to battery in time for reloading.

The former class contains the systems for which mobility is essential, and the latter contains those that are stationary. Some systems may be placed in an intermediate class, requiring mobility and permitting recoil of the piece relatively to its support.

FIRST CLASS. MACHINE GUNS.

Owing to the desire to simplify the supply of ammunition, these pieces are frequently fitted to the cartridge used by infantry. The conditions of transportation furnish the mass required. Their ballistic properties restrict their scope to the zone of infantry fire, within which their value consists in the small number of men required for their service

and the consequent possibility of selecting the coolest and most skillful men and of protecting them from the enemy's fire.

The difficulties of transportation on land appear to assign this class to the defence, for which service the concentration of its fire upon objects hidden by its own smoke, renders it most valuable.

To this class in the present U. S. Service belong the Gatling and the Gardner machine guns, which use the same ammunition as the service rifle, and the Hotchkiss Revolving Cannon, firing an explosive projectile such as described Chapter XVI.

THE GATLING GUN.

Construction.

This gun consists essentially of a cluster of parallel magazine bolt guns grouped cylindrically about an axial shaft to which they are attached by circular diaphragms called *barrel plates*. Each member of the combination is independent of the others except that the magazine is common to them all.

The rotation of a hand crank attached to the shaft brings each member in succession opposite to the mouth of the magazine where it is loaded, and operates the bolts so that they progressively perform the functions noted in Chapter XXVIII.

The manner in which this is accomplished is as follows; see figures 1, 2.

Fixed upon the shaft, *S*, in rear of the barrels is a composite fluted cylinder, *C*, each flute in which corresponds to the well of the receiver. Each flute carries a bolt, *L*, containing the essentials of the lock for a bolt gun. The firing pin is peculiar in that it passes through the bolt and terminates in a knob in rear.

From the rear of each bolt projects a radial lug corresponding to the handle of the ordinary rifle. These lugs enter a *cam groove* fixed within the casing that surrounds the fluted cylinder. This groove is essentially an oblique section of the casing, the highest point of the section being in rear.

The constraint of the flutes and of the groove commutes the continuous rotation of the cluster of bolts with respect to the axis, into intermittent reciprocating motion with respect to the barrel to which each bolt belongs.*

The upper and lower segments of the cam groove have their planes at right angles to the axis of revolution so that the developed groove is of the form shown in figure 3. The resulting arcs may be termed the *advancing* and *retiring segments*, *a* and *r*, and the *loading* and *firing flats*, *l* and *f*. The flats give the intermittent motion desired by interrupting the reciprocating motion of the bolts; in the first case to allow the cartridge time to fall before the bolt, and in the second case to allow for the event of a cartridge "hanging fire." Such concentric surfaces are frequently found in the cams used in machine guns.

The firing flat is at a distance in rear of the barrels exactly equal to the length of the bolt; the loading flat is in rear of the firing flat at a distance a little greater than that of the cartridge. See figure 2.

The exterior casing, which supports the ends of the shaft, the crank and the trunnions, is pierced above the front end of the cylinder to admit cartridges from a detachable magazine for which the casing provides a seat.

The sear is replaced by a grooved *cocking rib*, *R*, figure 2, into which the knob passes as the bolt advances along the

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* The principle has already been seen in the guide curve of the torsional testing machine, Chapter XV.

segment, *a*. Just as the coiled mainspring attains its maximum compression the lug on the bolt enters the firing flat, and the termination of the cocking groove allows the firing pin to fall.

The retraction of the firing pin and the extraction and ejection of the empty shell are effected by means similar to those already described for the small arm.

It is significant to observe how closely the latest type of the magazine arm approaches the principles of the Gatling gun, the essential features of which have not been changed since its appearance in 1865.

Remarks.

Each member is thus fired only once in each revolution, but the number of members (5 or 10) is such that while the individual feed is deliberate, the collective fire is rapid.

The bolts are interchangeable and can be readily removed and replaced. Being independent of each other, the removal of a disabled bolt affects only the intensity of the fire.

The crank handle may be attached directly to the shaft, or, when power rather than rapidity is required, to a worm gear on a shaft at right angles to the main shaft, as shown in figure 1. In the latter case each revolution of the crank causes one member to fire, whereas in the former case the number of shots fired in each revolution of the crank is equal to the number of the barrels.

Mounts.

The gun is mounted on a universal joint so that it may be *trained i. e.* moved in azimuth, without disturbing the position of the trail.

Instead of the wheeled carriage a tripod is sometimes used, as in mountain warfare; but, though light, its want of mobility is objectionable.

The Magazine.

With good ammunition the efficiency of the arm depends largely upon the efficiency of the feed, and this upon the form of magazine employed.

1. THE FEED CASE.

In the earlier models the magazine, or *feed case*, consisted of a tin prism of trapezoidal cross section containing 40 cartridges lying horizontally one above the other, as in the Lee magazine, inverted. These were surmounted by a weight provided with a projecting thumb-piece to which an assistant could apply the pressure necessary to prevent the dislocation of the column from the shock of falling and of firing.

In practice it was found difficult to prevent a cartridge from occasionally entering the flutes obliquely and thus jamming the gun; also, the longitudinal component of the weight varied with the inclination of the gun. The risk of jamming tended to retard the firing, which was further delayed by the time required to refill the empty feed cases.

2. THE BRUCE FEED. FIGURE 4.

This consists of two parts.

First, a coarsely toothed wheel revolving loosely in the hopper just below the mouth of the magazine. This guides the cartridges from the magazine to the fluted cylinder and keeps their axes parallel to that of the shaft. It may be used with all magazines.

Second, the detachable feed case is replaced by a semi-permanent bronze standard, figure 5. The front face of this contains two T shaped grooves so arranged as to hold two parallel columns of cartridges horizontally by the flanges

around the heads of the cartridges. The grooves are readily filled by entering into them the heads of the cartridges contained in the pasteboard boxes in which they are issued to the troops.

By pulling off the box at right angles to the grooves the cartridges are left hanging in the position desired. Under pressure from above, the grooved piece swings aside as each groove is alternately emptied and directs the remaining column into the hopper.

This arrangement enables the fire to be maintained as long as the general supply of ammunition for the troops is available.

3. THE ACCLES' FEED DRUM. FIGURE 5.

This consists of a cylinder having two heads which are at a distance apart about equal to the length of a cartridge. On the inside of each head is a spiral groove * the pole of which is in the axis of the drum, and the radial interval between the spires is equal to the diameter of that end of the cartridge in contact with the head.

Around the axis of the drum lie the radial arms of the *propeller*. These divide the interior of the drum into sectors, each of which contains a number of cartridges equal to the number of intersected spires. The ends of the arms project beyond the outer groove sufficiently to engage with the flutes of the cylinder, so that when this cylinder revolves the spiral groove is emptied from the outer end.

The feed is thus independent of gravity, and the connection between the fluted cylinder and the propeller arms is such that each cartridge as it emerges from the drum is carried parallel to itself and tangentially into the corresponding flute. The synchronism thus attained permits the gun to

* The spiral is not truly geometrical, as seen by the figure.

be fired with certainty at any angle of elevation, and at a speed that is limited only by the rapidity with which the crank can be turned. It has been fired at the rate of 2000 rounds per minute and also at a elevation of 87 degrees.

The objections to the drum refer to its bulk and to the time lost in refilling it. It contains 104 cartridges.

THE GARDNER GUN. FIGURES 6, 7.

This consists essentially of two or more bolt guns placed side by side, and surmounted by a magazine resembling the Bruce feed guide, but without the valve.

The bolts are operated by a transverse crank shaft, or *lock cam*, turned by a handle outside. The cranks are at 180 degrees from each other, and their crank pins, figure 7, *a*, work in vertical U shaped pieces in which the base of the bolts are formed.

The contact surfaces of the notches are partly concentric and partly eccentric with respect to the crank shaft; just as the cam groove of the Gatling is partly at right angles, and partly oblique to the axis of revolution. So long as the contact surfaces are eccentric the rotation of the crank will cause one bolt to advance and the other to retire.

When the crank pins are approaching the axis of the bore the contact surfaces become concentric; the bolts then become stationary; on one side to allow the cartridge to be fired, and on the other side to allow the next cartridge to fall into the well.

The reciprocating motion of the bolts operates a valve that allows the cartridges to descend in alternate succession from the grooves in the magazine. The valve swings horizontally under the lid. See figure 7.

The advantages of the arm are its lightness, and the simplicity and accessibility of the mechanism; as the lid cover-

ing the interior can be readily raised. On the other hand the rapidity of fire is much less than that of the Gatling, since the number of barrels is less; and the dependence of the feed upon the operation of *both* the bolts causes the gun to be disabled by accidents to its ammunition.

THE NORDENFELT MACHINE GUN.

This arm, principally used in the British Navy, has 2, 4, or more barrels arranged in a horizontal plane. In rear is a corresponding row of bolts moved back and forth together by a horizontal hand lever. The feed is by gravity through a series of hoppers, one of these standing over the well of each receiver.

The firing is practically by volley, although the interval between the discharges may be varied by the rapidity with which the lever is worked.

Accidents are said to have occurred from the explosion of the magazine by the flame escaping from defective cartridges, and igniting the ammunition in the adjacent magazine.

THE MAXIM AUTOMATIC MACHINE GUN. FIGURES 8, 9, 10, 11.

The distinguishing feature of this arm is its continuous operation by the force of the discharge, and the necessarily continuous supply of the ammunition afforded by the nature of the magazine.

Feed.

The magazine consists of a series of belts, made of two long strips of tape riveted together at intervals sufficient to hold single cartridges between them. Each belt contains 334 cartridges, and, as they may be linked together, the supply is continuous. The belt is fed transversely through the gun so that the cartridges occupy successively a fixed

position over the chamber. From this position they are automatically transferred to the chamber from which they are fired.

Immediately beneath the chamber is the *ejecting tube* through which the empty shells are successively passed forward and dropped to the ground.

Construction and Operation.

Omitting many important particulars, the operation of the arm is as follows:—

Construction.—The barrel is secured to a frame that has a slight sliding motion from front to rear in the exterior casing. In the rear end of this frame is journalled a double crank, figure 8. The crank pin is connected by a connecting rod with the breech-block in front. The breech-block slides longitudinally within the frame to a greater extent than the frame slides within the casing.

The ends of the crank shaft project through the casing and are utilized as follows:—

On one end is an arm, *d*, pointing downward and attached at its lower end by a short chain, *e*, to a powerful spiral spring, *f*, the front end of which is fixed to the casing.

On the other end is a curved arm, *l*, pointing upward and lying in front of a stop, *g*, rigidly fixed to the casing. On the same end of the crank shaft, but outside of *l* is a handle, *h*. This handle receives from the crank a reciprocating motion of nearly 180 degrees, being limited on one side by the flat spring, *b*, and on the other by the latch, *j*. The space, *o*, shows the distance to which the frame can recoil.

Before firing, the tension of the spring, *f*, brings the breech-block in contact with the barrel and forces the latter to its foremost position. The crank, crank pin and the connect-

ing rod are then in the horizontal plane containing the axis of the barrel.

Operation.—When the piece is fired (by pressing with the thumb on the trigger, *t*, meanwhile directing the aim with the handles, *m*,) the barrel and frame with the included parts begin to recoil together.

This motion brings the curved arm, *l*, against the stop, *g*, and rotates the crank downward with a motion which is accelerated by the curvature of *l*. In so doing the connection between the crank and the breech-block draws back the latter faster and further than the barrel moves, and thus leaves room for the cartridge to be loaded. At the same time the arm, *d*, winds up on itself the chain and puts on the spring, *f*, a tension that tends to restore the parts to their original position.

Also the handle, *h*, strikes the spring, *b*, which throws it back to the latch, *j*, thereby assisting the restoration of the parts by the spring, *f*. The cycle is now complete, as far as regards the parts named.

Feeding.—The feeding of the ammunition, which, from what precedes, is seen to be the motive power, is accomplished by a peculiar extractor, *x*. This slides up and down in front of the breech-block and, in an undercut groove on its front surface, holds two cartridges by their flanges. One cartridge is that last fired, and the other is that next to be fired.

When the block recoils the extractor pulls out these cartridges from the chamber and the belt respectively: and just before the block advances the extractor drops, so as to bring the fired case opposite to the ejector tube and the full cartridge opposite to the chamber.

After their entrance into their respective cavities the extractor rises, leaving the flange of the fired case unsupported, (as the undercut groove extends only over the upper portion

of the face of the extractor) and embraces another cartridge from the belt. The latter has meanwhile been moved along one interval by a pawl actuated by the preceding machinery.

If, as is usual, the pressure on the trigger be maintained during these motions another shot will be fired and the motions will be repeated as long as the ammunition is supplied.

Under these circumstances the rapidity of fire is 11 shots per second.

The operation of the lock may be understood from an inspection of figure 11. The cocking power is applied to the tumbler through a forked lever by which the connecting rod is attached to the block.

The handle, *h*, is used to start the gun firing or to overcome the stoppage resulting from a misfire.

As there is but one barrel, which in continuous firing might become heated to excess, it is surrounded with a water jacket. The recoil pumps a small quantity of water into this at every fire; it is provided with holes for the escape of the steam generated.

Advantages.

The principal advantages of this arm depend upon the absorption of so much of the energy of recoil by the working of the parts that the weight of the carriage may be greatly reduced without affecting its stability. The most important results from this principle will be found in its application to the field cannon hereafter described.

Another advantage results from the greatly increased dirigibility of the arm, since it may be pointed like a hose by the one man required for its service.

Considering the variety of the functions to be performed, the rapid motion of the parts and their restricted space, the construction cannot be called complicated. In such matters

it is better to seek simplicity of service than excessive simplicity of construction.

GENERAL REMARKS ON MACHINE GUNS.

Owing to the excitement prevailing during the few moments when rapidity of fire is of the greatest importance the efficiency depends mainly upon:—

1. The quality of the ammunition, the *food*.
2. The certainty of its supply, the *feed*.
3. The facility with which the aim may be changed, or the *dirigibility* of its fire.

1. The Food.

When compared with ordinary field artillery the range and power of the infantry ammunition is so limited, the correction of the aim so difficult, and the moral effect of solid projectiles so small that machine guns of small caliber, although rendered popular by their great mechanical efficiency, will probably in service be relegated to the subordinate purposes of the defense, and for use against an enemy unprovided with artillery.

When compared with infantry operating in broken or wooded ground their deficient mobility would often render them an obstruction rather than a help; since only under exceptional circumstances can they maintain their own defense.

Owing to the suddenness with which, in order to produce a decisive effect, their services are required it has been proposed to attach them to small tactical units of infantry and cavalry; but the objections above cited and the evident difficulty of administering such scattered commands would probably cause this plan to fail, as did the system of *battalion guns*, which, after trial in the Seven Years War, was abandoned.

The transport of the large amount of ammunition that machine guns require compels the use of wheeled draught (except in the mountain service) and therefore assigns them to the artillery. The question then arises, whether, having a given supply of men, horses and money, these means may not be better utilized in the legitimate sphere of the artillery rather than in providing a defensive arm of only occasional utility. The preponderance of opinion seems to be adverse to the general employment of machine guns of small caliber, and shows that mechanical perfection must be subordinated to tactical utility.

In the war of 1870 they were speedily silenced by the German artillery and have not since been organized for service in any European army. For naval purposes, where the difficulties of supply hardly exist, and where the deployment of a firing line is impossible, their value is greater, although here they are giving way to the rapid firing guns to be described.

2. The Feed.

The great rapidity with which the gun is operated is apt to lead to "jamming" in case a cartridge should from any cause fail to enter its proper chamber. The injury will be aggravated from the natural tendency in moments of excitement to overcome the obstruction by force. To this cause may be attributed grave complaints made against these guns during the recent fighting in the Soudan.

3. Dirigibility.

The dirigibility of these guns, except the Maxim, is impaired by the necessity of temporarily clamping them to resist the deflection due to the operation of the crank. The accuracy is also impaired by vibrations due to firing.

The fire may be distributed by oscillating the piece in a

plane parallel to the axle; but, if the line fired at be oblique to this plane it will be cut at but one point. The difficulty of sweeping a curved or oblique line is increased by the screen of smoke and by the uncertainty of the point of impact.

In the Maxim gun these objections only partly apply, since the pointing of the gunner is not interfered with by the motions of the men at the magazine and at the crank.

THE HOTCHKISS REVOLVING CANNON. FIGURES 12, 13.

In its mechanical construction this resembles the Gatling gun, but, as it fires an explosive projectile, it occupies a place intermediate between the machine guns proper and the *rapid firing* guns to be described.

It uses primed metallic ammunition of a caliber the inferior limit of which for explosive projectiles was, from reasons of humanity, fixed by International Convention at about one pound, and the superior limit of which is determined by the difficulties of the feed.

Operation.

The continuous revolution of the transverse crank shaft causes the cluster of five barrels to revolve intermittently in one direction. The interruption in the rotation enables the single breech mechanism, whose motion is continuous, to serve each barrel in succession, by simultaneously firing one barrel, loading another and extracting the empty cartridge case from a third, while the cluster is at rest. These operations are performed at every rotation of the crank, during half of which period the cluster is at rest.

The cartridge is supported by a stationary breech, in front of which the cartridges circulate and through which the firing pin passes at the lowest point.

The feed is by gravity through a trap door, which, being raised by the advance of the loading bolt, admits but one

cartridge at a time. The rapidity of fire is from 30 to 60 per minute.

Remarks.

The gun compensates for the relative infrequency of its fire by the use of an explosive projectile, each fragment of which may cause a hit. This permits the aim, which is normally deliberate, to be further corrected by observing the point at which the projectile explodes. A percussion fuze is used.

The strength of the parts of the breech mechanism and their accessibility compensate for the dependence of the performance of the gun as a whole upon that of each of its constituent parts.

Employment.

The Revolving Cannon was originally devised to defend vessels from the attacks of swift torpedo boats, the time allowed for firing at which, between their discovery and the impact of their torpedoes, might not exceed two minutes.

It has been attempted to introduce it into the field service; but the relation between the caliber and the weight of the piece and carriage has so far prevented its systematic adoption. For, while in the naval service perforation is of the utmost importance, and the operation of the percussion fuze is rendered certain by the nature of the object first struck; in the field the use of shrapnel ignited by an adjustable time fuze against animate objects is more important, since against defenses the Hotchkiss shell is powerless. Against field defenses indeed, it is curved fire with reduced charges and large projectiles that is required, rather than the flat trajectory needed for penetration.

For these reasons the Revolving Cannon has been principally confined to the naval service, in which during the war between France and China, in 1881, it was most effective.

An exception may be made as to its use in defending caponnières, for which purpose, as the range is constant and mobility is not necessary, the gun is well adapted. In this service canister is employed, and the barrels have each a different twist so as to distribute their fire with uniformity over the area to be swept.

SECOND CLASS. RAPID FIRING GUNS.

These are single barreled and may be classified according to the means by which they are loaded as:—

1. Hand served, or non-automatic.
2. Automatic, in varying degrees.

Object.

These arms may be said to have been developed from the later type of machine guns by the improvements in the offensive and defensive power of torpedo boats. Their proper employment on shore has yet to be determined; but in the naval service, in which their development is limited only by the weight of the ammunition to be handled, they have rapidly increased from the comparatively recent 1 pdr. and 6 pdr. to the 6 in. 100 pdr., and have made great changes necessary in the armament of vessels.*

* As the power of the gun increased, the weight of the armor increased so that but few vessels are able to carry enough armor to protect the entire surface exposed to fire. The armor is therefore often concentrated over the vital portions, and the ends left with little or no protection. Against the light armor required by these conditions, and now generally used, the rapid fire of guns small enough to be easily directed and large enough to carry an effective bursting charge, would be very destructive. Compared with a heavier caliber, the aim could be more easily corrected, the number of rounds for a given stowage capacity be increased, and the striking energy of the shell being sufficient to penetrate the target, their explosion would riddle it sufficiently to endanger the stability of the vessel. The effect would be increased by repeated impact in the same spot so that,

The Italian ironclad "Piemonte" built in England in 1889, is the most striking illustration of the principle involved. In a given time she can fire more than twice as great a weight of metal as any vessel afloat, including those costing five or six times as much. The 33 pounder with which she is largely armed has hit a target 6 feet square five times in succession in 31 seconds at a range of 1300 yards. Suppose the target to be a torpedo boat, advancing to attack the vessel carrying the gun at a speed of 20 knots per hour, (35 f. s.) and capable of effectively discharging its torpedo at a distance of 300 yards; she would be under fire for 86 seconds, during which time 20 shots might be fired, as against 2 or 3 shots from an ordinary breech loading cannon. A similar advantage would follow great rapidity of fire at the ports of an armored structure during the short time that they would be exposed to fire. There would be not only the chance of entering the port with shell, but that of disabling the gun, as it has been found that the chase of

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principally on account of their accuracy, the aggregate effect in a given time produced by a given weight of rapid fire guns would be greater than that of a smaller number of heavy cannon. The question involves the determination of the best utilization of that fraction of the entire displacement of the vessel that is allowed for its offensive power. This is small, being in the neighbourhood of 7 per cent.

For the reasons given Chapter XVI, page 21, armor piercing projectiles are becoming more and more restricted in their "mining power." It is thought that a 4 inch plate of the best quality will explode any high explosive fired against it as a bursting charge, before penetration is effected.

From what precedes it will appear that the present state of the question is that of a reaction from the idea which considers the ship as but a floating carriage for the heaviest gun that it can carry.

When tested in battle the truth will probably be found to lie between the two extremes, and the best results to be obtained by a proper application of the principle of the independence of function.

a modern gun can be penetrated by the armor-piercing projectiles fired from R. F. guns.

One great advantage of these guns would be, that as the attacking vessel approached and as therefore the flatness of the trajectory increased, the elevation would not require sensible adjustment between shots, and the target could be "followed" by the gunner; his eye being on the sights, his finger on the trigger, and the piece being directed by the motions of the body as hereafter described.

GENERAL PRINCIPLES.

The development of R. F. guns has followed improvements in the ammunition and in the *mounts*, as are called the means of supporting and directing the gun.

Ammunition.

Rapidity of fire results from the use of a simple fermeture in connection with self primed metallic ammunition constructed on the same principle as that used in small arms.

The size of the cartridges is now limited only by the difficulty of expeditiously handling them. That for the 33 pdr., containing about 12½ lb. of powder and weighing complete about 50 lbs., is as heavy as one man can load with ease. That for the 70 pdr. requires the united efforts of two men, the greatest number that can be employed about the breech without interfering with the gunner. For large calibers the projectile is sometimes loaded separately; in this case rapidity of fire depends solely on the mount.

Mounts.

The mounts are of two classes, depending upon the relation between the energy of the recoil and the character of the emplacement. When the latter is relatively strong *Non-Recoil Mounts* may be used, as in firing light guns from

ships and heavier guns from masonry emplacements. The structural elasticity of these mounts absorbs the recoil and returns the piece to battery.

When the emplacement is not rigid enough to resist the recoil, *Recoil Mounts* must be used, as in the field service. In these springs or hydraulic buffers distribute the pressure over a longer path than in the former case.

In all cases the aim is corrected by a crutch shaped *stock* which the gunner holds against his shoulder with one hand, while the other hand rests on the trigger.

The piece thus becomes almost as dirigible as a small arm, and is particularly adapted for firing from an oscillating platform at objects with relative motion, as in naval combats. In non-recoil mounts the stock may be fastened to the gun, but in the other class it must be fastened to the mount.

Figure 14 shows a *crinoline* or *cone* non-recoil mount; and figure 15 a type of the recoil mount. In this figure, *b*, is the buffer and, *s*, *s*, springs intended to return the piece to battery by their action on the bent levers, *l*, *l*.

Hotchkiss Mountain Carriage.

Figure 16 shows a non-recoil mount for a mountain carriage. The carriage as a whole is kept from sliding by the brake and by a transverse plate under the trail plate called the *spade*. The spade may be sunk into the ground by the weight of a man standing on the *wings* of the trail plate, and by that of the gunner on the seat.

The cheeks are unusually high, permitting the gun to be fought without wheels in exposed positions. A slight play in azimuth is allowed, since this greatly facilitates aiming, and the lateral component of the recoil is insufficient to affect the stability of the piece.

To facilitate transportation the stock is in two pieces.

The elevating gear is detachable; when used without it, the system resembles a large pistol on wheels.

Hotchkiss Field Carriage.

Figure 17 shows a recoil mount; the carriage consisting of three main parts and the wheels.

These parts are:—

1. The stock, *s*, secured to the axle.
2. The chassis, *c*, which has a limited motion in azimuth by means of the *training screw*, *T*.

   The top carriage, *t c*, to which is attached the elevating screw, *e*.

The top carriage is allowed considerable recoil on the chassis; being limited by the *buffer*, *b*, and restrained by three oblique spiral springs on each side. These are fastened at one end to the top carriage and at the other end to the chassis.

Powerful brakes, *B*, *B*, lock the wheels and are assisted by the spade. The brakes are connected by the brake bar. This is slung from the axle by two rods, so that in marching it may be hooked up under the trail.

CONSTRUCTION OF THE AMMUNITION.

Cartridge Case.

This may be either solid drawn as in small arms, or built up. In the latter case, for economy, the tube forming the body is bent in at the base and riveted between two cups by a wrought iron disc that forms the head, as shown in figure 18.* For greater certainty of fire the primer is renewed at every reloading.

Drill Cartridge.

That shown in figure 19 is of soft material formed to the full size of the cartridge about a section of the service musket

* It is now proposed (1890) to electro-weld the head to the bottom of the tube.

barrel. From this barrel the service cartridge may be fired at drill. This familiarizes the gunners with the service of the piece in action without requiring the great expense attending the use of the regular ammunition.

Projectiles.

Shell and shrapnel are provided. The former use a base percussion fuze and are either of cast iron or of forged and tempered steel according to the use required of them.

CONSTUCTION OF THE GUN AND FERMETURE.

Gun.

Except for the smallest calibers, this is a built up steel gun, the type of which for medium calibers is represented in figure 20.

The jacket carries both breech-block and trunnions so as to relieve the tube from longitudinal stress. The enlargement of the jacket in rear increases the bearing surface of the breech-block.

Fermeture. Figures 21, 22, 23, 24, 25.

This resembles that of the mountain gun already described; but it differs from it in that the motion of the block being vertical, its own weight helps to open it. The mechanism is contained in and about the block, and is situated so as to be most easily accessible for repairs.

It consists of the following parts:

B, the block, the left side of which contains three shallow grooves, viz.:

g g — the *guide groove.*
s g — the *stop groove.*
x g — the *extractor groove.*

Note.—To the late B. B. Hotchkiss, an American whose works were in France, is due the first practical application of the principle of rapid fire guns. His system is that generally used in this work to illustrate these principles. Many variations of his methods have arisen, some of the more important of which will be discussed.

On the right side is another *guide groove*, *g′ g′;* and *s w*, the *stud way;* beneath which is a wide triangular recess, *c w*, the *crank way*.

The double *lever*, *L*, is secured to the *crank shaft*, *c s*, which passes through the side of the breech, and terminates inside in the *crank*, *c*, on the upper end of which is the *stud*, *s*.

The hub of the double lever is formed into the *cocking toe*, *c t*.

The *hammer*, *h*, is secured to the *rocking shaft*, *r s*, by a *spline* (see Webster), so that it may be assembled with the rocking shaft and yet revolve certainly with it. The rocking shaft traverses the lower front quarter of the block. Its right hand extremity forms the *cocking arm*, *c a*, projecting so as to revolve in the same plane as the cocking toe above it.

The double-leaved *main spring*, *m s*, presses up the rear end of the hammer, and pulls it down in front through the *swivel*, *s v*, thus diminishing the friction of the rocking shaft in bearings. The main spring is kept from moving as a whole, by engaging its folded end in a notch in the rear portion of the block.

The *sear*, *s′*, is pivoted to the block just below the rocking shaft, and is constantly pressed up by the sear spring, *s s*.

When the block is in place, the extremity of the sear comes in contact with the front end of a bent lever forming the *trigger*, *T*. This trigger plays in the pistol grip which is rigidly connected with the breech.

Stock. Figure 14.

To the left side of the piece in non-recoil guns; or, in recoil guns, to that portion of the carriage which does not recoil, is fastened the *stock*.

A rubber tube forms a cushion for the gunner's left shoulder.

The handles beneath serve for grasping with the left hand at different elevations. The right hand then holding the pistol grip, the piece may be aimed by the motions of the body, and fired at the exact moment that the object is pierced by the line of sight.

OPERATION OF THE FERMETURE.

To open the Piece.

Turn the most convenient handle to the rear. In the first portion of its motion the crank stud travels in the concentric portion of the stud-way so that the block does not move; the direction of the stud-way then changing, further rotation causes the block to fall until it is arrested by the stop screw. The crank finally becomes nearly horizontal.

During the first portion of the rotation of the lever the cocking toe presses upon the cocking arm and revolves it, and with it the rocking shaft and hammer, until a notch upon the rocking shaft has engaged with the sear. At this moment the whole block, including the hammer, begins to fall.

The extractor operates as in the mountain gun already studied. Owing to the slight inclination of the groove $x\,g$ to $g\,g$, the first movements of the extractor are relatively slow, developing great power. Ejection follows when the abrupt change in the inclination of $x\,g$ reaches the stud on the extractor.

To close the Piece.

The rotation of the levers is reversed. The beveling of the front face of the block facilitates the insertion of the cartridge, as in the mountain gun.

To lock the Breech.

The weight of the block tends to keep the piece closed, since the vertical through *s* passes in front of the center of the crank shaft.

In recoil, the moment of the upper lever also tends to tighten the joint

THE NORDENFELT R. F. GUN. FIGURES 26, 27, 28.

This is principally intended to overcome an objection to the Hotchkiss system, that is based upon the *guillotining* action of its block, in case the cartridge is not fully inserted into the chamber before the breech is closed.

Its operation resembles that of the Hotchkiss, but in its construction the breech-block is divided by a plane of right section into the block proper, *B*, in front; and the wedge, *W*, in rear.

The upper rear portion of the wedge forms a convex cylindrical surface with its elements at right angles to the axis of the bore.

The first motion in opening the breech gives a downward movement to the wedge, the block remaining at rest. As the hand lever continues to move, the block and wedge revolve downward and backward together, so that when, as in closing, these motions are reversed, the cartridge is pressed home. This arrangement permits the cartridge to be actually

Note:—Observe the *face plate*, *f p*, and the detachable *hammer point* both good examples of independence of function.

The breech-block may be dismounted by opening the gun after removing the stop-screw. In this case the crank is revolved downward into a cavity formed for this purpose on the right face of the breech-block. The crank may be turned so as to emerge to the front from the groove; the block, being no longer supported on the stud, will then drop.

thrown into place, as its base may rebound several inches without affecting the rapidity of fire.

MAXIM R. F. GUN. FIGURES 29, 30, 31, 32.

Nomenclature.

The breech-block *C*, slides vertically in the barrel, *A*, and not in the jacket, as is usually the case.

The barrel slides back and forth for about six inches in the relatively thin jacket, *B*.

The jacket carries the trunnions, and beneath it is attached the cylinder of the hydraulic buffer, *D*. The piston rod of the buffer is attached to the barrel in rear, and is surrounded by a powerful helical spring lying within the cylinder. This spring gives the counter-recoil as in the smaller model.

Beneath the mouth of the chamber the barrel is traversed by the *rocking shaft*, *K*, to which are rigidly attached two curved arms of which but one, *J*, is shown. The arms are united in rear by a cross pin, *M*, that plays in a slot, *N*, passing through the block.

The extractor, in one piece, is composed of the upper *claw*, *X*, which engages with the rim of the cartridge, and of the *tail*, *H*. The extractor revolves about the horizontal axis, *Z*, lying between *X*, and *H*. Its upper portion, seen from the rear, is crescent shaped so as to engage both sides of the rim of the cartridge. Each of the arms of the crescent carries a projection, *V*, that may engage in a corresponding depression in the upper front edge of the block.

Operation.

When the gun is fired the barrel and block recoil together, with no relative motion between them. The hydraulic buffer acts as usual, and the counter-recoil spring is compressed.

As the spring draws the barrel forward, by means of some

mechanism in the *side box*, hereafter explained, the rocking shaft is given a small left handed rotation that causes *M* to slide through *N* and so depress the block. As the block passes *H* it presses it to the front, first extracting and then ejecting the cartridge by means of the gradually increasing curvature of *H*.

At the same time the hammer within the block is cocked, and other necessary functions are performed by means not herein described.

As soon as the cartridge has been ejected, the block is raised by the action of a spring which the act of recoil has compressed until it is caught by the projection *V*, *V*.

The entrance to the chamber being then clear, (or the door held open by the latch, *X*,) if a cartridge be inserted, *X* is pressed forward by the rim of the cartridge and *V* released. The spring before mentioned then fully closes the block.

By pulling the trigger, *T*, the piece may then be fired and the cycle be repeated. Or if, during the closing, the trigger be kept drawn back, the piece will be fired at the instant that the breech is closed. Under these circumstances the operation is completely automatic except as concerns the loading; the weight of the ammunition requires this to be done by hand.

A rapidity of 80 rounds per minute has been attained with the 3 pdrs., and guns have been designed up to a 40 pdr. caliber.

In commencing the firing, the rocking shaft may be revolved enough to open the breech by means of an independent handle outside. This may be also used when a cartridge misses fire.

Advantages.

A reduction of the work done on the carriage by the gun. A diminution of the jump, and of the vibrations, caused by

operating a hand lever. The reduction of the number of gunners required. One man can fire ten aimed rounds per minute, and can deliver unaimed fire as rapidly as he can throw the cartridges into the chamber.

The increase in efficiency of service due to the greater safety and certainty of operation, and the increased accuracy of this arm, compensate for its slightly greater complexity of structure as compared with other hand loaded Rapid Fire Guns.

Side Box.

As the barrel recoils, the shaft, *K*, figure 31, goes with it, carrying the triangular piece shown, upon the lower corner of which is the friction roller *e*.

The upper end at *a* slides under the lever, *c*, which is kept in close contact with *a* by the spring *d*.

When the counter recoil spring in *D* draws *K* forward, the notch in *a* engages with the rear end of *c*. This tilts the triangular piece into the position shown in figure 32; the spring, *b*, being compressed by the rising of the roller, *e*.

The partial rotation thus given to *K* causes the breech-block to descend as before explained.

The block is kept from rising under the pressure of *b*, until the projections *V*, *V*, figure 29, are unlatched. When this occurs the side mechanism resumes the position of figure 31.

Lock, etc.

As *J*, figure 29, swings downward, a cross pin, *M*, carries the swinging lever, *G*, to the rear. In the first part of the motion of *M* (the slot, *N*, being for a short distance concentric with *K*,*) the block does not descend. It is during this time that *G* withdraws the hammer, *E*, through which it

* See page 3.

passes, sufficiently to allow a spiral spring surrounding the firing pin, *R*, to retract its point sufficiently to clear the front face of the mortise in which works the block.

The continued motion of *G* compresses the mainspring, *L*, until the lower end of *G* engages with the hook on the end of the firing sear *I*. A projection, *O*, on *G*, also engages with the notch, *P*, in rear of the safety sear, *F*. During the closing of the block the lever, *G*, is thus held back by *two* sears: figure 30.

When the block is fully closed *M* strikes the front end of *F* from below, and releases *O* from *P*. By pulling the trigger, *g*, figure 30, *I* is depressed and the gun is fired.

The safety sear is intended to prevent a premature discharge when the trigger is kept drawn back during loading. See page 26.

CHAPTER XXX.

ACCURACY OF FIRE

INCLUDING POINTING; ACCURACY; PROBABILITY AND MANAGEMENT OF FIRE.

I. POINTING.

To point a gun is to give it such direction and elevation that when fired the projectile shall strike a given target. The target may, or may not be seen, and the subject therefore divides itself into *direct*, and *indirect* pointing.

DIRECT POINTING.

Principles.

Remembering that the line of sight is fixed by the position of the target and the eye, and includes the front sight point, the object of the rear sight is to determine the extent to which the breech must be depressed below the line of sight so as to give the angle of elevation required.

The rear sight is then a tangent scale, that may be graduated in ranges or in degrees according as the conditions of loading are, or are not, fixed.

The graduated portion, or *standard*, is parallel to, and generally out of the plane of fire at a distance from the axis determined by the position of the front sight.

Upon the standard moves the *slide;* this carries the rear sight point to which lateral motion in both directions may be given.

It is well to bear in mind that the deviation follows the motion of the rear sight point; thus, if we move it up further than is required, the projectile will strike high; if we move it too far to the right, the projectile will go to the same side of the plane of *sight.*

Horizontal Inclination.

Since the piece is elevated to correct for the action of gravity, the elevation must be taken in a vertical plane; so that if the trunnions be not horizontal the rear sight standard must be capable of being revolved to a vertical position.

It is also necessary that the axis about which the rear sight revolves shall be the *natural line of sight*, or that for which the elevation is 0. For then, since the natural line of sight is parallel to the axis of the piece, the inclination of the trunnions will cause it to describe a cylindrical surface, the inclination of any one of the elements of which is that of the axis. With a rear sight so arranged we may measure the vertical angle included between the natural line of sight and the actual or *artificial line of sight*, which is the angle required. Thus in figure 1, if the trunnions are inclined at the angle φ, the construction shows that if the front and rear sight points are f and r, the plane of sight, $S'S'$, will be inclined to the plane of fire at the angle $p\ f'\ r'$: and therefore, since the direction given to the gun is fixed by the line of sight, the gun will shoot to the left of the target. Also, when the rear sight is used, if h be the height of the slide, the vertical angle of elevation will be that subtended by only $h \cos \varphi$, or the gun will shoot low.

But if the height of the front sight $= f_{,} r_{,}$, or the *dispart*, (Chapter I) the planes of sight and of fire will become

parallel; and if the standard be revolved to a vertical position the adjustment will be complete.

Field Sight.

As a type of this class of sights, that adopted for the 3.2 inch field gun is here described.

The tangent scale, *A*, figure 2 is divided into degrees and ten minute intervals; and, by means of the attached vernier, *V*, it may be read to minutes. The vernier slide, *S*, carrying the *peep sight*, is moved to the desired elevation by the screw, *E*. The tangent scale admits of rotation about *C*, and may be placed in a vertical plane by observing the small spirit level *L*. It may also be moved laterally, by means of the screw, *W*, the scale, *D*, indicating the distance to the right or left of the natural line of sight passing through *C*.

Sights for Heavy Guns.

When the axis of the trunnions is maintained horizontal, as in the Siege and Sea Coast services, the operation of elevating the piece carries the natural line of sight in a plane parallel to the plane of fire, and the device described is not required.

For such pieces the standard may be replaced by graduated arcs and pointers for giving direction in both elevation and azimuth. For the former purpose the arc may be on the breech of the piece or on the cheek; and for the latter it may be concentric with the pintle. The horizontal arc is used for firing at objects not seen from the battery; but the bearing of which with reference to some meridian is known to an observer outside.

Refinements.

Accuracy of pointing may be increased by the use of tele-

scopes or of combined "peep sights" and cross wires. To avoid the delay in pointing resulting from the limited field of view a preliminary adjustment with the ordinary form of open sight is required.

Telescopic Sight.—This consists essentially of a small transit which for heavy cannon is mounted on a shelf fixed to one of the trunnions so that its upper plane shall be parallel to that made by the axis of the bore and that of the trunnions. The telescope is removed before firing. For field guns various models have been tried experimentally; but so far none has been found sufficiently simple for general adoption.*

Peep Sight.—In the 3.2 field rifle the front sight is as shown in figure 3, and the slide of the rear sight as in figure 2. The distance $a a'$ is equal to $b b'$, so that, having found the object by the sights a b, the aim may be corrected by the line passing through the peep hole b' and the cross wires a' contained in the cylindrical housing C.

INDIRECT POINTING.

This term applies to the method followed when the target cannot be seen from the piece. An example occurs in mortar practice.

I. The general practice consists in establishing an auxiliary *mark* (in mortar practice this is the front plummet) towards which the piece is pointed. The elevation is subse-

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* A simple form of telescopic sight for the field gun is now, 1890, under trial.

The vernier slide, S, is extended to the left as shown in figure 2, so as to form one support for a telescope, the other end of which rests on a similar crutch near the right trunnion.

When detached, the telescope may be used for watching the effect of the fire.

quently given by the level and corrected by an observer so situated as to see the effect of the fire.

II. Or else, the proper elevation of the piece having been determined by trial, a distant mark may be selected (or placed) in the plane of sight; the slide of the rear sight may then be moved until the rear sight point is on the prolongation of the line joining the mark and the front sight point. Whenever then, after firing, the new line of sight passes through the mark, the piece will have its primitive direction and elevation, provided that its new position does not differ materially from its primitive position. This method is serviceable in small arm firing.

III. In some cases it may not be possible to find a suitable mark. Under such circumstances the following method may be employed with cannon mounted on permanent platforms. Establish nearly parallel to the axis of the piece, when in its normal position, a vertical plane director. Establish by trial the piece in its proper direction and measure the shortest distance; 1st, from the plane director to the end of the axle arm; and 2nd, from the plane director to some definite point on the trail. When, after firing, these distances are restored, the piece is in its primitive direction.

By establishing the inclination of the plane director with reference to some common meridian, this method enables any piece of a battery to be oriented on the target by a distant observer in the manner indicated page 3.

Tables provided for this purpose give the difference of distance from the plane director for all inclinations to the common meridian likely to occur in practice.

II. ACCURACY OF FIRE.

Definitions.

The plane curve discussed in Chapter XX is called the *normal trajectory*. In practice the trajectory is a curve of

double curvature. The conditions of firing vary so greatly that it is impossible to secure the coincidence of two consecutive trajectories, so that, however constant these conditions may be made, the trajectories of any series of shots group themselves in the *sheaf of trajectories*, a sort of bent cone, the apex of which is at the muzzle and the axis of which is the *mean trajectory*. Figure 34.

Owing to the tendency of the individual trajectories to approach the mean trajectory, the density of the sheaf increases toward its axis, like the stream from a hose. Figure 33 shows approximately the manner in which the sheaf is subdivided: the *nucleus* contains the half lying nearest to the axis; the *envelope* surrounds the nucleus and contains 40 per cent of the whole, and the remaining 10 per cent constitute the *tailings*.

In order to hit the target we must know its position with reference to the mean trajectory, or the *deviation* of the gun at the range in question, and also the distribution of the trajectories throughout the sheaf, or the *errors*. We may accordingly study 1st, the causes of deviation and the corrections required to make the mean trajectory pass through the target; and 2nd, the measurement of the errors of the system comprising the gun, ammunition, carriage, gunner, etc., with a view of estimating the probability of striking a target of known dimensions at a given range.

I. CAUSES OF DEVIATION.

These may be either internal or external, as follows:—

Internal.

These are variations in the conditions of loading, in the temperature of the bore and in meteorological conditions. They affect the Initial Velocity, and for practical purposes

may be neglected, since the variations in velocity may be made less than 1 per cent.

External.

Among the external causes leading to deviation are:—

I. The error of the eye.

II. The drift, due to the rotation of the projectile. See Chapter XX.

III. The wind and weather.

IV. The inclination of the trunnions to the horizon, and the jump.

V. Errors in estimating the distance to the target, and also those due to variations in the angle of sight, or in the difference of level between the gun and the object.

VI. The rotation of the earth. This has no practical importance and may be neglected. In the northern hemisphere it carries the projectile to the right of the object.

These causes and their corrections are discussed as follows:—

I. The Error of the Eye.

The average error of the eye among a number of selected marksmen, determined by the method used in the preliminary instruction in small arms firing, is such that only about one half can be depended on to direct the line of sight invariably upon a 2 foot target at 1000 yards.

This error arises principally from variations in the height at which the line of sight pierces the front sight of the piece. It may be diminished by increasing the sight *radius*, by the use of telescopes and cross hairs, and by allowing for variations in the illumination of the front sight. When the front sight is obscured, too much of it will be taken, and when one side is brighter than the other the deviation will be *from* the light.

The nearer is the eye to the rear sight, the better will the front sight be seen, and the *finer* can the sight be taken.

The permissible error of the eye will depend upon the semi-height of the target and its distance; or, calling e the error, l the *sight radius* of the gun, R the range, and h the height of the target, we have, assuming from the principle of rigidity of the trajectory that it is a straight line, and from the similarity of the triangles in figure 4,

$$e : l :: \frac{h}{2} : R \text{ or } e = \frac{h\,l}{2\,R}.$$

II. Drift.

As the elevation increases, the deviation due to drift increases more rapidly than the range, so that the horizontal projection of the mean trajectory is convex toward the plane of fire.

Drift may be approximately compensated by giving to the rear sight leaf a permanent inclination to the plane of fire in the direction opposed to the drift. The angle of inclination is called the *permanent angle.* It may be shown that

$$\tan i = \frac{D}{R \sin e}.$$

in which i is the permanent angle; D, the deviation due to drift; R, the range, and e the angle of elevation.

Sin e increases more rapidly than the range, so that if D varied as R sin e, i would be constant.

Drift at ordinary ranges may be sufficiently compensated for by the permanent angle, and at long ranges a further correction may be given by the lateral motion of the rear sight slide.

With pointed projectiles the drift is in the direction of the rifling. In the U. S. service this is to the right.

Range tables, giving the ballistic properties of the gun as determined by experiment, furnish the angles of elevation

and the drift for the different ranges as well as the permanent angle.

RANGE TABLE FROM EXPERIMENT.

English 8 in. M. L. R. Howitzer (3)—(W=180 lbs.), w of R. L. G_2.

							5′ change of e or d changes. (5)		50 per cent zones $=2r$.(5)					
w	$I.\ V.$	$X.$	$e.$	drift	d = deflection of rear sight point for drift. (1)	ω.	$X,$	$Y,$ or $Z,$	x(4)	$y.$	$z.$	$v.$	$T.$	Fuze Scale. (2)
lbs.	f. s.	yds.	°′	yds.	°′	°′	yds.	yds.	yds.	yds.	yds.	f.s.	sec.	
11.5	956	1200	2.44	1.9	0.06	4.00	25.0	1.74	16.4	0.40	1.21	876	4.0	
		1600	5.06	3.8	0.09	5.36	23.8	2.32	21.3	0.56	2.14	852	5.4	
		2300	7.33	8.7	0.13	8.42	23.8	3.34	29.7	0.87	4.64	814	7.9	
		2400	7.54	9.5	0.13	9.12	22.7	3.49	30.9	0.92	5.07	809	8.2	
		2500	8.16	10.4	0.14	9.42	22.7	3.63	32.1	0.98	5.50	804	8.6	
10.5	920	1200	4.12	2.3	0.07	4.36	20.8	1.74	15.5	0.45	1.31	845	4.1	
		1600	5.48	4.5	0.10	6.21	20.8	2.32	20.3	0.62	2.31	823	5.6	
		2300	8.36	10.5	0.16	9.51	20.8	3.34	28.3	0.99	4.99	787	8.3	
		2400	9.00	11.5	0.17	10.24	20.8	3.49	29.4	1.05	5.44	782	8.7	
		2500	9.24	12.7	0.18	10.57	20.2	3.63	30.5	1.12	5.90	777	9.1	
3.5	473	1200	16.33	11.4	0.33	17.30	4.6	1.74	21.7	1.50	6.9	438	8.4	
		1600	24.21	22.3	0.48	26.00	3.6	2.32	30.4	2.18	14.9	433	11.8	
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	

III. The Wind.

For any range the deviation of a projectile caused by a wind of which the velocity is one mile per hour, acting at

(1) The rear sight of this piece is not set at the permanent angle.

(2) The actual time of burning varies in flight from that at rest. See Chapter XVIII, page 7.

(3) Range tables for guns assume a constant value of w. They give in addition to the above, the jump, the inclination of ω as $\frac{1}{x}$, and the penetration in wrought iron at the different ranges.

(4) For guns this is much more uniform. See page 52.

(5) See *post.*

right angles to the plane of fire, is called the *coefficient of the deviation* for that range. So that, calling u the velocity of the wind in miles per hour as shown by the anenometer, ψ the inclination of the wind to the plane of fire as shown by the wind vane, and k the coefficient above defined, we have

$$D = k\,u \sin \psi.$$

The relation between k and R may be recorded by an empirical curve, from the indications of which the rear sight may be set to the right or left of its normal position when the force and the direction of the wind are known.

The Weather.

The relative effect of varying meteorological conditions is provided for by the coefficients explained in Chapter XX, pp. 9, 11.

IV. Inclination of the Trunnions.

This has been explained page 2.

V. ESTIMATION OF DISTANCES.*

The importance of this correction varies inversely with the height of the target and the flatness of the trajectory. Chapter XX, Eq. (30).

The distance of the target may be ascertained:—

1st. By the eye.

Constant practice is necessary, even at short ranges, and the results are much affected by varying atmospheric conditions and by the nature of the intervening ground. At long ranges it even requires experience to determine whether a shell bursts short or beyond the target.

2nd. By rapid measurements with different *range finders.*†

* The extent of this sub-head requires a variation in the method of using type. The subject extends to page 22.

† For 3rd see page 21.

RANGE FINDERS OR TELEMETERS.

These may be divided into two general classes, the *acoustic* and the *optical* instruments. The former depend upon the assumed constancy of the velocity of sound, and the latter upon the rapid solution of plane triangles.

1. ACOUSTIC TELEMETERS.

These measure the time elapsing between the flash of an enemy's gun and the arrival of the report. Then, taking the velocity of sound at 1100 f. s. we have $R = vt$. The following methods are in use.

1. The Le Boulengé Telemeter.

This consists of a graduated glass tube filled with some non-freezing transparent liquid, through which, when the tube is vertical, a metallic index falls with a slow *uniform* motion.

To use the instrument, hold it horizontally with the index at 0: the instant that the flash or smoke is seen turn it vertical; when the sound is heard return the tube to a horizontal position and take the reading in meters.

2. A simple method is to count the number, n, of cadenced steps made during this interval. If these are at the rate of 110 per minute, then

$$R \text{ in yards} = n \frac{1100 \times 60}{3 \times 110} = 200\, n.$$

2. OPTICAL TELEMETERS.

General Principles.

If we consider the distant object as a point, its *parallax* is the apparent difference of direction of the object as seen from two different points of view; and conversely, if the dimensions of the object are considerable, its parallax may

be measured by the angle that it subtends from a single point of view.

The triangle formed in the first instance by the object and the two points of view, and in the second case by the point of view and the observed extremities of the object is the triangle to be solved. Its solution is facilitated and sufficient accuracy for first approximations attained by considering it either a right angled triangle or an isoceles triangle, of which, in the first case the distance between the two points of view, and in the second case the known or assumed dimensions of the object forms the base. The base or the point of view should evidently be selected so as to diminish the error of this assumption.

If the triangle to be solved is a right angled triangle, figure 5, we may measure AC by multiplying the length of the base AB by the reciprocal of the sine of the parallax at C when this is found; or, as the parallax is generally small, the reciprocal of the tangent may be used, giving CB practically equal to AC, figure 6.

If the triangle is isoceles the length of a side will be given by $\frac{AB}{2} \times \frac{1}{\sin\frac{C}{2}}$. Some instruments use a fixed parallax and vary the position of the observer at the extremities of the base until the desired triangle is formed. As the parallax is so chosen that $\frac{1}{\sin\frac{C}{2}}$ is a whole number, say 20, it is only necessary to multiply the length of the base by 10 to obtain the range. This can easily be done mentally.

Classification.

Accordingly instruments may be classified as follows:—

I. Fixed base, variable parallax.

II. Fixed parallax, variable base.

III. Variable parallax, variable base.

The same instrument may sometimes be used in more than one method, and the parallax may be either vertical or horizontal.

In order to diminish their size and to permit the simultaneous observation of two distant points from an unsteady support, many instruments employ the principle of the sextant by using two reflecting surfaces, the angle between which is one half of that between the incident and the second reflected ray, figure 7. The relative inclination of the reflecting surfaces m, m' may be fixed, or, as in the sextant, variable.

The conflicting claims of portability, rapidity of operation and accuracy have caused many forms of range finders to be devised. No one kind has yet been generally adopted.

The ingenuity of inventors has been principally exercised in increasing the scale on which the parallax is measured and in avoiding all but the simplest computations in the field.

Vertical Parallax.

These instruments require the vertical base to be known. The base may be, 1, either the height of the object, as the average height of a man, or that of the mast or funnel of an enemy's vessel, or, 2, the height of the instrument above the horizontal plane on which the object is situated, as in coast batteries having considerable command.

1. Stadia. *Figure* 8.

This instrument was formerly used when the ranges of small arms were so short that the parallax for an object the height of a man was of appreciable magnitude.

The button b is held between the teeth, and the Stadia S is held at a distance from the eye regulated by the cord c. The head and feet of the man whose distance is to be measured are brought tangent to the lines oh, ot, and the slide S' brought to the corresponding position along the

scale. The average height of a man in this service is taken at 5 ft. 8 in.

The objections to this particular instrument are evident, but the principle is applicable against vessels. A very convenient surveying instrument of this nature uses fixed parallel horizontal wires inside a telescope, and a graduated staff, from which the distance to the staff may be read off directly.

2. The Depression Range Finder.

This consists of a telescope, the height of which above the water level is known. The distance and direction of the object can be read directly from scales; so that, if the observer be remote from the smoke of the battery, and be screened by natural objects from the enemy's view, he may safely direct the continuous firing of pieces provided with the pointing apparatus described page 3.

The instrument admits of a correction for the stage of tide, etc.

HORIZONTAL PARALLAX.

CLASS I. FIXED BASE; VARIABLE PARALLAX.

1. The Plane Table.

This is used in plotting the effects of fire over water, and the principle is applicable in warfare.

For target practice the base is established so that straight lines drawn from its extremities to the point of impact shall intersect as nearly as possible at right angles. The position of the base with reference to the piece having been established on a chart, the various points of fall can be plotted in their relative positions and the ranges and deviations determined by a scale. In this practice the base may be established parallel to the plane of fire and near the target; but as in war this would not be possible, the equivalent of

the plane table is established so near the piece that the difference of its distance from the object may be neglected. In such cases the length of the base should increase with the range, so as to neutralize errors in observing the parallax; but since in practice a base of suitable length cannot always be obtained, the errors due to the use of a short base may be corrected by refinements in the construction of the instrument, as in the following cases.

2. Berdan Range Finder. *Figure* 9.

In this instrument we find a fixed base only 6 feet long, so that it is contained within the limits of the vehicle on which the whole apparatus is carried. At one end is the fixed telescope B with its line of collimation exactly at right angles to the base $A\ B$. The inclination of the movable telescope A to the axis of B, or the parallax of the object C, can be read off from a scale giving $A\ C$ directly. This instrument is rapid and accurate; but is necessarily confined to the Artillery.

3. The Fiske Range Finder. *Figure* 10.

This is intended to be used on naval vessels in which the length of the base is necessarily restricted. It operates on the same general principle as the Berdan; but permits the solution of oblique angled triangles. In order to magnify the scale it employs the principle of the Wheatstone Bridge.

Let $A\ B$ be the base, at each end of which is a telescope pivoted at A and B, which are the centers of the graduated arcs, $G\ H\ I$, $J\ K\ L$. Let C be the object. Draw $A\ E$ parallel to $B\ C$. Then

$$A\ C = A\ B \frac{\sin A\ B\ C}{\sin C}$$

The angle at B can be easily measured, and the angle

$H A I =$ arc $J K -$ arc $G H$, is determined substantially as follows.

Let C, figure 11, be the target and A and B the metallic pivots of the two telescopes connected with the battery at L, and through the telescopes and their graduated traverse circles $D E F G$, $H I J$, with the galvanometer K. The letters a, b, c, d, refer to the usual nomenclature of the arms of the Wheatstone Bridge.

The angle $A B C$ can be read off directly from the traverse circle $H I J$, and the angle $E A F =$ the parallax may be measured in two ways.

First; Supposing the galvanometer to be under the eye of the observer at A, he has merely to swing his telescope until the galvanometer balances. Then, since $a : b :: c : d$ the change in the angle at A will be the parallax, the range for which, corresponding to an angle $A B C = 90°$, can be read off directly from the traverse circle and then be corrected by multiplying by sin B. This correction is mechanically performed in the act of sighting from A.

Second; Suppose the galvanometer to be under the eye of a distant observer, as at the gun, and let him be provided with means for determining with sufficient exactness the angle $A B C$, or sin B. The observers at the telescopes may then simply follow the object with their instruments and the observer at the gun may determine the range by introducing into the circuit a rheostat graduated for ranges.*

CLASS II. FIXED PARALLAX; VARIABLE BASE.

1. The Weldon Range Finder. *Figure* **7.**

This uses two small, thin, triangular prisms having one side silvered and the angles between two of the reflecting surfaces $= 44° 17'$.

* This description is much abbreviated and omits many interesting details.

Therefore, in figure 6, $A = B = 88° \, 34'$ and tangent $\frac{C}{2} = \frac{1}{40}$. The instrument requires two observers, each of whom holds the prism so as to see C by double reflection, and simultaneously the eye of the other observer by direct vision over the prism.

A single observer may use the instrument by establishing the direction of the base line from A; marking the spot and finding some spot B upon the base line from which C can be seen by double reflection and A by direct vision. Then

$$\frac{A\,B}{2} \times \frac{1}{\frac{1}{40}} = 20 \times A\,B = A\,D.$$

2. The Pratt Range Finder. *Figure* 12.

Each instrument contains two pairs of mirrors, called the upper and the lower pair. The upper pair are inclined at 45° to each other, and the lower pair at an angle of less than 45°. Suppose this angle to be fixed, as in the Weldon at 44° 17′, although it may be set at various other angles.

1. Suppose the distance $B\,C$, figure 13, is to be measured. Standing at B, with the *upper* mirrors and by direct vision establish some distant and well defined mark D, so taken that $C\,B\,D = 90°$, and mark the point B.

Move in the prolongation of $D\,B$ till A is reached, from which point D seen by direct vision coincides with the image of C reflected in the *lower* pair of mirrors. Then

$$A\,B \times 40 = B\,C.$$

2. The operation may be reversed by moving from A to B. In this case, as the observer's back will be turned towards A while changing his station, the direction AD should be preserved by selecting an auxiliary mark d' between D and B.

3. It is evident that the instrument may be used for solving isoceles triangles in the same manner as the Weldon.

4. The instrument may be used to measure the distance between two objects, both of which are inaccessible, as in determining on shore, the distance between a friendly vessel and an enemy's. To illustrate, let BC, figure 14, be the distance to be measured and let the observer be at A. Establish AD at right angles to AB and with the lower pair of mirrors determine the point D so that $AD=\frac{AB}{40}$. The distance AB need not be measured.

Similarly, determine the point E so that $AE=\frac{AC}{40}$.

Then, since ADE and ABC are similar triangles,

$$BC=40\ ED.$$

Remark.

Instruments of Class II are simple, portable and fairly accurate; but in broken or wooded country time may be lost in finding a base suited to the parallax.

CLASS III. VARIABLE BASE; VARIABLE PARALLAX.

1. Gautier's Telemeter.

This is one of the earliest and best forms of this class. Its essential principles may be illustrated as follows.

Suppose we have a form of protractor such as the dividers shown in figure 15, and provided with sight points at a, b, c. Let it be required to find the distance AC, figure 16.

Select some distant point D, such that $CBD=cab=90°$. Then advance to A on the line BD and sight along ab at C; ac will then point to D'. If now we move ac so that while ab still points to C, ac will point to D, the change cac' will be the parallax and $AC=AB\times\frac{1}{\sin cac'}$.

In practice $c\,a\,b = A\,B\,C$ may vary 8° from 90° without causing the resulting error in range to exceed $\frac{A\,C}{100}$. This permits slight modifications in the length of the base to allow for the nature of the ground.

Construction.

The instrument resembles in appearance a telescope, the object glass being replaced by a glass prism, the angle of which is 6°. This is set in a graduated ring that may be turned axially about the line of collimation of the eye piece *E*.

The prism causes rays entering from the front to be deviated by an amount $I\,T\,I'$, equal to half the angle of the prism, or 3°. So that by turning the graduated ring the refracted ray will describe a cone, the angle of which is 6°. The ring is graduated for the reciprocal of the sines of the intermediate angles corresponding to whole numbers varying by 10, 20, etc.

Operation. *Figure* 16.

The variable inclination of the mirrors permits the points *B* and *D D* to be selected according to the ground. Having caused the image of *C* in the mirror *M* to come just below some well defined point of *D* seen through the prism when set at "Infinity" or $\frac{1}{\sin 0}$, we may then advance along *B D* to *A* and restore coincidence by turning the graduated ring, the reading of which will give the multiplier for the base.

Or else, we may set the ring to some simple factor as 20, and advance until coincidence is made.

Like the other instruments, the Gautier admits of a variety of applications.

2. The Gordon Range Finder. *Figure* 18.

This resembles in principle the Gautier. It consists of an opera glass in front of which is mounted a frame containing two plane mirrors, $M\,M'$, M lying just below the line of collimation.

M may be turned by a thumb screw to an extent measured by an enlarged scale, the 0 of graduation of which corresponds to an inclination of the mirrors of 45°.

Having established a right angle at B, figure 16, we restore coincidence at A, and pass from the reading to a table of factors. The telescope aids in defining distant objects and is useful in watching the effects of the fire, by which after all the elevation is mainly regulated.

3. The Nolan Range Finder. *Figure* 19.

This consists of a pair of instruments, each of which comprises a tripod upon which are mounted two telescopes of different lengths and powers and placed one above the other with their lines of collimation set at about 90°. This angle may be varied and the difference from 90° read from a decimal scale.

Having established a base that will form with the object nearly an isoceles triangle, we direct the short telescopes, s and s', on each other, and the long telescopes, l, l', on the *same point* of C. The sum of the displacements of A and B is the parallax, and considering the triangle isoceles,

$$\log\left(A\,C = B\,C = \frac{A\,B}{2} \times \frac{1}{\sin\frac{c}{2}}\right) = \log\frac{A\,B}{2} + \operatorname{colog}\sin\tfrac{c}{2}.$$

To avoid calculation by ordinary methods a *reckoning cylinder*, consisting of a series of concentric discs, is used. This applies the principle of the *slide rule;* since of two successive rings, one is graduated in terms of $\log\frac{A\,B}{2}$, and

the other in terms of colog $\sin\frac{C}{2}$. By setting the 0 of the scale of sines to the reading of the scale of bases, if we take the point on the scale of bases corresponding to the reading of the scale of sines (or for simplicity, to the sum of the readings on the two decimal scales) we have the range at once.*

To avoid having the tripods to transport, the instruments may be mounted on the cheeks of the flank pieces of a battery.

Returning to the Estimation of Distances page 10, we have as the next method—

3rd.† By trial with smaller pieces, using percussion shell, as described Chapter XVIII, page 12. The sights for these guns should be graduated in yards.

4th. By establishing a *fork* by changing the elevation until about one half of the projectiles fall short. There are two methods of doing this.

1st. The *progressive method*, or feeling the way from the first range by ± increments until the target is enclosed in the fork.

2nd. The method of *successive means*. Having established a large fork by throwing one shot short and one beyond the target, we take the mean of the two ranges for the next range; and, according as this is beyond or short of the target, take the mean of it and that one of the first two that was short or beyond,

* The simplicity of this principle will become apparent by taking two scales of equal parts held in the same plane and sliding one along the other.

† This article follows 2nd, page 10.

and so on, continually narrowing the fork. This method is preferred when time permits its employment.

In some services the number of turns of the elevating screw required to change the range by a nearly constant increment at the different ranges is known. This number varies less rapidly than the range.

Variations in the Angle of Sight.*

As seen in Chapter XX, page 24, for the same elevation the range varies somewhat with the angle of sight. The correction for this with the flat trajectories now common may be included with that for deviations in range.

II. ERRORS.

CAUSES OF ERROR.

It is usual to consider in gunnery practice that there are two main causes of error, one tending to increase or diminish the range, or, what is the same thing, tending to raise or lower the trajectory, and producing range errors; and the other tending to move the trajectory to the right or left, producing lateral errors.

Lateral errors evidently follow the same laws as range errors.

MEASUREMENT OF ERRORS.

The errors of a piece † at a given range are determined by firing the piece under as constant conditions as possible.

For small arms the gun is clamped in a fixed rest, and the shots received on an iron target, the image of which is

* See page 7.

† This will be considered to include the errors due to the carriage, ammunition, and to other causes of deviation that cannot be controlled.

projected by a camera on a sheet of paper ruled to scale. The target may then be computed as hereafter described.

The resulting accuracy is compared with that of alternate targets made with a standard arm, and with ammunition carefully prepared in such large quantities that its uniformity from day to day may be considered as constant.

We may thus eliminate all variables but the particular one under consideration.

For cannon, however, for which it is often impracticable to procure vertical targets of sufficient size, the firing is often conducted over water and its results plotted with a plane table.

COMPUTATION.

Definitions.

Let n be the number of shots fired. Let $X\ Y\ Z$ be the axes of coördinates, drawn through the point aimed at; X being taken parallel to the plane of fire, Y at right angles thereto, and Z vertical. Let x', x''; y', y''; z', z''; etc., be the coördinates of the shot marks taken with their proper signs, and r', r'' their radial distances from the origin. Let T be the distance to the target. Then we have

Mean deviation in range. $X_{\prime} = \dfrac{\pm x' \pm x'' \pm \text{etc.}}{n}$.

Mean lateral deviation $Y_{\prime} = \dfrac{\pm y' \pm y'' \pm \text{etc.}}{n}$.

Mean vertical deviation $Z_{\prime} = \dfrac{\pm z' \pm z'' \pm \text{etc.}}{n}$

Mean range $R = T \pm X_{\prime}$

Mean radial deviation or "*string*," $S = \dfrac{r' + r'' + r''' + \text{etc.}}{n}$

Mean point of impact, or center of impact. This is the point of which the coördinates are $(X_{\prime}\ Y_{\prime})$ or $(Y_{\prime}\ Z_{\prime})$. This point determines the *mean trajectory*, *mean line of fire* and

mean plane of fire. It is the center of the group of shot marks, and the origin for the measurement of *errors*.*

Range error for a single shot $v_1 = X_, \sim x'$. †
Lateral error for a single shot $v_2 = Y_, \sim y'$.
Vertical error for a single shot $v_3 = Z_, \sim z'$.

Mean error in range. $$\varepsilon_1 = \frac{v_1' + v_1'' + \text{etc.}}{n}.$$

Mean lateral error. $$\varepsilon_2 = \frac{v_2' + v_2'' + \text{etc.}}{n}.$$

Mean vertical error. $$\varepsilon_3 = \frac{v_3' + v_3'' + \text{etc.}}{n}.$$

Mean absolute error. $$\varepsilon_0 = \frac{r_{,} + r_{,,} + r_{,,,} + \text{etc.}}{n}.$$

In which $r_{,}$, $r_{,,}$, etc., are measured radially from the *mean point of impact*, so that

$$r_{,} = \sqrt{v_1^2 + v_2^2} \text{ or } = \sqrt{v_2^2 + v_3^2}$$

according to the position of the target.

Of these mean errors the first three are used to analyze the causes of error. Thus if ε_1 or ε_3 should be found to greatly exceed ε_2 it would be assumed that either the powder, or the weight of the projectile had varied, or that there had been an irregular wind in the direction of the plane of fire. On the other hand, if ε_2 should greatly exceed ε_1 or ε_3, it

* While deviations are measured from the point aimed at, errors are measured from the mean point of impact.

† The sign, $\sim$, means that the coördinate distance between the mean point of impact and the shot mark is taken. The confusion arising when these lie on opposite sides of an axis may be avoided by selecting, instead of the target, an arbitrary origin outside the group; so that all deviations shall be +.

would be likely that there had been an irregular wind across the plane of fire.

The mean absolute error is principally employed to measure the accuracy of fire arms when tested under the conditions given, page 23.

Vertical Projections.

In firing over water, z', z'' are unknown or not observed. Therefore, in order to determine where the projectile would have passed through a vertical target in the plane YZ, it is necessary to assume that the angle of fall, ω, is constant for the particular range under consideration. Therefore if, as in Chapter XX, we assume that the trajectory for the small distance involved is a straight line, we may write

$$z = x \times \tan \omega, \text{ etc.}$$

Similarly if we assume that the mean trajectory passes through the mean point of impact in both the vertical and the horizontal planes, we may write $\varepsilon_3 = \varepsilon_1 \tan \omega$.

The following system is evidently applicable to computing the results of firing against either horizontal or vertical planes.

EXAMPLE.

Suppose we fire 10 shots over water with the sight fixed at the estimated distance of the target, or 1000 yards. The first shot falls 50 yards beyond and 10 yards to the left, and so on as shown in the following table. Shots falling beyond the origin and those going to the right of it are estimated positively. Suppose that we obtain the following record.

COMPUTATION.

No. of Fire.	DEVIATIONS. In Range. +	DEVIATIONS. In Range. −	DEVIATIONS. Lateral. +	DEVIATIONS. Lateral. −	ERRORS. In Range. +	ERRORS. In Range. −	ERRORS. Lateral. +	ERRORS. Lateral. −	SQUARES OF ERRORS. In Range. v_1^2	SQUARES OF ERRORS. Lateral. v_2^2	r, etc. $= (v_1^2+v_2^2)^{\frac{1}{2}}$
1.	50			10	25			8	625	64	26
2.		30	5			55	7		3025	49	55
3.	20		15			5	17		25	289	18
4.	100			6	75			4	5625	16	75
5.		70		4		95		2	9025	4	95
6.	150			20	125			18	15625	324	126
7.		60	—	—		85	2		7225	4	85
8.	—	—	10			25	12		625	144	28
9.	90			10	65			8	4225	64	66
10.	—	—	—	—	—	25	2		625	4	25
Total	410 —160 10)250 +25.0	160	30	50 —30 10)20 —2.0	290 +290 10)580 ε_1=58.0	290	40 +40 10)80 ε_2=8.0	40	Σv_1^2=46650	Σv_2^2=962	10)600 ε_0=60.0

Mean deviation in range= + 25.0 yds.
Mean lateral deviation = —2.0 yds.

Mean error in range = 58.0 yds.
Mean lateral error = =8.0 yds.

From the above computation and figure 20 it appears that the mean range is 1025 yards, so that if the elevation had been taken for 975 yards the target would have been more probably hit.

The shaded rectangle of figure 20 represents the plan of a vessel: it applies to the subject of Probability of Fire yet to be discussed.

Note.—When the mean range is correctly found the + errors are equal to the — errors of the same kind. This tests the accuracy of the work. It is not necessary however that fractional parts of yards should enter into the value of the mean range.

Swiss Method.

When the number of shot marks is very great, as in volley firing with small arms, it may be very difficult to measure all the individual shot marks without error. The following method may then be employed. See figure 20′.

Count the total number of shot marks $= n$. Then holding a straight edge horizontally slide it over the face of the target until $\frac{n}{4}$ marks lie above it; indicate this by a horizontal line. Draw similar lines for $\frac{2n}{4}$ and $\frac{3n}{4}$, and repeat the process with the straight edge vertical.

We may thus approximate closely to the measurement of all the elements of the computation; but we must assume that all the shots strike the target.

III. PROBABILITY.

By the probability of hitting a target *under certain circumstances* is meant the ratio of the number of times that the target would be hit to the whole number of shots fired, supposing the number of shots fired under these circumstances to be infinite.

Here unity is the mathematical symbol for *certainty*, so that a probability of $\frac{1}{4}$ or 25 per cent, signifies that one hit may reasonably be expected from four independent shots fired under the same circumstances.

If the same ratio held with a finite number of shots we could calculate with certainty beforehand the number of shots necessary to yield a certain number of hits. But in practice the ratio is not certain, but only approximate ; the approximation increasing with the number of shots upon which the calculations are based.

Probability Curve.

Suppose we take as an origin a point at a distance from the gun equal to the mean range, and lay off the errors to the right and left of this point according as they are positive or negative. If then, corresponding to each error in range as an abscissa we draw an ordinate of a length proportionate to the probability of that error, the coördinates so obtained will be those of the points of a curve known as *Probability Curve*, figure 21.

The general form of this curve will appear from considering;

1. That small errors are more probable than large ones, and therefore that the ordinates will be greatest for the smallest errors.*

2. That postive and negative errors are equally probable, and therefore that the probability curve will be symmetrical about the axis of S.

3. That large errors are not practically to be expected; since such errors would come under the head of avoidable mistakes.

4. That the curve will have the axis of X as an asymptote; since, theoretically, the only error that can never be committed is one that is infinitely great.

The equation of the probability curve is

$$s = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2}, \qquad (1)$$

in which s is the probability of an error of the magnitude x; e is the base of the Napierian system of logarithms, and h is the *measure of precision*, the value of which is found to be

$$h = \sqrt{\frac{n-1}{2 \sum v^2}}, \qquad (2)$$

* See remark as to the density of the sheaf of trajectories, page 6.

in which n is the number of shots fired, and Σv^2 is the sum of the squares of the errors in the direction in which these are estimated.

By multiplying both members of Eq. (1) by dx and integrating between limits $x_{\prime} = OM$ and $x_n = ON$ we may obtain the probability of committing an error in range between the two errors $x_{\prime}$ and x_n, or

$$P = \frac{h}{\sqrt{\pi}} \int_{x_{\prime}}^{x_n} e^{-h^2x^2}\, dx = \text{area } RMNL. \qquad (3)$$

The total area between the curve and the axis of X is unity, so that, for a particular curve in which range errors only are considered, if the area $RMNL = \frac{1}{10}$, the meaning of the case would be, that when the center of the group of shot marks was at O, a target, the length of which measured in the plane of fire is MN, and the width of which at right angles to the plane of fire is infinite, and which is situated at the distance OM from the center of the group could reasonably be expected to be struck by one tenth the number of shots fired at it; the certainty of prevision increasing with the accuracy with which h is known, or with n, Eq. (2).

It is evident that the above discussion applies as well to lateral errors as to errors in range, and that for each gun and range an infinite number of probability curves exists, depending upon the directions assumed for the axes. Of these we shall consider but two, but it is well to remember that the others together form the *surface of probability*,†

Note.—For the deduction of these equations and for the fuller treatment of the subjects herein discussed see the work on "The Accuracy and Probability of Fire" by Ensign J. H. Glennon, U. S. N. The subject is treated in other works on Probability, and Least Squares.

† Consider an infinite number of shots fired against a target. Imagine that after firing all the projectiles are arranged upon the points on

figure 22, the volume between which and that portion of the plane of reference bounded by any plane figure is proportional to the probability of striking an object of the dimensions and position of the figure at the range and under the circumstances in question.

Since, from the symmetry of the probability curve the integral between the limits $-x$ and $+x$ Eq. (3) is twice the integral from $-x$ to O, or from O to $+x$, we have

$$P = \frac{h}{\sqrt{\pi}} \int_{-x}^{+x} e^{-h^2 x^2}\, dx = \frac{2h}{\sqrt{\pi}} \int_0^x e^{-h^2 x^2}\, dx. \quad (4)$$

as the probability that a $\pm$ error in range taken without regard to its sign is numerically less than x.

Taking hx as one variable, the values of P corresponding to its different numerical values have been calculated and are arranged in the following Table I.

Example.—Suppose it be desired to compute the chance of hitting the deck of a ship 300 feet long and 36 feet wide (100 yds by 12 yds.), the keel of the ship being in the plane of fire and the center of the ship being at the mean point of impact. The circumstances being those given in the preceding example.

We have $h_1 = \sqrt{\frac{n-1}{2 \Sigma v_1^2}} = \sqrt{\frac{9}{93300}} = \frac{1}{102} = 0.0098.$

Therefore, supposing for the present that the ship is of indefinite width or that we are firing at a belt or zone at right angles to the plane of fire and limited by parallel lines at 50 yards on each side of the mean point of impact;

---

which they struck, being superposed one upon the other upon the elementary surfaces upon which they fell. The mass of projectiles will thus form a volume bounded by the *surface of probability* and the target.

TABLE I.

PROBABILITY OF ERRORS.

$$P = \frac{2}{\sqrt{\pi}} \int_0^{hx} e^{-h^2 x^2} \, d\,(hx).$$

hx	*P*	*hx*	*P*	*hx*	*P*	*hx*	*P*	*hx*	*P*
0.00	0.00000	0.40	0.42839	0.80	0.74210	1.20	0.91031	1.60	0.97635
.02	.02256	.42	.44747	.82	.75381	1.22	.91553	1.62	.97804
.04	.04511	.44	.46622	.84	.76514	1.24	.92050	1.64	.97962
.06	.06762	.46	.48465	.86	.77610	1.26	.92523	1.66	.98110
.08	.09008	.48	.50275	.88	.78669	1.28	.92973	1.68	.98249
.10	.11246	.50	.52050	.90	.79691	1.30	.93401	1.70	.98379
.12	.13476	.52	.53790	.92	.80677	1.32	.93806	1.72	.98500
.14	.15695	.54	.55494	.94	.81627	1.34	.94191	1.74	.98613
.16	.17901	.56	.57161	.96	.82542	1.36	.94556	1.76	.98719
.18	.20093	.58	.58792	.98	.83423	1.38	.94902	1.78	.98817
.20	.22270	.60	.60386	1.00	.84270	1.40	.95228	1.80	.98909
.22	.24429	.62	.61941	1.02	.85084	1.42	.95537	1.82	.98994
.24	.26570	.64	.63458	1.04	.85865	1.44	.95830	1.84	.99073
.26	.28690	.66	.64938	1.06	.86614	1.46	.96105	1.86	.99147
.28	.30788	.68	.66378	1.08	.87333	1.48	.96365	1.88	.99216
.30	.32863	.70	.67780	1.10	.88020	1.50	.96610	1.90	.99279
.32	.34912	.72	.69143	1.12	.88679	1.52	.96841	1.92	.99338
.34	.36936	.74	.70468	1.14	.89308	1.54	.97058	1.94	.99392
.36	.38933	.76	.71754	1.16	.89910	1.56	.97263	1.96	.99443
.38	.40901	.78	.73001	1.18	.90484	1.58	.97455	1.98	.99489
								2.0	.99532
								3.0	.99998
								∞	1.00000

the permissible error is $\pm$ 50 yds $= x \therefore hx = 0.49$ and p_1* from the table is about 0.51.

Similarly for the lateral precision;—

$$h_2 = \sqrt{\frac{9}{1924}} = 0.0684 \text{ and } h_2 x = 0.41,$$

and p_2 from the Table is about 0.43.

That is, that about 43 per cent of the shots will fall within a belt 6 yds. wide on each side of the plane of fire.

Now, from the theory of probabilities, the probability of the concurrence of two events is the product of the probability of each of the two events when considered separately; the probability that a shot will fall within both belts is

$$p_0 = 0.51 \times 0.43 = 0.22$$

OTHER MEASURES OF INACCURACY.

Probable Error.

In case that hx is so chosen that P in Eq. (4) is equal to $\frac{1}{2}$, the probability of committing an error numerically less than x is equal to that of committing an error numerically greater than x. Such a value of x is called the *probable error*, or r. By interpolating in Table I it is found that for $P = \frac{1}{2}$, $hx = hr = 0.4769$, or

$$r = \frac{0.4769}{h} = 0.4769 \sqrt{\frac{2 \Sigma v_2}{n-1}} = 0.6745 \sqrt{\frac{\Sigma v^2}{-1}} \cdot$$

The term probable error is a misnomer. Its true meaning may be illustrated by reference to figure 21 in which if $x = r = OM$ be so taken that $OSMR = \frac{1}{4}$ then, from the symmetry of the curve $2\,OSMR = \frac{1}{2}$; or $2r$ will be the width of the space, measured equally in both directions from the mean point of impact, within which the shots are

* The symbols $p_1\, p_2\, p_3$ correspond to the nomenclature page 24.

as likely to fall as not, or within which there is an even chance of striking. The accuracy varies inversely with the magnitude of this error.

True Mean Error.

The probable error must not be confused with the mean error already found. In the example the probable error in range is $r = 48.5$ yds. and the mean error is 58 yds. The latter value is only an approximation to the *true mean error*,* for which the symbol is $\bar{x}$. From the equation of the probability curve this has been found

$$\bar{x} = \frac{1}{h\sqrt{\pi}},$$

or in the example, $\bar{x} = 57$ yards: very nearly the mean error.

By combining the expressions for $\bar{x}$ and r we find the ratio

$$\frac{r}{\bar{x}} = 0.8453. \qquad (5)$$

Probable Zones and Rectangles.

From the foregoing example we see that the true mean error does not differ materially from the computed mean error obtained with a fair number of shots.

Also, that since the positive and negative errors are equal, if r be the probable error in either direction from the mean point of impact, $2r = 1.69\ \bar{x}$ will be the width of a zone measured at right angles to the direction of the error that will contain half the number of shot marks.

This measure is much used in the British service in which the mean error in range $\times$ 1.69 gives very nearly

* In the mathematical course this is called the mean error.

the width (in the plane of fire) of the *length zone*, figure 23. See page 38.

Similarly, the mean lateral error $\times 1.69$ gives the width (at right angles to the plane of fire) of the *breadth zone*, figure 24.

The mean vertical error $\times 1.69 =$ the mean error in range $\times \tan \omega \times 1.69$, gives the width (vertically) of the *height zone.*

Referring to the horizontal plane, the intersection of the length and breadth zones gives a rectangle probably containing $\frac{1}{2}$ of $\frac{1}{2}$, or $\frac{1}{4}$ of the whole number of shots. See pages 27 and 32. This is called the 25 *per cent rectangle.*

The *Probable Rectangle* is the 50 per cent rectangle, or is the rectangle of which (the center being at the mean point of impact, and the sides being parallel to the directions in which the errors are measured) the sides are respectively proportional to the probable errors in the same directions, and the probability of hitting within it in *either* direction is

$$P = \sqrt{\frac{1}{2}} = 0.7071.$$

By reference to Table I, we find that for $P = 0.7071$, $hx = 0.7438$.

By substituting the value of h (Eq. 2) we have for the sides of the probable rectangle —

$$\text{length} = 2\,x_1 = 2.104\sqrt{\frac{\Sigma\,v_1^2}{n-1}}, \tag{6}$$

$$\text{breadth} = 2\,x_2 = 2.104\sqrt{\frac{\Sigma\,v_2^2}{n-1}} \tag{7}$$

Remark.

The probable rectangle is seen to be useful in comparing the accuracy of two guns by comparing the magnitudes of the errors which they will make with equal facility.

Its determination is based on the Theory of Probabilities, but is only indirectly connected with the probability of hitting at a single shot a target the dimensions and position of which are known.

Accuracy of a Gun.

The probable rectangle in the horizontal plane will vary in dimensions for different guns and different mean ranges, and its size will be an inverse measure of the accuracy of the gun for the mean range for which it is calculated.

Unless the angle of fall is the same for the different guns in question, the relative accuracy as determined by the size of the probable rectangle in the vertical plane will be very different from that determined in the horizontal plane. A gun having a very flat trajectory is placed at a disadvantage when its accuracy is measured by the size of its probable rectangle in the horizontal plane; and, in general, the more nearly the plane of the target coincides with direction of the fall of the projectiles, the greater is this disadvantage. On the other hand, guns are placed on the same footing, and each shows best its accuracy when the target plane for each gun is chosen perpendicular to the trajectory at the mean point of impact. Against vulnerable horizontal targets, as the decks of ships, better results may therefore be expected with rifled mortars than with high powered guns; unless these latter are mounted in a commanding position, or the muzzle velocity is so reduced, as to give a large angle of fall on the horizontal plane in question.

Probability of Hitting an Object of any Form.

Table I may be used for finding the probability of hitting a target. As, however, the probable errors, laterally and in range, would usually be given, a more convenient table

is one in which the argument is $\frac{x}{r}$. This is readily formed from Table I by remembering the relation between h and r, or $\frac{1}{r} = \frac{h}{0.4769}$ whence,

$$\frac{x}{r} = \frac{hx}{0.4769}, \tag{8}$$

Table II, known as Chauvenet's table, is to be used after the manner of a table of logarithms. Thus for a value of $\frac{x}{r} = 0.40$ the corresponding probability is 0.21: Conversely, a probability of 0.535, or $53\frac{1}{2}$ per cent, corresponds to a value of $\frac{x}{r} = 1.08$.

In figure 25 is plotted the curve expressing the relation between $\frac{x}{r}$ and P.

TABLE II.

PROBABILITY OF ERRORS.

$$P = \frac{2}{\sqrt{\pi}} \int_0^t e^{-t^2} dt, \quad t = hx = 0.4769 \frac{x}{r}.$$

P.	0	1	2	3	4	5	6	7	8	9
0.0	0	.02	.04	.06	.07	.09	.11	.13	.15	.17
.1	.18	.20	.22	.24	.26	.28	.30	.32	.34	.36
.2	.38	.40	.41	.43	.45	.47	.49	.51	.53	.55
.3	.57	.59	.61	.63	.65	.67	.70	.72	.74	.76
.4	.78	.80	.82	.84	.86	.89	.91	.93	.95	.98
.5	1.00	1.02	1.04	1.07	1.09	1.12	1.14	1.17	1.19	1.22
.6	1.25	1.27	1.30	1.33	1.36	1.39	1.42	1.45	1.48	1.51
.7	1.54	1.57	1.60	1.64	1.67	1.71	1.74	1.78	1.82	1.86
.8	1.90	1.94	1.98	2.03	2.08	2.13	2.18	2.24	2.30	2.37
.9	2.44	2.52	2.60	2.69	2.78	2.91	3.04	3.22	3.45	3.82

RECTANGULAR OBJECT.

In figure 26 suppose O is the mean point of impact of a number of shots in the horizontal plane, OX the direction in which range errors are measured, and OY perpendicular to OX, the direction in which lateral errors are measured. Required the probability of hitting the rectangle $ABCD$, of which O is the center, AB and BC being parallel to OX and OY.

Let r_1 and r_2 be the probable errors of the gun in range and laterally under the circumstances.

The probability of committing an error in range less numerically than OE is found in the table for

$$\frac{x}{r} = \frac{OE}{r_1}.$$

The probability of committing a lateral error less numerically than OF is found likewise in the same table for $\dfrac{OF}{r_2}$. The probability of the concurrence of these two events is the probability of hitting inside the rectangle $ABCD$, which is therefore the product of the two probabilities found.

The probability of hitting inside the rectangle $OFBE$ is $\frac{1}{4}$ the probability of hitting inside the rectangle $ABCD$; since numerically equal positive and negative errors are equally probable, and the probability of hitting $ABCD$ is the sum of the probabilities of hitting the four smaller rectangles into which it is divided.

The probability of hitting inside the rectangles $OFMN$ and $OLRE$ is found in the same way as that of hitting inside $OFBE$.

The probability of hitting inside $NMBE$ is that of hitting inside $OFBE$ minus that of hitting inside $OFMN$. Similarly for $LFBR$, $LFMK$ and $NKRE$.

The probability of hitting $KMBR$ is that of hitting $NMBE$ minus that of hitting $NKRE$.

ANY PLANE FIGURE.

Any plane figure may be divided (approximately) into small rectangles and the probability of hitting each rectangle found. The sum of the separate probabilities may then be found, and this will be the probability of hitting the figure.

In what has preceded we have used horizontal targets. There is nothing in the method, however, that will not apply equally well to vertical targets if we substitute vertical for range errors.

EXAMPLES IN THE PROBABILITY OF FIRE.

Although the value of the probable error may be ascertained, the computation may be abbreviated by assuming that the true mean error, $\bar{x}$, does not differ materially from the mean error ε, determined as shown on page 24. So that in Table II the ratio $\frac{x}{r}$ may be taken as equal to

$$\frac{2x}{2r} = \frac{2x}{2 \times 0.845\,x} = \frac{2x}{1.69\,\varepsilon},$$

from which the corresponding probability may be found.

The same results are obtained by the English method of using the 50 per cent zones, as follows:

This considers that the width of the 50 per cent zone is unity, see Table II, and therefore, (the probable error of the gun being constant for the same range and circumstances) that the width of other zones containing a per cent of shots greater or less than 50 per cent is proportional to the cor-

responding value of $\frac{2x}{2r}$; $2r$ representing the width of the 50 per cent zone. And conversely, that when the ratio $\frac{x}{r}$ is known, the probability of striking within the given zone is that given by the corresponding value of P in Table II.

Data.

For a given gun under given conditions we have—

$R = 4945$ yds.; $\omega = 7° \ 25'$: $\varepsilon_1 = 19.4$ yds.; $\varepsilon_2 = 1.8$ yds.;

$\varepsilon_3 = 19.4 \times \tan 7° \ 25' = 2.5$ yds.

Suppose that unless otherwise stated the mean point of impact is at the center of the target.

1. Determine the probability of striking a raft 12 yds. square at the range R.

We have $\frac{2x_1}{2r_1} = \frac{12}{19.4 \times 1.69} = 0.366 \therefore p_1 = 0.19$, and

$\frac{2x_2}{2r_2} = \frac{12}{1.8 \times 1.69} = 4 \therefore p_2 = 1$, or practical certainty; and about 1 out of 5 shots may be expected to strike the raft.

2. Suppose the target to be of the same dimensions and vertical: we have $p_2 = 1$; $p_3 = 0.945 = p_0$. See page 35.

3. If a zone of a certain width catches 20 per cent of the shots fired, how much wider must another zone be to catch 80 per cent?

From Table II we have for $p = 20$ per cent, $\frac{x}{r} = 0.38$ and for $p = 80$ per cent, $\frac{x'}{r} = 1.90$, and since r is constant

$$\frac{x'}{r} : \frac{x}{r} :: 1.90 : 0.38 \quad \text{or} \quad x' = 5x.$$

4. Suppose that the mean point of impact is at the middle of the lower edge of a target 8 feet square. (The case corresponds to firing at the water line of a vessel.)

We suppose the target to be extended downward by an amount equal to its height, and take for p_3 half the probability of striking the whole target: we have

$$p_2 = 0.45; \; p_3 = \frac{0.60}{2} \quad \therefore \quad p_0 = 0.135.$$

5. What would be the effect of raising the mean point of impact 2 feet on the above target?

Suppose that target to be extended downward 4 feet, as in figure 27. Then the target may be supposed to be composed of two portions a and b, the former of which is half the height of a target 12 feet high, and the other portion half the height of a target 4 feet high.

We find for a $\quad p_3' = \frac{0.48}{2} = 0.24$

and for b $\quad p_3' = \frac{0.17}{2} = 0.085$

$\therefore \quad p_3 = \quad 0.325$

and $p_0 = 0.146.$

6. Suppose, as in figure 28, that there are two targets 8 feet square and 8 feet apart fired at from the above gun and at the above range. Which plan would give the greater number of hits in the two targets collectively:

1st. To aim at the center (a) of one?

2nd. To aim at the center (b) of the space between the two?

We will suppose the mean point of impact will fall on the point aimed at, and, as the mean vertical error will be constant, we may neglect it in making the comparison desired.

First Case.

For the target aimed at we have $p_2' = 0.45$.

Now, consider the target $(4+8+8) \times 2 = 40$ feet wide, and take one half the resulting value of p_2 or $p_2'' = 0.50$.

Similarly consider the target $(4+8) \times 2 = 24$ feet wide and find $p_2''' = 0.47$.

Then, $p_2'' - p_2''' = 0.03$ is the probability of hitting the target not aimed at, and $p_0 = p_2' + (p_2'' - p_2''') = 0.48$ will be the probability for both targets.

Second Case.

Considering the target 24 feet wide $\quad p_2' = 0.947$
and for the middle space $\quad p_2'' = 0.45$

$$\therefore \quad p_0 = \overline{0.497}.$$

RIGHT LINE METHOD.

The abscissa of the center of gravity of the area included between the axes OX and OS and the probability curve $A\ B\ C$, figure 29, is evidently the *true mean error*, $\bar{x}$, or the arithmetical mean of an infinite number of errors.

If we draw $D\ E$ so that the centers of gravity of $D\ O\ E$ and $O\ A\ B\ C \infty\ O$ shall coïncide, and call $O\ E = m$, then $D\ O\ E = O\ A\ B\ C \infty O = \frac{1}{2}$, and $D\ O = \frac{1}{m}$.

From the properties of the triangle the abscissa of the center of gravity of $D\ O\ E = \frac{m}{3} = \varepsilon$: and if the probability curve were considered to be a right line we would have $\bar{x} = \varepsilon$. Given then the mean error, ε, the line $D\ E$ would be determined.

In case the right line method were strictly accurate,

there would be no possibility of committing an error greater them m, which would therefore be an extreme error.

Let us see from the probability curve what would be the probability of committing an error less than m. We have, assuming $\varepsilon = \bar{x}$. See page 33.

$$m = 3 \times \text{mean error} = \frac{3}{h\sqrt{\pi}},$$

whence $$hm = \frac{3}{\sqrt{\pi}} = 1.6925.$$

Referring to Table I, we find the corresponding probability 0.983; that is, 98 per cent of the shots will make an error less than three times the mean error: a close approximation.

Also making $x = 0$, in Equation (1.)

$$s = h\,\pi^{-\frac{1}{2}}\,e^{-h^2x^2}$$

we have the value of the maximum ordinate of the probability curve. It is

$$s_0 = \frac{h}{\sqrt{\pi}} = \frac{h\sqrt{\pi}}{\pi} = \frac{1}{\pi \times \textit{mean error}}.$$

Similarly the corresponding ordinate in the case of the right line is

$$s_0' = \frac{1}{m} = \frac{1}{3 \times \textit{mean error}},$$

or slightly greater than the probability curve.

These differences are not material. The approximation in other cases is shown by figure 29; the probability of committing an error between O and $+ x$ $(= OF)$ being, according to the probability curve, the area $OABF$, and according to the right line the area $ODGF$. Experience also shows that, in firing projectiles, the extreme error is a little more than three times the mean error, but this is not of great importance; all that is necessary is that the proba-

bility of exceeding this error should be small enough to be neglected, and we cannot have any doubt in this respect.

Equation of the Right Line.

In the two triangles ODE and FGE, tan $DEO = \frac{GF}{FE} = \frac{DO}{OE}$ or substituting for GF, FE, DO and OE their values, s, $m - x$, $\frac{1}{m}$ and m, respectively, we have

$$\frac{s}{m-x} = \frac{1}{m^2}, \text{ or } s = \frac{m-x}{m^2}, \tag{9}$$

as the equation of the line DE.

The probability of hitting within a small length dx, at x is then

$$p = s\,dx = \frac{m-x}{m^2}\,dx. \tag{10}$$

Probability of Hitting any Plane Figure.

Suppose positive errors in range are measured in the direction OX figure 32, from the mean point of impact O, OM being equal to m, the extreme error in range.

Following out the supposition of the two causes of error, suppose positive lateral errors are measured in the direction of the axis OY, ON being equal to n, the extreme lateral error.

If DC represent dx, $p_x = \frac{m-x}{m^2}\,dx$, is the probability of hitting somewhere between the lines AD and BC, distant $+x$ in range from the mean point of impact.

If KR represent dy, $p_y = \frac{n-y}{n^2}\,dy$, is the probability of hitting between the lines RT and KS, distant $+y$ laterally from the mean point of impact.

The small area $A' B' C' D'$ fulfills both these conditions and the probability of hitting it is therefore

$$p_{xy} = \frac{m - x}{m^2} \, dx \;\; \frac{n - y}{n^2} \, dy.$$

If we denote by $D'' D$ and $A'' D$, figure 32, by y_1 and y_2 respectively, the probability of hitting between two lines drawn through the points D'' and A'' parallel to $O X$ is

$$p_y' = \int_{y_1}^{y_2} \frac{n - y}{n^2} \, dy.$$

If however, we limit the target in the direction of the range errors to the length dx at $+ x$, the probability of hitting the target, which is represented by $A'' B'' C'' D''$ is

$$p'_{xy} = \frac{m - x}{m^2} \, dx \int_{y_1}^{y_2} \frac{n - y}{n_2} \, dy.$$

If $y_1 = f_1 (x)$ and $y_2 = f_2 (x)$ are the equations of two curves $H D'' G$ and $E A'' F$ we can write

$$P_y = \frac{m - x}{m^2} \, dx \int_{y_1}^{y_2} \frac{n - y}{n^2} \, dy$$

$$= \frac{m - x}{m^2} \, dx \int_{f_1 (x)}^{f_2 (x)} \frac{(n - y)}{n^2} \, dy = F (x) \, dx,$$

as the probability of hitting any elementary area

$A'' B'' C'' D''$.

Calling $O L$, a_1, and $O K$, a_2, the probability of hitting the figure $E F G H$ is

$$P_{xy} = p_o = \Sigma_{a_1}^{a_2} p_0 = \int_{a_1}^{a_2} F (x) \, dx,$$

$$\text{or} \quad p_0 = \frac{1}{m^2 \, n^2} \int_{a_1}^{a_2} (m - x) \, dx \int_{y_1}^{y_2} (n - y) \, dy. \qquad (11)$$

Remark.

Caution is necessary in using the right line method. The ordinates of the prolonged line $D E$, figure 29, to the right of E do not represent probabilities, nor do they to the left of D. As a consequence, in the figure, first a_1, a_2, y_1, y_2 must be positive; second, when either of these exceeds the extreme error in its direction, it must be placed equal to that extreme error. The probability of hitting a figure, part of which is in each of the quarters of the extreme rectangle surrounding the mean point of impact, is obtained by adding together the probability of hitting the parts, calculated separately. Distances measured along the axes OX and OY from the mean point of impact are regarded as positive in each quarter.

It will be noted that this method dispenses with the use of the tables.

Abbreviation of the Right Line Method.

In figure 30 the probability of an error less than x will be the quotient of the area of the trapezoid above x by the area of the triangle above m. Call these areas T and Δ and let ε be the abscissa of the center of gravity of the triangle, then $3\,\varepsilon = m$.

We have

$$T = \frac{s + s'}{2} x.$$

From the similarity of the triangles

$$s : m :: s' : m - x \quad \therefore \quad s' = \frac{s\,(m - x)}{m} = \frac{s\,(3\,\varepsilon - x)}{3\,\varepsilon} \text{ or}$$

$$T = s\left(x - \frac{x^2}{6\,\varepsilon}\right),$$

also

$$\Delta = \frac{s\,m}{2} = \frac{3\,s\,\varepsilon}{2}.$$

Therefore

$$p_0 = \frac{T}{\Delta} = \frac{2}{3}\ \frac{x}{\varepsilon} - \frac{1}{9}\left(\frac{x}{\varepsilon}\right)^2.$$

This gives a formula that can be very easily memorized. See below.

Application to Different Figures.

Rectangle.—In figure 31, denoting OF by a, and OE by b, the probability of hitting the rectangle $EAFO$ is

$$p_0 = \frac{1}{m^2\,n^2}\int_0^a (m-x)\,dx \int_0^b (n-y)\,dy$$

$$= \frac{1}{m^2\,n^2}\left(m\,a - \frac{a^2}{2}\right)\left(n\,b - \frac{b^2}{2}\right),$$

or

$$p_0 = \frac{a\,b}{4\,m\,n}\left(2 - \frac{a}{m}\right)\left(2 - \frac{b}{n}\right).$$

The probability of hitting the rectangle $DABC$ of which the center is at the mean point of impact, is four times the probability for $EAFO$; or

$$p_0 = \frac{a\,b}{m\,n}\left(2 - \frac{a}{m}\right)\left(2 - \frac{b}{n}\right)$$

$$= \left(\frac{2\,a}{m} - \frac{a^2}{m^2}\right)\left(\frac{2\,b}{n} - \frac{b^2}{n^2}\right) \qquad (12)$$

or, since $m = 3\,\varepsilon_1$ and $n = 3\,\varepsilon_2$,

$$p_0 = \left[\frac{2\,a}{3\,\varepsilon_1} - \frac{1}{9}\left(\frac{a}{\varepsilon_1}\right)^2\right] \times \left[\frac{2\,b}{3\,\varepsilon_2} - \frac{1}{9}\left(\frac{b}{\varepsilon_2}\right)^2\right].$$

Applying this method to example page 30, we have, since $m = 58 \times 3 = 174$ and $n = 8 \times 3 = 24$

$$p_0 = \left[\frac{100}{174} - \left(\frac{50}{174}\right)^2\right] \times \left[\frac{12}{24} - \left(\frac{6}{24}\right)^2\right]$$

$$= 0.4922 \times 0.4375 = 21.54 \text{ per cent.}$$

a close approximation to the longer method.

Other Figures.

Similar applications are given in the work of Ensign Glennon, for triangles, trapezoids and rhombs, so that such apparently difficult problems as the probability of hitting the gable end of a house may be computed. These methods are too elaborate for this course; but the following formula may be used for small arm firing, the targets in which are ellipses whose axes are determined by the relation between the mean vertical and lateral errors of the arm and ammunition.

Ellipse:—Calling a and b the semi-axes of the ellipses corresponding in direction to m and n, we find in Glennon

$$p_0 = \frac{2\,a\,b}{m\,n}\left(\frac{\pi}{2} - \frac{2\,a}{3m} - \frac{2\,b}{3\,n} + \frac{a\,b}{4\,m\,n}\right). \tag{13}$$

Circle.—If we make $b = a = r$ the ellipse becomes a circle, and

$$p_0 = \frac{2\,r^2}{m\,n}\left(\frac{\pi}{2} - \frac{2\,r}{3m} - \frac{2\,r}{3\,n} + \frac{r^2}{4\,m\,n}\right). \tag{14}$$

Remarks.

These methods have a practical application in determining the supply of ammunition necessary to produce a given result. Also, when time is limited, as in the attack of torpedo boats, and the rate of fire of the guns is known, the number of guns to be mounted on a certain front may be calculated.

In solving problems for ranges at which the mean errors are not given, these may be taken as proportional to the squares of the nearest ranges for which the errors are given.

Animate Objects.

The following table may be used in connection with problems in the probability of small arm fire. It is derived from the measurement of photographs of a number of Italian soldiers, of average size and build, taken naked.

TABLE III.

OBJECT.	Mean h't, ft.	Mean width, ft.	Vertical Area sq. ft.
Foot soldier, standing, front.	5.3	0.95	5.0
Foot soldier, standing, side.	5.3	0.57	3.0
Foot soldier, kneeling and firing, front.	3.4	1.00	3.4
Foot soldier, lying down and firing, front.	1.4	1.18	1.7
Foot soldier, lying down close.	0.7	1.64	1.3
Horseman and horse, front.	8.0	1.52	12.2
Horseman and horse, side.	8.0	2.4	19.4
Horse, front.	—	—	9.0
Horse, side.	—	—	17.0

IV. MANAGEMENT OF FIRE.

THE COLLECTIVE FIRE OF SMALL ARMS.

Advantages.

The need of reserving fire for the critical moments of an action, and the difficulty of controlling the fire of troops in dispersed order have led to the use of *collective fire* from groups of men under the direction of subordinate officers.

Careful experiments have shown that the superiority of very good individual marksmen over ordinary marksmen diminishes rapidly as the range increases; while skill in collective fire maintains it superiority at all ranges. This skill depends less upon natural aptitude than upon practice under the disturbing circumstances of collective fire, and thus demands "fire discipline". This is even more important for troops armed with magazine arms than for those armed with single breech loaders.

Collective fire is regulated by watching the dust thrown up by the hits, at least one half of which should fall short.

The ricochet at low elevations considerably increases the efficiency of the fire.

Definitions.

The intersection of the nucleus and envelope of the sheaf of trajectories by the horizontal plane through the *line of sight* forms the *beaten zone:* the *grazed zone* corresponds to the dangerous space for the lower element of the envelope and is taken to be equal to that of the mean trajectory: the sum of the beaten and grazed zones forms the *dangerous zone*, as seen in figure 33.

This figure represents the projection of the sheaf and the zones on the plane of sight. The line of sight is supposed to coincide with the surface of the ground; the firer lying down and aiming at the enemy's feet. He was formerly supposed to stand up and aim at the waist.

Constancy of the Beaten Zone.

It has been observed that as the range increases the increase in longitudinal diviation resulting from accidental differences in the angles of departure, is approximately compensated for by the diminished obliquity of the horizontal section of the sheaf resulting from the increased angle of fall. See lines $b' z' = b'' z''$, figure 34. Consequently the depth of the beaten zone is approximately constant at all ranges, although the width of the beaten zone, is, except at extreme ranges, approximately $\frac{1}{100}$ of the range. With arms and ammunition of the type of the service rifle, cal. 0.45, it is about 100 yards for the nucleus, and about 300 yards for both the nucleus and envelope, as shown in the figure. See extract from Range Table, page 52.

Remarks.

These data are derived from target firing; in battle the deviations may be eight times as great.

The depth of the beaten zone may be advantageously increased by varying the elevation of the pieces in a group.

Variations in Dangerous Zone. Vulnerability.

The depth of the dangerous zone varies with;—

1. The flatness of the trajectory.
2. The height of the target.
3. The range.
4. The inclination of the ground to the line of sight.

1. FLATNESS OF TRAJECTORY.

This principally affects the grazed zone. The maximum grazed zone for a man kneeling is in round numbers as follows, Springfield 350 yds., new Hebler 550 yds. See example 7; Chapter XX. The maximum grazed zone is a convenient measure of the power of the arm and ammunition. Within this zone adjustment of the rear sight is unnecessary.

2. HEIGHT OF THE TARGET.

The maximum grazed zone for a man standing is, Springfield 420 yds., Hebler 450 yds. In connection with the effect of change of position upon the probability of being struck it may be stated that at medium ranges the *vulnerability*, or chance of being hit with a given expenditure of ammunition, is as follows: Lying down : kneeling : standing :: 1 : 2 : 3. See ratio of areas in Table III.

Owing to the increase in the angle of fall at long ranges these differences disappear.

3. RANGE.

Beyond the maximum grazed zone, the depth of the dangerous zone diminishes as the range increases; and so does the vulnerability, which is approximately diminished one half for every increase in range of 250 yards.

4. INCLINATION OF THE GROUND TO THE LINE OF SIGHT.

The effect of variations in the slope of the ground is shown by figure 34 which represents, but not to scale, the nucleus of a 1000 yard trajectory.

The beaten zones *BZ* at different ranges are practically 100 yards in depth, if measured on the line of sight. See page 49.

The included figures show the variations in the beaten zone arising from varying the inclination of the ground respectively $\frac{1}{10}$; $\frac{1}{20}$; $\frac{1}{50}$ above and below the line of sight.*

The limit of efficiency is reached when the ground is parallel to the tangent at and beyond the object aimed at, *B*. In this case we shall have the largest grazed zone and a large beaten zone making together a deep dangerous zone. Beyond *B* there will be one or two grazed zones, depending on the height of the target *T*.

For a given slope the *safe zone*, $x\ x'$, will increase as the enemy approaches from *O* to *B* and as his trajectory consequently becomes flatter. Troops behind cover will therefore be better protected by *advancing* toward the cover as the enemy approaches.

This explains the vulnerability of deep columns on descending slopes and the insecurity of apparently good cover in rear of the point at which the enemy is directing his fire. This was noticed at the battle of Inkerman, in which great destruction was wrought among the horses of an English battery which were out of sight of the enemy, but in rear

* These slopes are found about West Point as follows:

$\frac{1}{10}$ Road to Post-office, near its intersection by the Dragoon path.

$\frac{1}{20}$ From S. W. corner of Academic Building, for a short distance to the west.

$\frac{1}{50}$ From crosswalk in front of sally port of Academic Building to the north.

of the guns and on the reverse slope of a hill, the slope of the hill being nearly tangent to the enemy's trajectory.

Effect of Increasing the Rapidity of Fire of Small Arms.

Owing to the diminished accuracy of rapid fire, its effect in a given time increases less rapidly than does its rate. It appears from experiment that doubling the rate increases the efficiency about $1\frac{1}{2}$ times; viz—

Kind of Fire.	Rate per Min.	Relative Efficiency
Common	6	1.0
Rapid	12	1.5
Magazine	24	2.25

The factor of efficiency may be determined from the following equation in which e is the factor; n is the number of shots fired at a given target and range, h is the number of hits, and t is the time in minutes required to fire the shots, n.

Then since the efficiency is directly proportional to h, and inversely proportional to n and to t, we have

$$e = \frac{h}{n} \times \frac{h}{t} = \frac{h^2}{n \times t}. \tag{15}$$

Range Table. Extract.

Hotchkiss 6 pdr. R. F. Gun. See page 49. $IV = 1818$, $w = 2$ lbs., $W = 6$ lbs.

Range.—Yds.	Mean errors in range.—Yds.
100	20
600	18
1100	17
1600	16
2100	16
2600	17
3100	18
3600	19
4000	19